oliveboard & HandaKaFunda

present

Shortcuts, Formulas & Tips

For MBA, Banking, Civil Services & Other Entrance Examinations

Vol. 2: Algebra & Modern Math

Quadratic and Other Equations

For a quadratic equation, $ax^2 + bx + c = 0$, its roots

- ⇨ $\alpha \text{ or } \beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- ⇨ Sum of roots = $\alpha + \beta = \frac{-b}{a}$
- ⇨ Product of roots = $\alpha\beta = \frac{c}{a}$

Discriminant $\Delta = b^2 - 4ac$

Condition	Nature of Roots
$\Delta < 0$	Complex Conjugate
$\Delta = 0$	Real and equal
Δ > 0 and a perfect square	Rational and unequal
Δ > 0 and not a perfect square	Irrational and unequal

Tip: If c = a, then roots are reciprocal of each other

Tip: If b =0, then roots are equal in magnitude but opposite in sign.

Tip: Provided a, b and c are rational

- ⇨ If one root is p + iq, other root will be p – iq
- ⇨ If one root is p + $\sqrt{q}$, other root will be p – $\sqrt{q}$

Cubic equation $ax^3+bx^2+cx+d = 0$

- ⇨ Sum of the roots = - b/a
- ⇨ Sum of the product of the roots taken two at a time = c/a
- ⇨ Product of the roots = -d/a

Biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$

- ⇨ Sum of the roots = - b/a
- ⇨ Sum of the product of the roots taken two at a time = c/a
- ⇨ Sum of the product of the roots taken three at a time = -d/a
- ⇨ Product of the roots = e/a

Inequalities

If a > b and c > 0,

- ⇨ a + c > b + c
- ⇨ a - c > b - c
- ⇨ ac > bc
- ⇨ a/c > b/c

If a, b ≥ 0, then $a^n > b^n$ and $1/a^n < 1/b^n$, where n is positive.

a < b and x > 0, then $\frac{a+x}{b+x} > \frac{a}{b}$

a > b and x > 0, then $\frac{a+x}{b+x} < \frac{a}{b}$

Modular Inequalities

|x- y| = |y - x|
|x. y| = |x| . |y|
|x+ y| < |x| + |y|
|x+ y| > |x| - |y|

Quadratic Inequalities

(x – a) (x – b) > 0 {a < b}

⇨ (x < a) U (x > b)

(x – a) (x – b) < 0 {a > b}

⇨ a < x < b

For any set of positive numbers: AM≥GM≥HM

⇨ $(a_1+a_2+ \ldots.+a_n)/n \geq (a_1.a_2. \ldots..a_n)^{1/n}$

If a and b are positive quantities, then

⇨ $\frac{a+b}{2} \geq \sqrt{ab}$

If a,b,c,d are positive quantities, then

⇨ $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a} \geq 4$

⇨ $a^4 + b^4 + c^4 + d^4 \geq 4abcd$

If a,b,c …. k are n positive quantities and m is a natural number, then Continued >>

⇨ $\frac{a^m+b^m+c^m....+k^m}{n} > \left(\frac{a+b+c...+k}{n}\right)^m$

$\left(\frac{a+b+c+\cdots+k}{n}\right)^n > a.b.c.d \ldots .k$

Tip: $\frac{a^m+b^m}{2} > \left(\frac{a+b}{2}\right)^m \quad [m \leq 0 \text{ or } m \geq 1]$

$\frac{a^m+b^m}{2} < \left(\frac{a+b}{2}\right)^m \quad [0 < m < 1]$

Tip: For any positive integer n, $2 \leq \left(1+\frac{1}{n}\right)^n \leq 3$

Tip: $a^m b^n c^p$........will be greatest when $\frac{a}{m} = \frac{b}{n} = \frac{c}{p}$

Tip: If a > b and both are natural numbers, then

⇨ $a^b < b^a$ {Except $3^2 > 2^3$ & $4^2 = 2^4$}

Tip: $(n!)^2 \geq n^n$

Tip: If the sum of two or more positive quantities is constant, their product is greatest when they are equal and if their product is constant then their sum is the least when the numbers are equal.

⇨ If x + y = k, then xy is greatest when x = y

⇨ If xy = k, then x + y is least when x = y

Logarithm

$$Log(ab) = Log(a) + Log(b)$$

$$Log\left(\frac{a}{b}\right) = Log(a) - Log(b)$$

$$Log(a^n) = n\ Log(a)$$

$$Log_b(a) = \frac{log_c(a)}{log_c(b)}$$

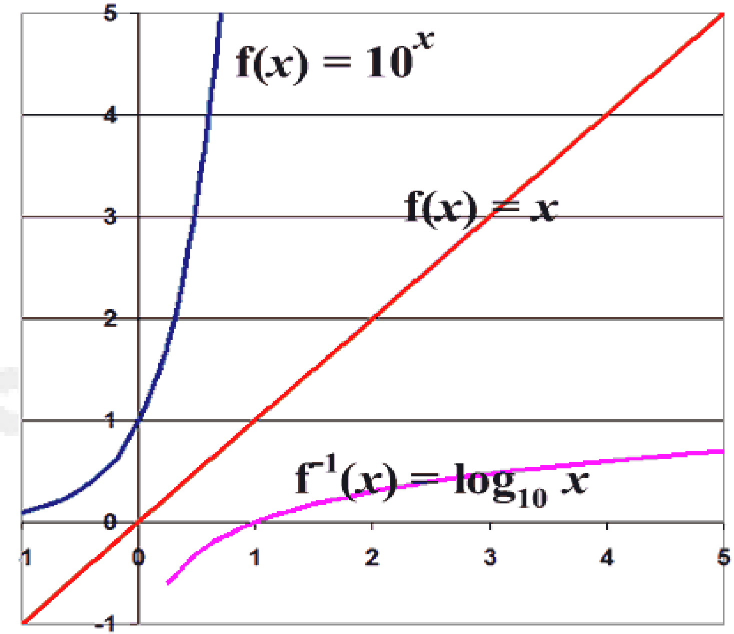
$$Log_b\ b = 1$$

$$Log_b\ 1 = 0$$

$$Log_b\ b^x = x$$

Ln x means $\log_e x$

$$x = b^{\log_b x}$$



Domain: Set of real and finite values that the independent variable can take.

Range: Set of real and finite values that the dependent variable can have corresponding to the values of the independent variable

Co-Domain: Set of real and finite values that the dependent variable can have.

One to One: Every element in the Domain has one and only one image in the Co-Domain. Every element in Co-Domain has one and only one pre-image in the Domain.

Tip: Range is a subset of Co-Domain. Co-domain may or may not have values which do not have a pre-image in the domain.

Tip: It is not a function if for some value in the domain, the relationship gives more than one value. Eg: f(x) = $\sqrt{x}$ (At x = 4, f(x) could be both +2 and -2)

Tip: Domain cannot have any extra value ie the values at which the function does not exist.

Many to One: If at least two elements in Domain have the same image in the co-domain.

Onto Function: If for every element in the Co-Domain there is at least one pre-image in the Domain. In this case, Range = Co-Domain

Into Function: If there is at least one element in the Co-Domain which does not have a pre-image in the Domain. In this case, Range is a proper subset of Co-Domain.

Even Function: f(x) is even if and only if f(-x) = f(x) for all values of x. The graph of such a function is symmetric about the Y-Axis

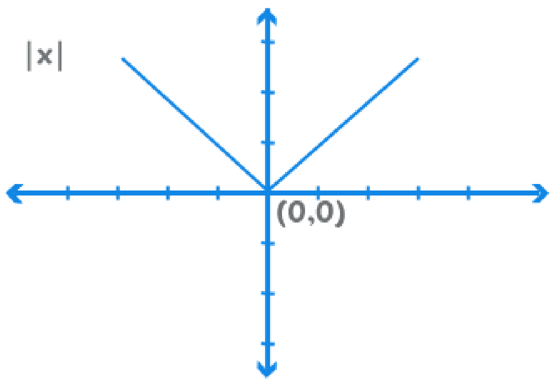
Odd Function: f(x) is odd if and only if f(-x) = - f(x) for all values of x. The graph is symmetric about the origin

Tip: If f(x) is an odd function and f(0) exists
⇨ f(0) = 0

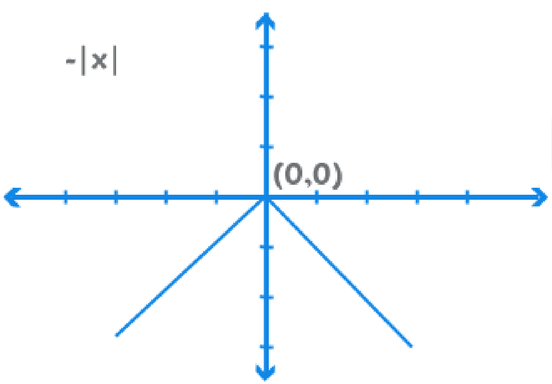
Graphs

Continued >>

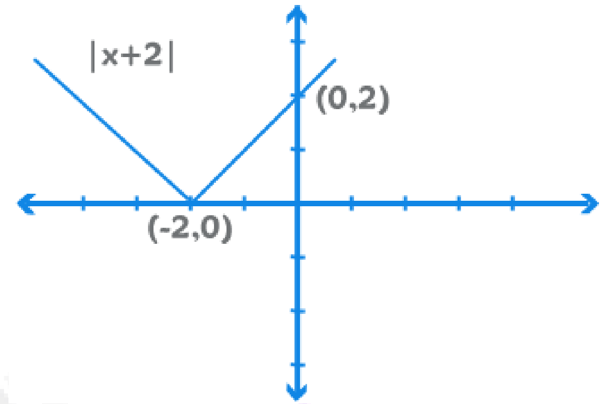
f(x) = |x|



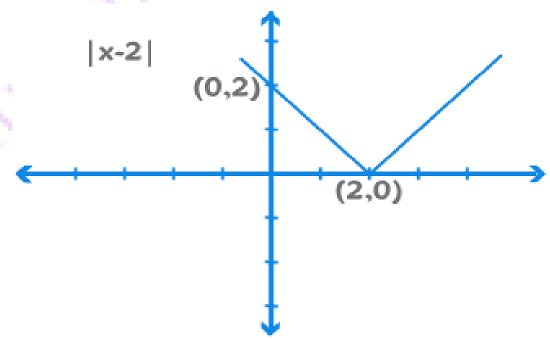
If we consider –f(x), it gets mirrored in the X-Axis.



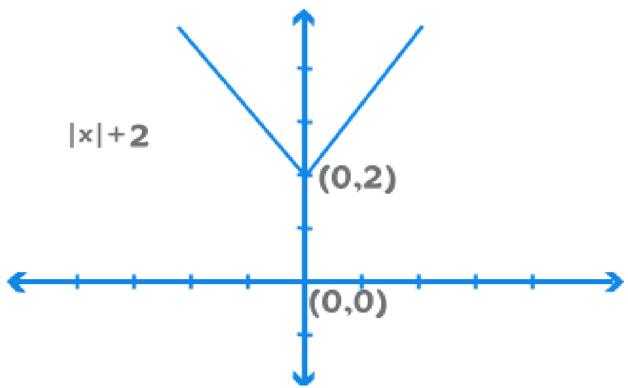
If we consider f(x+2), it shifts left by 2 units



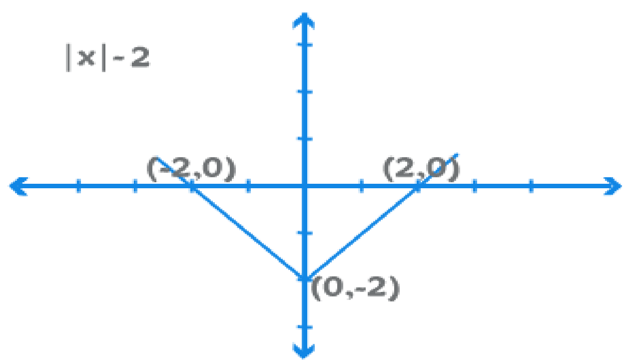
If we consider f(x-2), it shifts right by 2 units.



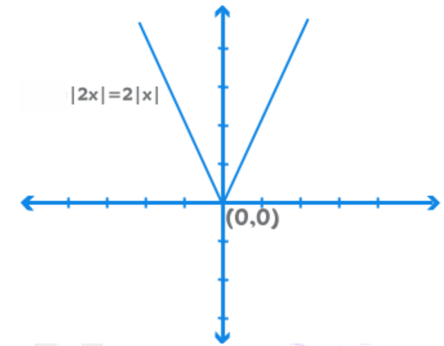
If we consider f(x) + 2, it shifts up by 2 units.



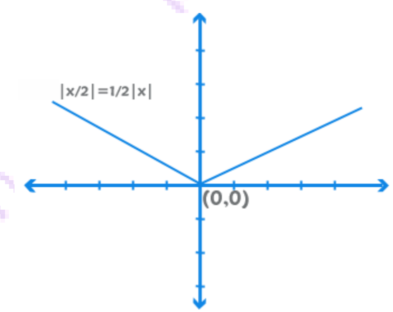
If we consider f(x) – 2, it shifts down by 2 units.



If we consider f(2x) or 2f(x) ,the slope doubles and the rise and fall become much sharper than earlier



If we consider f(x/2) or ½ f(x), the slope halves and the rise and fall become much flatter than earlier.



Set Fundamentals

The number of elements in a set is called its **cardinal number** and is written as n(A). A set with cardinal number 0 is called a null set while that with cardinal number ∞ is called an infinite set.

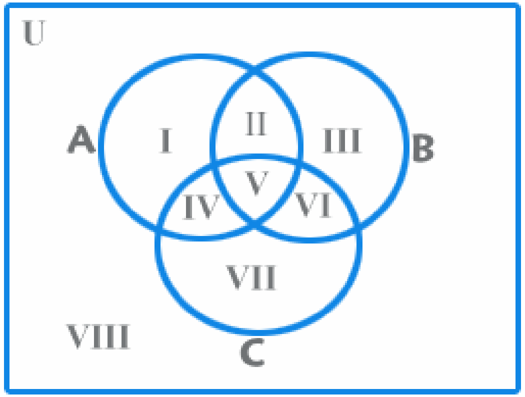
Set A is said to be a **subset** of Set B if each and every element of Set A is also contained in Set B. Set A is said to be a **proper subset** of Set B if Set B has at least one element that is not contained in Set A. A set with 'n' elements will have 2^n subsets (2^n – 1 proper subsets)

The **Universal set** is defined as the set of all possible objects under consideration.

Tip: Any set is a subset of itself, but not a proper subset. The empty set, denoted by ∅, is also a subset of any given set *X*. The empty set is always a proper subset, except of itself. Every other set is then a subset of the universal set.

Union of two sets is represented as A ∪ B and consists of elements that are present in either Set A or Set B or both. **Intersection of two sets** is represented as A ∩ B and consists of elements that are present in both Set A and Set B. **n(A∪B) = n(A) + n(B) — n(A∩B)**

Venn Diagram: A venn diagram is used to visually represent the relationship between various sets. What do each of the areas in the figure represent?



I – only A; II – A and B but not C; III – Only B; IV – A and C but not B; V – A and B and C; VI – B and C but not A; VII – Only C

n(A∪B∪C) = n(A) + n(B) + n(C) — n(A∩B) — n(A∩C) - n(B∩C) + n(A∩B∩C)

Binomial Theorem

For some basic values:

$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$
$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$$

Theorem

$$\boldsymbol{(a+b)^n = {}^n_0C\, a^n b^0 + {}^n_1C\, a^{n-1} b^1 + {}^n_2C\, a^{n-2} b^2 \ldots + {}^n_nC\, a^0 b^n}$$

$$(x+1)^n = x^n + nx^{n-1} + {}^n_2C\, x^{n-2} \ldots + nx + 1$$

$$ {}^n_0C + {}^n_1C + {}^n_2C \ldots + {}^n_nC = 2^n$$

$$ {}^n_0C + {}^n_2C + {}^n_4C \ldots = {}^n_1C + {}^n_3C + {}^n_5C \ldots = \frac{2^n}{2} = 2^{n-1}$$

Some basic properties

Tip: There is one more term than the power of the exponent, n. That is, there are terms in the expansion of $(a + b)^n$.

Tip: In each term, the sum of the exponents is n, the power to which the binomial is raised.

Tip: The exponents of a start with n, the power of the binomial, and decrease to 0. The last term has no factor of a. The first term has no factor of b, so powers of b start with 0 and increase to n.

Tip: The coefficients start at 1 and increase through certain values about "half"-way and then decrease through these same values back to 1.

Tip: To find the remainder when $(x + y)^n$ is divided by x, find the remainder when y^n is divided by x.

Tip: $(1+x)^n \cong 1 + nx$, when $x \ll 1$

Permutation & Combination

When two tasks are performed in succession, i.e., they are connected by an **'AND'**, to find the total number of ways of performing the two tasks, you have to **MULTIPLY** the individual number of ways. When only one of the two tasks is performed, i.e. the tasks are connected by an '**OR**', to find the total number of ways of performing the two tasks you have to **ADD** the individual number of ways.

Eg: In a shop there are'd' doors and 'w' windows.
*Case1: If a thief wants to enter via a door or window, he can do it in – **(d+w) ways**.*
*Case2: If a thief enters via a door and leaves via a window, he can do it in – **(d x w) ways**.*

Linear arrangement of 'r' out of 'n' distinct items (^nP_r):

The first item in the line can be selected in 'n' ways AND the second in (n — 1) ways AND the third in (n — 2) ways AND so on. So, the total number of ways of arranging 'r' items out of 'n' is

$$(n)(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

Circular arrangement of 'n' distinct items: Fix the first item and then arrange all the other items linearly with respect to the first item. This can be done in **(n — 1)!** ways.

Tip: In a necklace, it can be done in $\frac{(n-1)!}{2}$ ways.

Selection of r items out of 'n' distinct items (^nCr): Arrange of r items out of n = Select r items out of n and then arrange those r items on r linear positions.

$$^nP_r = {^nC_r} \times r! \rightarrow {^nC_r} = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Continued >>

Dearrangement If 'n' things are arranged in a row, the number of ways in which they can, be deranged so that none of them occupies its original place is

$$n!\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\ \dots+\frac{(-1)^n}{n!}\right)$$

Tip: Number of ways of arranging 'n' items out of which `p' are alike, 'q' are alike, 'r' are alike in a line is given by = $\frac{n\,!}{p!q!r!}$

Partitioning

'n' **similar** items in 'r' **distinct** groups	No restrictions	${}^{n+r-1}C_{r-1}$
	No group empty	${}^{n-1}C_{r-1}$
'n' **distinct** items in 'r' **distinct** groups	No restrictions	r^n
	Arrangement in a group important	$\frac{(n+r-1)!}{(r-1)!}$
'n' **similar** items in 'r' **similar** groups	List the cases and then find out in how many ways is each case possible	
'n' **similar** items in 'r' **similar** groups	List the cases and then find out in how many ways is each case possible	

Probability

$$P(A) = \frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ outcomes}$$

For **Complimentary Events:** *P(A) + P(A') = 1*

For **Exhaustive Events:** *P(A) + P(B) +P(C)… = 1*

Addition Rule:

P (A U B) = P(A) + P(B) – P(A ∩ B)

For **Mutually Exclusive Events** P(A ∩ B) = 0

⇨ *P (A U B) = P(A) + P(B)*

Multiplication Rule:

P(A ∩ B) = P(A) P(B/A) = P(B) P(A/B)

For **Independent Events** P(A/B) = P(B) and P(B/A) = P(B)

⇨ *P(A ∩ B) = P(A).P(B)*
⇨ *P (A U B) = P(A) + P(B) – P(A).P(B)*

Tip: If the probability of an event occurring is P, then the probability of that event occurring 'r' times in 'n' trials is = $^nC_r \times P^r \times (1-P)^{n-r}$

Odds

$$\text{Odds in favor} = \frac{Number\ of\ favorable\ outcomes}{Number\ of\ unfavorable\ outcomes}$$

$$\text{Odds against} = \frac{Number\ of\ unfavorable\ outcomes}{Number\ of\ favorable\ outcomes}$$

Sequence, Series & Progression

Arithmetic Progression

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$$

Tip: Number of terms $= \frac{a_n - a_1}{d} + 1$

Geometric Progression

$$a_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Sum till infinite terms = $\frac{a}{1-r}$ (Valid only when r<1)

Sum of first n natural numbers

⇨ $1 + 2 + 3 \ldots + n = \frac{n(n+1)}{2}$

Sum of squares of first n natural numbers

⇨ $1^2 + 2^2 + 3^2 \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Sum of cubes of first n natural numbers

⇨ $1^3 + 2^3 + 3^3 \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Tip: Sum of first n odd numbers

⇨ $1 + 3 + 5 \ldots + (2n-1) = n^2$

Tip: Sum of first n even numbers

⇨ $2 + 4 + 6 \ldots 2n = n(n+1)$

Tip: If you have to consider 3 terms in an AP, consider {a-d,a,a+d}. If you have to consider 4 terms, consider {a-3d,a-d,a+d,a+3d}

Tip: If all terms of an AP are multiplied with k or divided with k, the resultant series will also be an AP with the common difference dk or d/k respectively.

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