oliveboard & HandaKaFunda

present

Shortcuts, Formulas & Tips

For MBA, Banking, Civil Services & Other Entrance Examinations

Vol. 1: Number System & Arithmetic

Glossary

Natural Numbers: 1, 2, 3, 4.....

Whole Numbers: 0, 1, 2, 3, 4.....

Integers: ....-2, -1, 0, 1, 2 .....

Rational Numbers: Any number which can be expressed as a ratio of two integers for example a p/q format where 'p' and 'q' are integers. Proper fraction will have (p<q) and improper fraction will have (p>q)

Factors: A positive integer 'f' is said to be a factor of a given positive integer 'n' if f divides n without leaving a remainder. e.g. 1, 2, 3, 4, 6 and 12 are the factors of 12.

Prime Numbers: A prime number is a positive number which has no factors besides itself and unity.

Composite Numbers: A composite number is a number which has other factors besides itself and unity.

Factorial: For a natural number 'n', its factorial is defined as: n! = 1 x 2 x 3 x 4 x .... x n (Note: 0! = 1)

Absolute value: Absolute value of x (written as |x|) is the distance of 'x' from 0 on the number line. |x| is always positive. $|x| = x$ *for* $x > 0$ OR $-x$ *for* $x < 0$

Tip: The product of 'n' consecutive natural numbers is always divisible by n!

Tip: Square of any natural number can be written in the form of 3n or 3n+1. Also, square of any natural number can be written in the form of 4n or 4n+1.

Tip: Square of a natural number can only end in 0, 1, 4, 5, 6 or 9. Second last digit of a square of a natural number is always even except when last digit is 6. If the last digit is 5, second last digit has to be 2.

Tip: Any prime number greater than 3 can be written as $6k \pm 1$.

Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{\left(\frac{1}{m}\right)} = \sqrt[m]{a}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\left(\frac{m}{n}\right)} = \sqrt[n]{a^m}$$

$$a^0 = 1$$

Tip: If $a^m = a^n$, then m = n

Tip: If $a^m = b^m$ and $m \neq 0$;
Then $a = b$ if m is Odd
Or $a = \pm b$ if m is Even

Last digit of a^n

n(Right) a(Down)	1	2	3	4	Cyclicity
0	0	0	0	0	1
1	1	1	1	1	1
2	2	4	8	6	4
3	3	9	7	1	4
4	4	6	4	6	2
5	5	5	5	5	1
6	6	6	6	6	1
7	7	9	3	1	4
8	8	4	2	6	4
9	9	1	9	1	2

Tip: The fifth power of any number has the same units place digit as the number itself.

HCF and LCM

For two numbers, HCF x LCM = product of the two.

HCF of Fractions = $\frac{HCF\ of\ Numerator}{LCM\ of\ Denominator}$

LCM of Fractions = $\frac{LCM\ of\ Numerator}{HCF\ of\ Denominator}$

Tip: If a, b and c give remainders p, q and r respectively, when divided by the same number H, then H is HCF of (a-p), (b-q), (c-r)

Tip: If the HCF of two numbers 'a' and 'b' is H, then, the numbers (a+b) and (a-b) are also divisible by H.

Tip: If a number N always leaves a remainder R when divided by the numbers a, b and c, then N = LCM (or a multiple of LCM) of a, b and c + R.

Relatively Prime or Co-Prime Numbers: Two positive integers are said to be relatively prime to each other if their highest common factor is 1.

Factor Theory

If N = $x^a y^b z^c$ where x, y, z are prime factors. Then,

Number of factors of N = **P** = (a + 1)(b + 1)(c + 1)

Sum of factors of N = $\frac{x^{a+1}-1}{x-1} X \frac{y^{b+1}-1}{y-1} X \frac{z^{c+1}-1}{z-1}$

Number of ways N can be written as product of two factors = P/2 or (P+1)/2 if P is even or odd respectively

The number of ways in which a composite number can be resolved into two co-prime factors is 2^{m-1}, where m is the number of different prime factors of the number.

Number of numbers which are less than N and co-prime to $\emptyset(N) = N\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)\left(1-\frac{1}{z}\right)$ {Euler's Totient}

Tip: If N = $(2)^a(y)^b(z)^c$ where x, y, z are prime factors
Number of even factors of N = (a)(b+1)(c+1)
Number of odd factors of N = (b+1)(c+1)

Divisibility Rules

A number is divisible by:

2, 4 & 8 when the number formed by the last, last two, last three digits are divisible by 2,4 & 8 respectively.

3 & 9 when the sum of the digits of the number is divisible by 3 & 9 respectively.

11 when the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.

6, 12 & 15 when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.

7, if the number of tens added to five times the number of units is divisible by 7.

13, if the number of tens added to four times the number of units is divisible by 13.

19, if the number of tens added to twice the number of units is divisible by 19.

Algebraic Formulae

$\mathbf{a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)}$. Hence, *$a^3 \pm b^3$ is divisible by $(a \pm b)$ and $(a^2 \pm ab + b^2)$.*

$\mathbf{a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})}$[for all n]. Hence, *$a^n - b^n$ is divisible by $a - b$ for all n.*

$\mathbf{a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b2 \dots - b^{n-1})}$[n-even] Hence, *$a^n - b^n$ is divisible by $a + b$ for even n.*

$\mathbf{a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})}$[n-odd] Hence, *$a^n + b^n$ is divisible by $a + b$ for odd n.*

$\mathbf{a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)}$ Hence, *$a^3 + b^3 + c^3 = 3abc$ if $a + b + c = 0$*

For ex., check divisibility of 312 by 7, 13 & 19

- **For 7:** 31 + 2 x 5 = 31 + 10 = 41 ***Not divisible***
- **For 13:** 31 + 2 x 4 = 31 + 8 = 39 ***Divisible***.
- **For 19:** 31 + 2 x 2 = 31 + 4 = 35 ***Not divisible***.

Remainder / Modular Arithmetic

$$\boldsymbol{Rem\left[\frac{a*b*c\ldots}{d}\right] = Rem\left[\frac{a}{d}\right]*Rem\left[\frac{b}{d}\right]*Rem\left[\frac{c}{d}\right]\ldots}$$

$$\boldsymbol{Rem\left[\frac{a+b+c\ldots}{d}\right] = Rem\left[\frac{a}{d}\right]+Rem\left[\frac{b}{d}\right]+Rem\left[\frac{c}{d}\right]\ldots}$$

Case 1 – When the dividend (M) and divisor (N) have a factor in common (k)

⇨ $\boldsymbol{Rem\left[\frac{M}{N}\right] = Rem\left[\frac{ka}{kb}\right] = k\,Rem\left[\frac{a}{b}\right]}$

Example: $Rem\left[\frac{3^{15}}{15}\right] = 3\ Rem\left[\frac{3^{14}}{5}\right] = 3*4 = 12$

Case 2 – When the divisor can be broken down into smaller co-prime factors.

⇨ $Rem\left[\frac{M}{N}\right] = Rem\left[\frac{M}{a*b}\right]$ *{HCF (a,b) = 1}*

⇨ Let $Rem\left[\frac{M}{a}\right] = r_1$ & $Rem\left[\frac{M}{b}\right] = r_2$

⇨ $\boldsymbol{Rem\left[\frac{M}{N}\right] = axr_2 + byr_1}$ *{Such that ax+by = 1}*

Example: $Rem\left[\frac{7^{15}}{15}\right] = Rem\left[\frac{7^{15}}{3*5}\right]$

⇨ $Rem\left[\frac{7^{15}}{3}\right] = 1$ & $Rem\left[\frac{7^{15}}{5}\right] = Rem\left[\frac{2^{15}}{5}\right] = 3$

⇨ $Rem\left[\frac{7^{15}}{15}\right] = 3*x*3+5*y*1$

{Such that 3x+5y=1}

⇨ Valid values are x = -3 and y = 2

⇨ $Rem\left[\frac{7^{15}}{15}\right] = 9x+5y = -17 \equiv 13$

Case 3 – Remainder when $f(x) = ax^n + bx^{n-1} + cx^{n-2}$ … is divided by $(x-a)$ the remainder is $f(a)$

Tip: If f(a) = 0, (x-a) is a factor of f(x)

Continued >>

Remainder Related Theorems

Euler's Theorem:

Number of numbers which are less than N = $a^p * b^q * c^r$ and co-prime to it are

⇨ $\emptyset(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$

If M and N are co-prime ie HCF(M,N) = 1

⇨ $\boldsymbol{Rem\left[\frac{M^{\emptyset(n)}}{N}\right] = 1}$

Example: $Rem\left[\frac{7^{50}}{90}\right] =?$

⇨ $\emptyset(90) = 90\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$

⇨ $\emptyset(90) = 90 * \frac{1}{2} * \frac{2}{3} * \frac{4}{5} = 24$

⇨ $Rem\left[\frac{7^{24}}{90}\right] = 1 = Rem\left[\frac{7^{48}}{90}\right]$

⇨ $Rem\left[\frac{7^{50}}{90}\right] = Rem\left[\frac{7^{2}}{90}\right] * Rem\left[\frac{7^{48}}{90}\right] = 49 * 1 = 49$

Fermat's Theorem:

If N is a prime number and *M and N are co-primes*

⇨ $\boldsymbol{Rem\left[\frac{M^{N}}{N}\right] = M}$

⇨ $\boldsymbol{Rem\left[\frac{M^{N-1}}{N}\right] = 1}$

Example: $Rem\left[\frac{6^{31}}{31}\right] = 6$ & $Rem\left[\frac{6^{30}}{31}\right] = 1$

Wilson's Theorem

If N is a prime number

⇨ $\boldsymbol{Rem\left[\frac{(N-1)!}{N}\right] = N-1}$

⇨ $\boldsymbol{Rem\left[\frac{(N-2)!}{N}\right] = 1}$

Example: $Rem\left[\frac{30!}{31}\right] = 30$ & $Rem\left[\frac{29!}{31}\right] = 1$

Tip: Any single digit number written (P-1) times is divisible by P, where P is a prime number >5.

Examples: 222222 is divisible by 7
444444….. 18 times is divisible by 19

Base System Concepts

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Converting from base ‘n’ to decimal

⇨ $(pqrst)_n = pn^4 + qn^3 + rn^2 + sn + t$

Converting from decimal to base ‘n’

The example given below is converting from 156 to binary. For this we need to keep dividing by 2 till we get the quotient as 0.

```
2)156   0
 2)78   0
 2)39   1
 2)19   1
  2)9   1
  2)4   0
  2)2   0
  2)1   1
    0
```

Starting with the bottom remainder, we read the sequence of remainders upwards to the top. By that, we get $156_{10} = 10011100_2$

Tip: $(pqrst)_n \times n^2 = (pqrst00)_n$

$(pqrst)_n \times n^3 = (pqrst000)_n$

Averages

Simple Average = $\frac{\text{Sum of elements}}{\text{Number of elements}}$

Weighted Average = $\frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n}$

Arithmetic Mean = $(a_1 + a_2 + a_3 \ldots.a_n) / n$

Geometric Mean = $\sqrt[n]{a_1a_2\cdots a_n}$

Harmonic Mean = $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$

For two numbers a and b

- ⇨ AM = (a + b)/2
- ⇨ GM = $\sqrt{a.b}$
- ⇨ HM = $\frac{2ab}{a+b}$

Tip: AM ≥ GM ≥ HM is always true. They will be equal if all elements are equal to each other. If I have just two values then GM^2 = AM x HM

Tip: The sum of deviation (D) of each element with respect to the average is 0

⇨ $D = (x_1 - x_{avg}) + (x_2 - x_{avg}) + (x_3 - x_{avg}) \ldots + (x_1 - x_{avg}) = 0$

Tip: $x_{avg} = x_{assumed\ avg} + \frac{Deviation}{No.of\ elements}$

Median of a finite list of numbers can be found by arranging all the observations from lowest value to highest value and picking the middle one.

Mode is the value that occurs most often

Percentages

Fractions and their percentage equivalents:

Fraction	%age	Fraction	%age
1/2	50%	1/9	11.11%
1/3	33.33%	1/10	10%
1/4	25%	1/11	9.09%
1/5	20%	1/12	8.33%
1/6	16.66%	1/13	7.69%
1/7	14.28%	1/14	7.14%
1/8	12.5%	1/15	6.66%

Tip: r% change can be nullified by $\frac{100r}{100+r}$% change in another direction. Eg: An increase of 25% in prices can be nullified by a reduction of [100x25/(100+25)] = 20% reduction in consumption.

Tip: If a number 'x' is successively changed by a%, b%, c%...

⇨ Final value = $x\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)\ldots$

Tip: The net change after two successive changes of a% and b% is $\left(a+b+\frac{ab}{100}\right)\%$

Interest

Amount = Principal + Interest

Simple Interest = PNR/100

Compound Interest = $P(1+\frac{r}{100})^n - P$

Population formula $P' = P(1 \pm \frac{r}{100})^n$

Depreciation formula = Initial Value x $(1 - \frac{r}{100})^n$

Tip: SI and CI are same for a certain sum of money (P) at a certain rate (r) per annum for the first year. The difference after a period of two years is given by

$$\Rightarrow \Delta = \frac{PR^2}{100^2}$$

Growth and Growth Rates

Absolute Growth = Final Value – Initial Value

Growth rate for one year period =

$$\frac{\text{Final value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

$$\text{SAGR or AAGR} = \frac{\text{Final value} - \text{Initial Value}}{\text{No. of years}} \times 100$$

$$\text{CAGR} = \left(\frac{\text{Final value} - \text{Initial Value}}{\text{Initial Value}}\right)^{\frac{1}{no.of\ years}} - 1$$

Tip: If the time period is more than a year, CAGR < AAGR. This can be used for approximating the value of CAGR instead of calculating it.

Profit and Loss

%Profit / Loss = $\frac{\text{Selling Price – Cost Price}}{\text{Initial Value}} \, x\ 100$

In case false weights are used while selling,

% Profit = $\left(\frac{\text{Claimed Weigth} - \text{Actual Weight}}{\text{Actual Weight}} - 1\right) x\ 100$

Discount % = $\frac{\text{Marked Price –Selling Price}}{\text{Marked Price}}$ x 100

Tip: Effective Discount after successive discount of a% and b% is (a + b – $\frac{ab}{100}$). Effective Discount when you buy x goods and get y goods free is $\frac{\text{y}}{\text{x+y}}$ x 100.

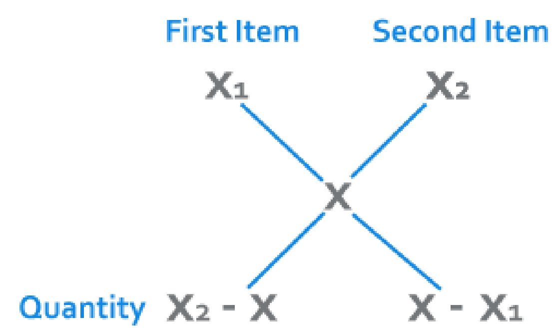
Mixtures and Alligation

Successive Replacement – Where *a* is the original quantity, *b* is the quantity that is replaced and *n* is the number of times the replacement process is carried out, then

$$\frac{\text{Quantity of original entity after n operation}}{\text{Quantity of mixture}} = \left(\frac{\text{a} - \text{b}}{\text{a}}\right)^{\text{n}}$$

Alligation – The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture

$$\Rightarrow \frac{\text{Quantity of first item}}{\text{Quantity of second item}} = \frac{x_2 - x}{x - x_1}$$



Ratio and Proportion

Compounded Ratio of two ratios a/b and c/d is ac/bd,
Duplicate ratio of a : b is $a^2 : b^2$
Triplicate ratio of a : b is $a^3 : b^3$
Sub-duplicate ratio of a : b is √a : √b
Sub-triplicate ratio of a : b is ³√a : ³√b
Reciprocal ratio of a : b is b : a

Componendo and Dividendo

If $\frac{a}{b} = \frac{c}{d}$ & $a \neq b$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$
Four (non-zero) quantities of the same kind a,b,c,d are said to be in proportion if a/b = c/d.

The non-zero quantities of the same kind a, b, c, d.. are said to be in continued proportion if a/b = b/c = c/d.

Proportion

a, b, c, d are said to be in proportion if $\frac{a}{b} = \frac{c}{d}$
a, b, c, d are said to be in continued proportion if
$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Tip: If a/b = c/d = e/f = k

⇨ $\frac{a+c+e}{b+d+f} = k$

⇨ $\frac{pa+qc+re}{pb+qd+rf} = k$

⇨ $\frac{pa^n+qc^n+re^n}{pb^n+qd^n+rf^n} = k^n$

Given two variables *x* and *y*, *y is* ***(directly) proportional*** *to x* (*x and y* ***vary directly***, or *x and y are in* ***direct variation***) if there is a non-zero constant *k* such that y = kx. It is denoted by $y \propto x$

Two variables are **inversely proportional** (or **varying inversely**, or in **inverse variation**, or in **inverse proportion** or **reciprocal proportion**) if there exists a non-zero constant *k* such that y = k/x.

Time Speed and Distance

Speed = Distance / Time

1 kmph = 5/18 m/sec; 1 m/sec = 18/5 kmph

$$\text{Speed}_{\text{Avg}} = \frac{Total\ Distance\ Covered}{Total\ Time\ Taken} = \frac{d_1 + d_2 + d_3 \ldots d_n}{t_1 + t_2 + t_3 \ldots t_n}$$

If the distance covered is constant then the average speed is Harmonic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

- ⇨ $\text{Speed}_{\text{Avg}} = \dfrac{n}{1/s_1 + 1/s_2 + 1/s_3 \ldots 1/s_n}$
- ⇨ $\text{Speed}_{\text{Avg}} = \dfrac{2s_1s_2}{s_1 + s_2}$ (for two speeds)

If the time taken is constant then the average speed is Arithmetic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

- ⇨ $\text{Speed}_{\text{Avg}} = \dfrac{s_1 + s_2 + s_3 \ldots s_n}{n}$
- ⇨ $\text{Speed}_{\text{Avg}} = \dfrac{s_1 + s_2}{2}$ (for two speeds)

Tip: Given that the distance between two points is constant, then

- ⇨ If the speeds are in *Arithmetic Progression*, then the times taken are in *Harmonic Progression*
- ⇨ If the speeds are in *Harmonic Progression*, then the times taken are in *Arithmetic Progression*

For Trains, time taken = $\dfrac{Total\ length\ to\ be\ covered}{Relative\ Speed}$

For Boats,

$\text{Speed}_{\text{Upstream}} = \text{Speed}_{\text{Boat}} - \text{Speed}_{\text{River}}$

$\text{Speed}_{\text{Downstream}} = \text{Speed}_{\text{Boat}} + \text{Speed}_{\text{River}}$

$\text{Speed}_{\text{Boat}} = (\text{Speed}_{\text{Downstream}} + \text{Speed}_{\text{Upstream}}) / 2$

$\text{Speed}_{\text{River}} = (\text{Speed}_{\text{Downstream}} - \text{Speed}_{\text{Upstream}}) / 2$

For Escalators,The difference between escalator problems and boat problems is that escalator can go either up or down.

Races & Clocks

Linear Races

Winner's distance = Length of race

Loser's distance = Winner's distance – (beat distance + start distance)

Winner's time = Loser's time – (beat time + start time)

Deadlock / dead heat occurs when beat time = 0 or beat distance = 0

Circular Races

Two people are running on a circular track of length L with speeds a and b in the same direction

⇨ Time for 1st meeting = $\frac{L}{a-b}$

⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}\right)$

Two people are running on a circular track of length L with speeds a and b in the opposite direction

⇨ Time for 1st meeting = $\frac{L}{a+b}$

⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}\right)$

Three people are running on a circular track of length L with speeds a, b and c in the same direction

⇨ Time for 1st meeting = LCM $\left(\frac{L}{a-b}, \frac{L}{a-c}\right)$

⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right)$

Clocks To solve questions on clocks, consider a circular track of length 360°. The minute hand moves at a speed of 6° per min and the hour hand moves at a speed of ½° per minute.

Tip: Hands of a clock coincide (or make 180°) 11 times in every 12 hours. Any other angle is made 22 times in every 12 hours.

Time and Work

If a person can do a certain task in t hours, then in 1 hour he would do 1/t portion of the task.

A does a particular job in 'a' hours and B does the same job in 'b' hours, together they will take $\frac{ab}{a+b}$ hours

A does a particular job in 'a' hours more than A and B combined whereas B does the same job in 'b' hours more than A and B combined, then together they will take $\sqrt{ab}$ hours to finish the job.

Tip: A does a particular job in 'a' hours, B does the same job in 'b' hours and C does the same job in 'c' hours, then together they will take $\frac{abc}{ab+bc+ca}$ hours.

Tip: If A does a particular job in 'a' hours and A&B together do the job in 't' hours, the B alone will take $\frac{at}{a-t}$ hours.

Tip: If A does a particular job in 'a' hours, B does the same job in 'b' hours and ABC together do the job in 't' hours, then

- ⇨ C alone can do it in $\frac{abt}{ab-at-bt}$ hours
- ⇨ A and C together can do it in $\frac{bt}{b-t}$ hours
- ⇨ B and C together can do it in $\frac{at}{a-t}$ hours

Tip: If the objective is to fill the tank, then the *Inlet pipes* do **positive work** whereas the *Outlet pipes* do **negative work**. If the objective is to empty the tank, then the *Outlet pipes* do **positive work** whereas the *Inlet Pipes* do **negative work**.

Visit

www.oliveboard.in

Personalized Online Preparation for MBA, Banking & Engineering Exams

www.handakafunda.com

Live online classes and videos for MBA and Banking Exams