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Shortcuts, Formulas & Tips

For MBA, Banking, Civil Services & Other Entrance Examinations

Vol. 3: Geometry

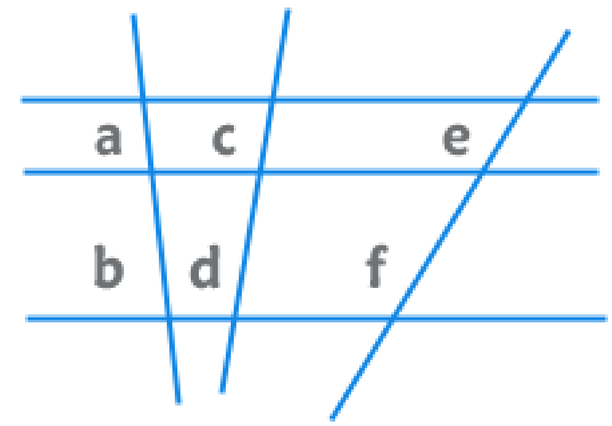
Lines and Angles

Sum of the angles in a straight line is 180°

Vertically opposite angles are congruent (equal).

If any point is equidistant from the endpoints of a segment, then it must lie on the **perpendicular bisector**

When two **parallel lines** are intersected by a **transversal**, **corresponding angles** are equal, **alternate angles** are equal and **co-interior angles** are supplementary. (*All acute angles formed are equal to each other and all obtuse angles are equal to each other*)



Tip: The ratio of intercepts formed by a transversal intersecting three parallel lines is equal to the ratio of corresponding intercepts formed by any other transversal.

$$\Rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

Triangles

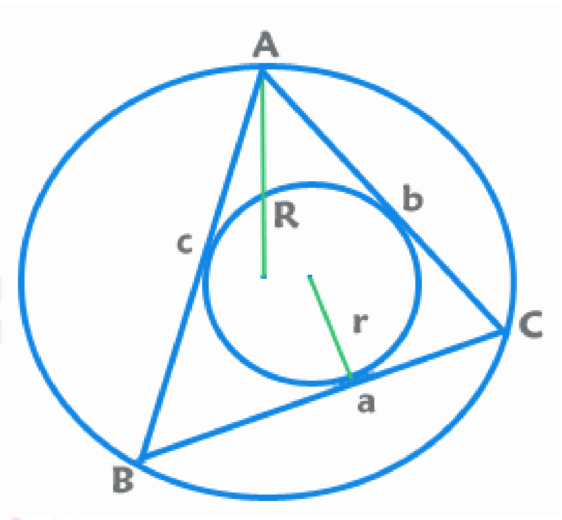
Sum of interior angles of a triangle is 180° and sum of exterior angles is 360°.

Exterior Angle = Sum of remote interior angles.

Sum of two sides is always greater than the third side and the difference of two sides is always lesser than the third side.

Side opposite to the biggest angle is longest and the side opposite to the smallest angle is the shortest.

Area of a triangle:



= ½ x Base x Height

= ½ x Product of sides x Sine of included angle

$=\sqrt{s(s-a)(s-b)(s-c)}$; here s is the semi perimeter [s = (a+b+c)/2]

= r x s [r is radius of incircle]

$=\frac{abc}{4R}$ [R is radius of circumcircle]

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A **Median** of a triangle is a line segment joining a vertex to the midpoint of the opposing side. The three medians intersect in a single point, called the **Centroid** of the triangle. Centroid divides the median in the ratio of 2:1

An **Altitude** of a triangle is a straight line through a vertex and perpendicular to the opposite side or an extension of the opposite side. The three altitudes intersect in a single point, called the **Orthocenter** of the triangle.

A **Perpendicular Bisector** is a line that forms a right angle with one of the triangle's sides and intersects that side at its midpoint. The three perpendicular bisectors intersect in a single point, called the **Circumcenter** of the triangle. It is the center of the circumcircle which passes through all the vertices of the triangle.

An **Angle Bisector** is a line that divides the angle at one of the vertices in two equal parts. The three angle bisectors intersect in a single point, called the **Incenter** of the triangle. It is the center of the incircle which touches all sides of a triangle.

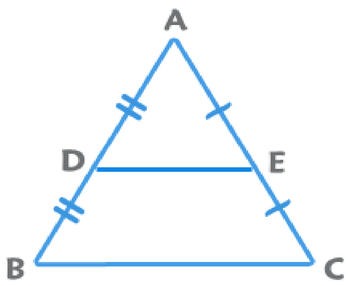
Tip: *Centroid* and *Incenter* will always lie inside the triangle.

- For an **acute angled triangle**, the *Circumcenter* and the *Orthocenter* will lie inside the triangle.
- For an **obtuse angled triangle**, the *Circumcenter* and the *Orthocenter* will lie outside the triangle.
- For a **right angled triangle** the *Circumcenter* will lie at the midpoint of the hypotenuse and the *Orthocenter* will lie at the vertex at which the angle is 90°.

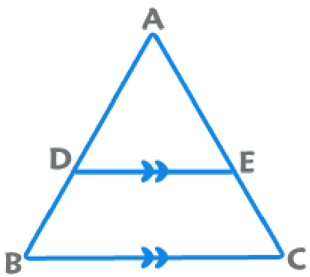
Tip: The *orthocenter*, *centroid*, and *circumcenter* always lie on the same line known as **Euler Line**.

- The orthocenter is twice as far from the centroid as the circumcenter is.
- If the triangle is **Isosceles** then the incenter lies on the **same line**.
- If the triangle is **equilateral**, all four are the **same point**.

Theorems



Mid Point Theorem: The line joining the midpoint of any two sides is parallel to the third side and is half the length of the third side.



Basic Proportionality Theorem: If DE || BC, then AD/DB = AE/EC

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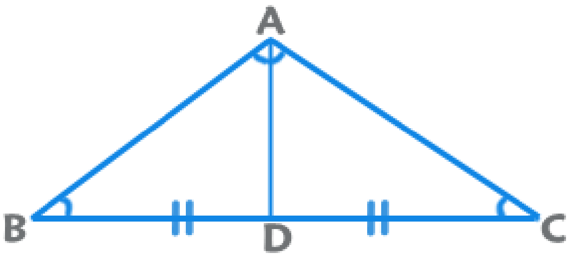
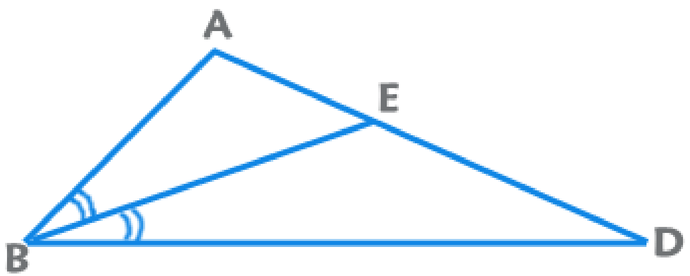


Apollonius' Theorem: $AB^2 + AC^2 = 2\,(AD^2 + BD^2)$



Interior Angle Bisector Theorem: AE/ED = BA/BD

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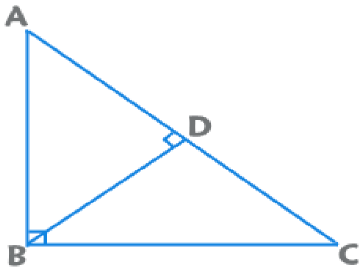
Special Triangles

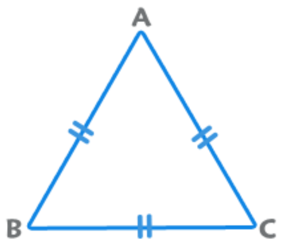
Right Angled Triangle:



$\Delta ABC \approx \Delta ADB \approx \Delta BDC$

$BD^2 = AD \times DC$ and $AB \times BC = BD \times DC$

Equilateral Triangle:



All angles are equal to 60°. All sides are equal also.

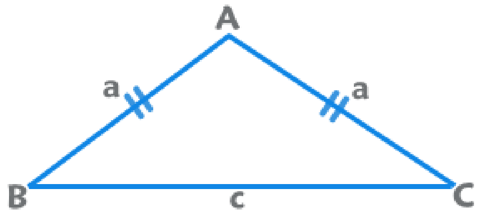
Height = $\frac{\sqrt{3}}{2}$ x Side

Area = $\frac{\sqrt{3}}{4}$ X $Side^2$

Inradius = 1/3 Height

Circumradius = 2/3 Height.

Isosceles Triangle:



Angles equal to opposite sides are equal.

Area = $\frac{c}{4}\sqrt{4a^2 - c^2}$

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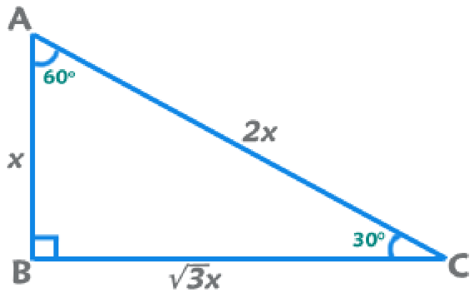
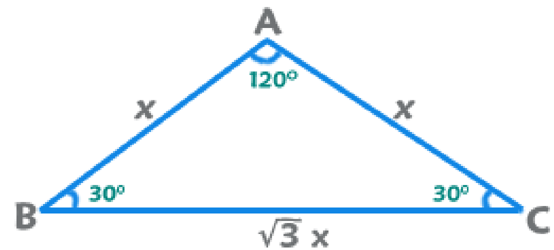
30°-60°-90° Triangle



Area = $\frac{\sqrt{3}}{2} * x^2$

30°-30°-120° Triangle



Area = $\frac{\sqrt{3}}{4} * x^2$

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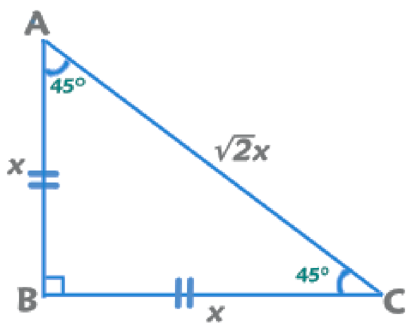
45°-45°-90° Triangle



Area = $x^2/2$

Similarity of Triangles

Two triangles are similar if their corresponding angles are congruent and corresponding sides are in proportion.

Tests of similarity: (AA / SSS / SAS)

For similar triangles, if the sides are in the ratio of a:b

- ⇨ Corresponding **heights** are in the ratio of a:b
- ⇨ Corresponding **medians** are in the ratio of a:b
- ⇨ **Circumradii** are in the ratio of a:b
- ⇨ **Inradii** are in the ratio of a:b
- ⇨ **Perimeters** are in the ratio of a:b
- ⇨ **Areas** are in the ratio $a^2 : b^2$

Congruency of Triangles

Two triangles are congruent if their corresponding sides and angles are congruent.

Tests of congruence: (SSS / SAS / AAS / ASA)

All ratios mentioned in similar triangle are now 1:1

Polygons

Sum of interior angles = (n-2) x 180° = (2n-4) x 90°
Sum of exterior angles = 360°

Number of diagonals = $^nC_2 - n = \frac{n(n-3)}{2}$

Number of triangles which can be formed by the vertices = $^nC_3$

Regular Polygon:

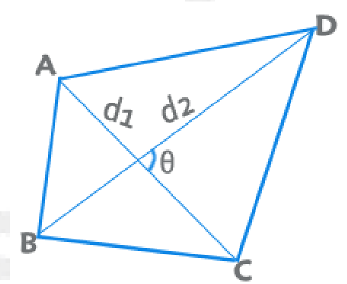
If all sides and all angles are equal, it is a regular polygon.

All regular polygons can be inscribed in or circumscribed about a circle.

Area = ½ x Perimeter x Inradius *{Inradius is the perpendicular from centre to any side}*

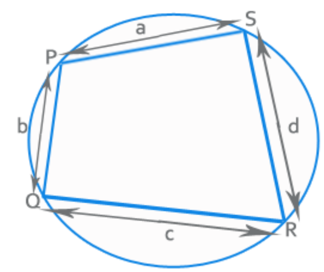
Each Interior Angle = $\frac{(n-2)180°}{n}$; Exterior = 360°/n

Quadrilaterals:



Sum of the interior angles = Sum of the exterior angles = 360°
Area for a quadrilateral is given by ½ $d_1 d_2 \sin\theta$.

Cyclic Quadrilateral



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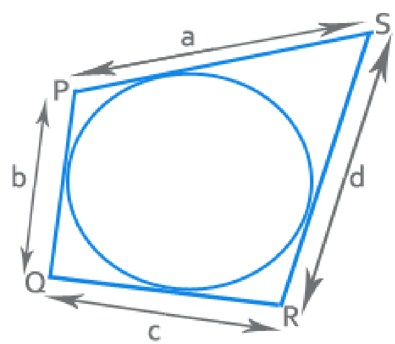
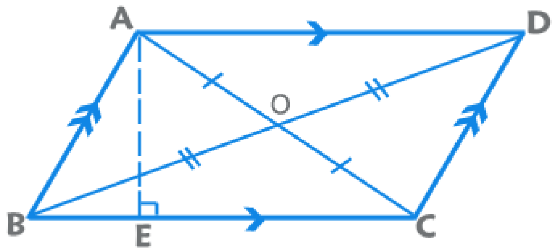
If all vertices of a quadrilateral lie on the circumference of a circle, it is known as a cyclic quadrilateral.
Opposite angles are supplementary
Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi perimeter $s = \frac{a+b+c+d}{2}$

Tip: Sum of product of opposite sides = Product of diagonals



Tip: If a circle can be inscribed in a quadrilateral, its area is given by = $\sqrt{abcd}$

Parallelogram



Opposite sides are parallel and congruent.

Opposite angles are congruent and consecutive angles are supplementary.

Diagonals of a parallelogram bisect each other.

Perimeter = 2(Sum of adjacent sides);
Area = Base x Height = AD x BE

Tip: A parallelogram inscribed in a circle is always a *Rectangle*. A parallelogram circumscribed about a circle is always a *Rhombus*.

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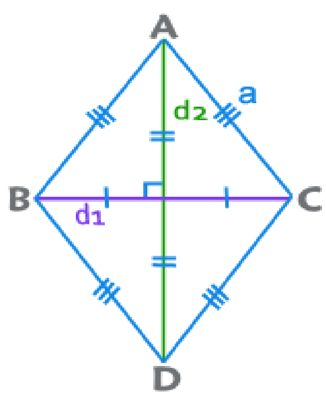
Tip: Each diagonal divides a parallelogram in two triangles of equal area.

Tip: Sum of squares of diagonals = Sum of squares of four sides

⇨ $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Tip: A *Rectangle* is formed by intersection of the four angle bisectors of a parallelogram.

Rhombus



A parallelogram with all sides equal is a Rhombus. Its diagonals bisect at 90°.

Perimeter = 4a; Area = ½ $d_1 d_2$; Area = d x $\sqrt{a^2 - (\frac{d}{2})^2}$

Rectangle

A parallelogram with all angles equal (90°) is a Rectangle. Its diagonals are congruent.

Perimeter = 2(l+b); Area = lb

Square

A parallelogram with sides equal and all angles equal is a square. Its diagonals are congruent and bisect at 90°.

Perimeter = 4a; Area = a^2; Diagonals = $a\sqrt{2}$

Tip: From all quadrilaterals with a given area, the square has the least perimeter. For all quadrilaterals with a given perimeter, the square has the greatest area.

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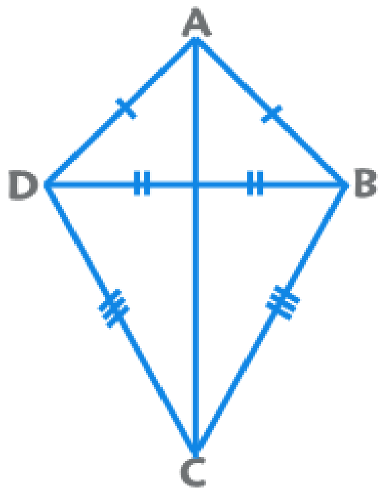
Kite

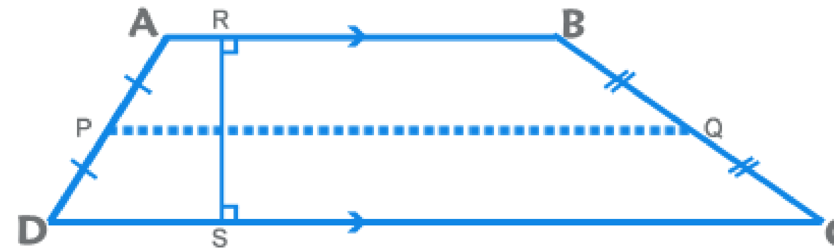


Two pairs of adjacent sides are congruent.

The longer diagonal bisects the shorter diagonal at 90°.

Area = Product of Diagonals / 2

Trapezium / Trapezoid



A quadrilateral with exactly one pair of sides parallel is known as a Trapezoid. The parallel sides are known as bases and the non-parallel sides are known as lateral sides.

Area = ½ x (Sum of parallel sides) x Height

Median, the line joining the midpoints of lateral sides, is half the sum of parallel sides.

Tip: Sum of the squares of the length of the diagonals = Sum of squares of lateral sides + 2 Product of bases.

⇨ $AC^2 + BD^2 = AD^2 + BC^2 + 2 \times AB \times CD$

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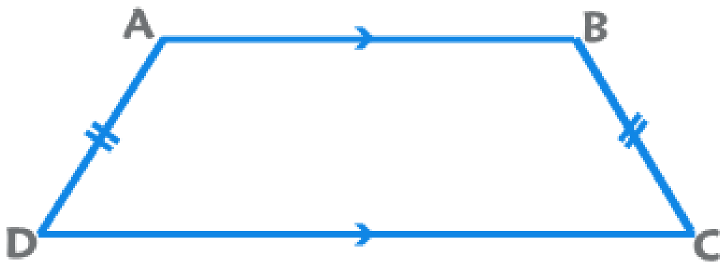
Isosceles Trapezium



The non-parallel sides (lateral sides) are equal in length. Angles made by each parallel side with the lateral sides are equal.

> **Tip:** If a trapezium is inscribed in a circle, it has to be an isosceles trapezium. If a circle can be inscribed in a trapezium, Sum of parallel sides = Sum of lateral sides

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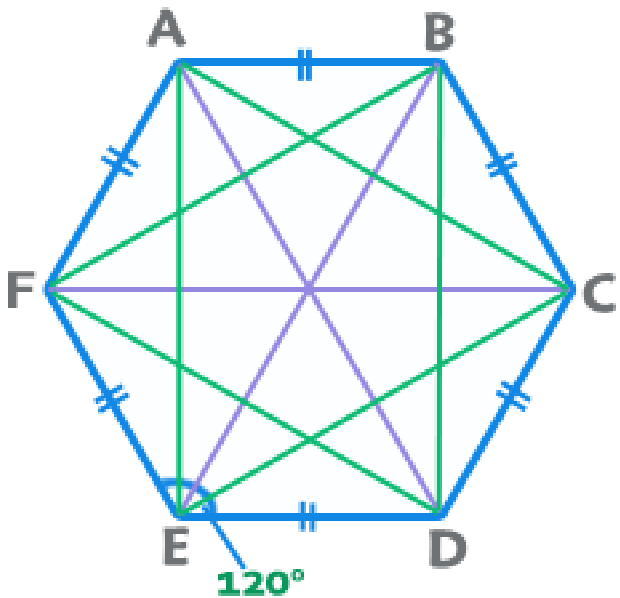
Hexagon (Regular)



Perimeter = 6a; Area = $\frac{3\sqrt{3}}{2}$ x a^2

Sum of Interior angles = 720°.

Each Interior Angle = 120°. Exterior = 60°

Number of diagonals = 9 {3 big and 6 small}

Length of big diagonals (3) = 2a

Length of small diagonals (6) = $\sqrt{3}$ a

Tip: A regular hexagon can be considered as a combination of six equilateral triangles. All regular polygons can be considered as a combination of 'n' isosceles triangles.

Area of a **Pentagon** = 1.72 a^2

Area of an **Octagon** = $2(\sqrt{2} + 1)\ a^2$

Circles

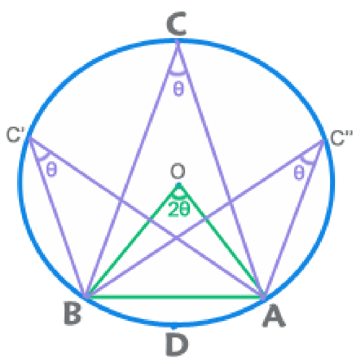
Diameter = 2r; Circumference = $2\pi r$; Area = πr^2

Chords equidistant from the centre of a circle are equal.

A line from the centre, perpendicular to a chord, bisects the chord.

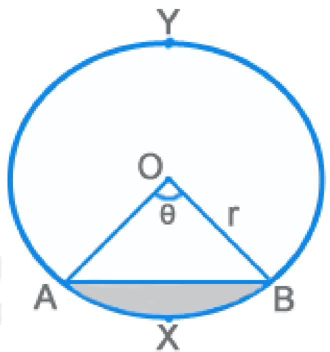
Equal chords subtend equal angles at the centre.

The diameter is the longest chord of a circle.



A chord /arc subtends equal angle at any point on the circumference and double of that at the centre.

Chords / Arcs of equal lengths subtend equal angles.



Chord AB divides the circle into two parts: Minor Arc AXB and Major Arc AYB.

Measure of arc AXB = ∠ AOB = θ

Length (arc AXB) = $\frac{\theta}{360°} \times 2\pi r$

Area (sector OAXB) = = $\frac{\theta}{360°} \times \pi r^2$

Area of Minor Segment = Shaded Area in above figure

⇨ Area of Sector OAXB - Area of Δ OAB

⇨ $r^2 \left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2} \right]$

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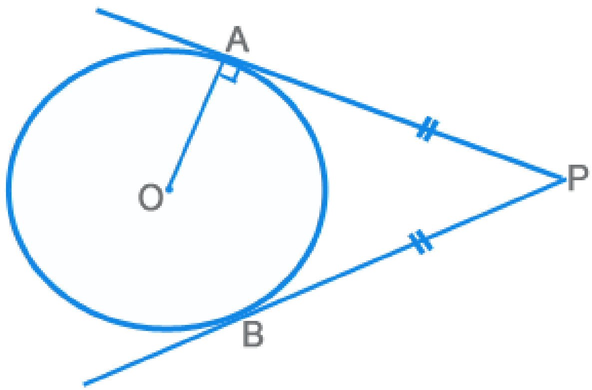
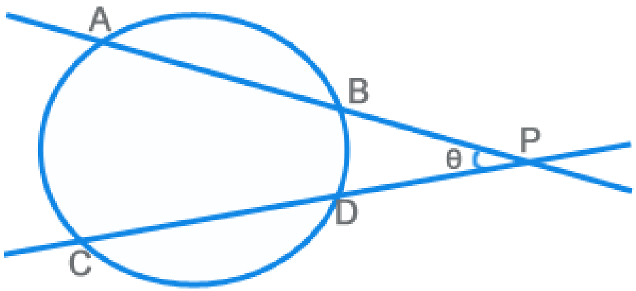
Properties of Tangents, Secants and Chords



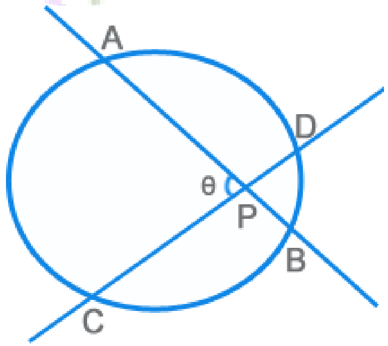


PA x PB = PC x PD

θ = ½ [m(Arc AC) – m(Arc BD)]

The radius and tangent are perpendicular to each other.

There can only be two tangents from an external point, which are equal in length **PA = PB**



PA x PB = PC x PD
θ = ½ [m(Arc AC) + m(Arc BD)]

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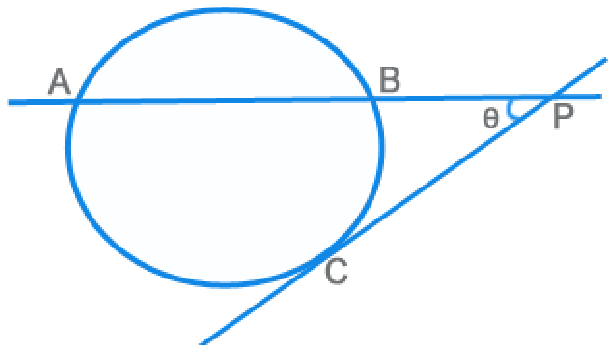
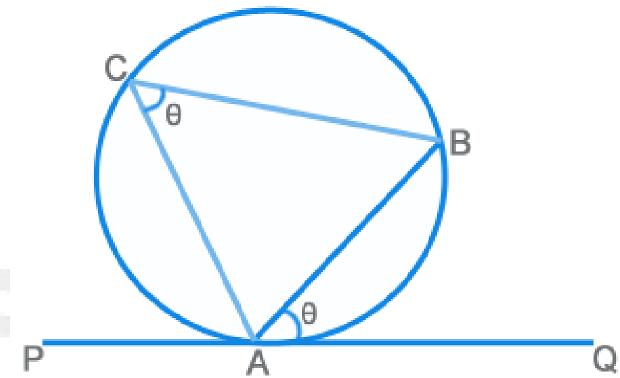
Properties *(contd.)*



PA x PB = PC^2

θ = ½ [m(Arc AC) - m(Arc BC)]

Alternate Segment Theorem



The angle made by the chord AB with the tangent at A (PQ) is equal to the angle that it subtends on the opposite side of the circumference.

⇨ ∠ BAQ = ∠ ACB

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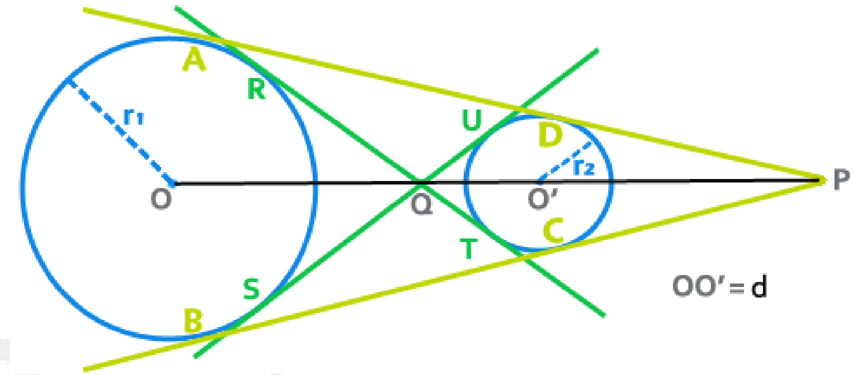
Common Tangents

Two Circles	No. of Common Tangents	Distance Between Centers (d)
One is completely inside other	0	< r1 - r2
Touch internally	1	= r1 - r2
Intersect	2	r1 - r2 < d < r1 + r2
Touch externally	3	= r1 + r2
One is completely outside other	4	> r1 + r2



Length of the Direct Common Tangent (DCT)

⇨ AD = BC = $\sqrt{d^2 - (r1 - r2)^2}$

Length of the Transverse Common Tangent (TCT)

⇨ RT = SU = $\sqrt{d^2 - (r1 + r2)^2}$

Tip: The two centers(O and O'), point of intersection of DCTs (P)and point of intersection of TCTs (Q) are collinear. Q divides OO' in the ratio $r_1 : r_2$ internally whearea P divides OO' in the ratio $r_1 : r_2$ externally.

Solid Figures

	Volume	Total Surface Area	Lateral / Curved Surface Area
Cube	$Side^3$	$6 \times Side^2$	$4 \times Side^2$
Cuboid	L x B x H	2(LB + LH + BH)	2 (LH + BH)
Cylinder	$\pi r^2 h$	$2\pi r (r + h)$	$2\pi rh$
Cone	$(1/3)\ \pi r^2 h$	$\pi r (r + L)$	πrl {where $L = \sqrt{r^2 + h^2}$}
Sphere	$(4/3)\ \pi r^3$	$4\ \pi r^2$	$4\ \pi r^2$
Hemisphere	$(2/3)\ \pi r^3$	$3\ \pi r^2$	$2\ \pi r^2$

Tip: There are 4 body diagonals in a cube / cuboid of length ($\sqrt{3}$ x side) and $\sqrt{l^2 + b^2 + h^2}$ respectively.

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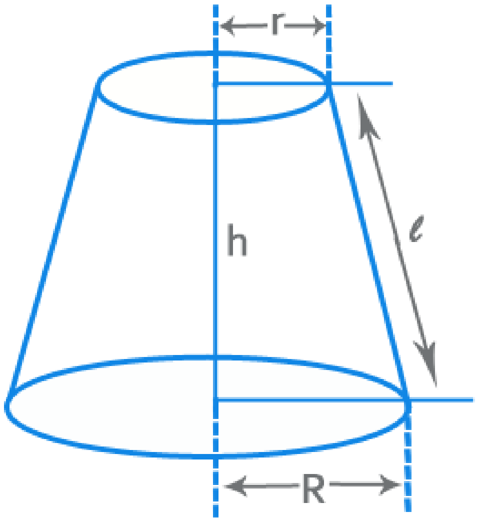
Frustum / Truncated Cone

It can be obtained by cutting a cone with a plane parallel to the circular base.

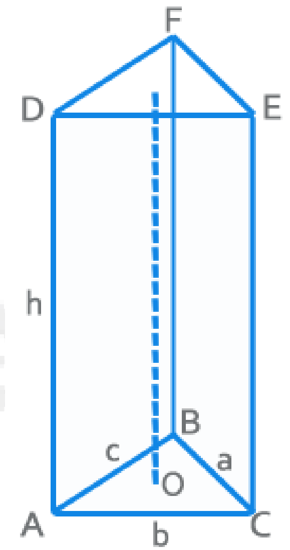


Volume = $1/3\ \pi h (R^2 + r^2 + Rr)$

Lateral Surface Area = π (R+r) L

Total Surface Area = π (R+r) L + π (R^2+r^2)

Prism



It is a solid with rectangular vertical faces and bases as congruent polygons (of n sides). It will have '2n' Vertices; 'n+2' Faces and '3n' Sides / Edges.

Lateral Surface Area = Perimeter x Height

Total Surface Area = Perimeter x Height + 2 $Area_{Base}$

Volume = $Area_{Base}$ x Height

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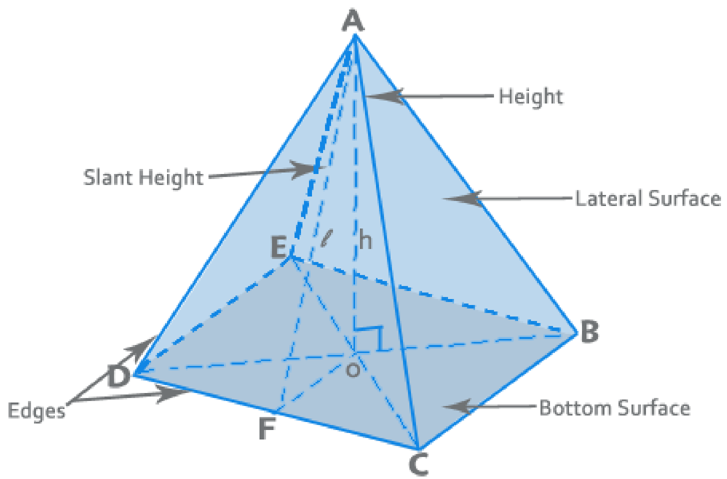
Pyramid



It is a figure in which the outer surfaces are triangular and converge at a point known as the apex, which is aligned directly above the centre of the base.

Lateral Surface Area = ½ x Perimeter x Slant Height

Total Surface Area = ½ x Perimeter x Slant Height + $Area_{Base}$

Volume = $^1/_3$ x $Area_{Base}$ x Height

Tip: If a sphere is inscribed in a cube of side a, the radius of the sphere will be a/2. If a sphere is circumscribed about a cube of side a, the radius of the sphere will be $\sqrt{3}$ a /2.

Tip: If a largest possible sphere is inscribed in a cylinder of radius 'a' and height h, its radius r will be

⇨ r = h/2 {If 2a > h}
⇨ r = a {If 2a < h}

Tip: If a largest possible sphere is inscribed in a cone of radius r and slant height equal to 2r, then the radius of sphere = $r/\sqrt{3}$

Tip: If a cube is inscribed in a hemisphere of radius r, then the edge of the cube = $r\sqrt{\frac{2}{3}}$

Co-ordinate Geometry

Distance between two points P(x_1,y_1) and Q(x_2,y_2) is given by = $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

If a point R (x,y) divides P(x_1,y_1) and Q(x_2,y_2) internally in the ratio of m:n, the coordinates of R ie (x,y) are given by = $\frac{mx_2 + nx_1}{m+n}$, $\frac{my_2 + ny_1}{m+n}$

If a point R (x,y) divides P(x_1,y_1) and Q(x_2,y_2) externally in the ratio of m:n, the coordinates of R ie (x,y) are given by = $\frac{mx_2 - nx_1}{m-n}$, $\frac{my_2 - ny_1}{m-n}$

Tip: The X axis divides the line joining P(x_1,y_1) and Q(x_2,y_2) in the ratio of $y_1 : y_2$

Tip: The Y axis divides the line joining P(x_1,y_1) and Q(x_2,y_2) in the ratio of $x_1 : x_2$

Slope(m) of a line is the tangent of the angle made by the line with the positive direction of the X-Axis.
For a general equation ax + by + c = 0; slope (m) = -a/b.
For a line joining two points, P (x_1,y_1) and Q(x_2,y_2), the slope(m) is = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope(m)	Type of line	Angle with X-Axis
> 0 (+ive)	Rising	Acute
0	Parallel to X-Axis	0^0
< 0 (-ive)	Falling	Obtuse
∞	Parallel to Y-Axis	90^0

Equation of a line parallel to X-axis is y = a {Of X-Axis is y = 0}
Equation of a line parallel to Y-Axis is x = a {Of Y-Axis is x = 0}

The intercept of a line is the distance between the point where it cuts the X-Axis or Y-Axis and the origin. Y-Intercept is often denoted with the letter ‘c’.

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Equation of a line

General form: ax + by + c = 0

Slope Intercept Form: Slope is m, y-intercept is c

⇨ y = mx + c

Slope Point Form: Slope is m, point is x_1,y_1

⇨ $y - y_1 = m(x - x_1)$

Two Point Form: Two points are x_1,y_1 and x_2,y_2

⇨ $y - y_1 = [\frac{y_2 - y_1}{x_2 - x_1}](x - x_1)$

Two Intercept Form: X-intercept is a, Y-intercept is b.

⇨ $\frac{x}{a} + \frac{y}{b} = 1$ OR bx + ay = ab

Acute angle between two lines with slope m_1 and m_2 is given by

⇨ Tan θ = $\left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

⇨ For parallel lines, θ = 0°; $m_1 = m_2$

⇨ For perpendicular lines, θ = 90°; $m_1 m_2 = -1$

Distance of a point P (x_1,y_1) **from a line** ax + by + c = 0

⇨ d = $\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

⇨ From origin, d = $\left| \frac{c}{\sqrt{a^2 + b^2}} \right|$

Distance between two parallel lines, $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$

⇨ d = $\left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$

Tip: If we know three points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ of a parallelogram, the fourth point is given by

⇨ $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$

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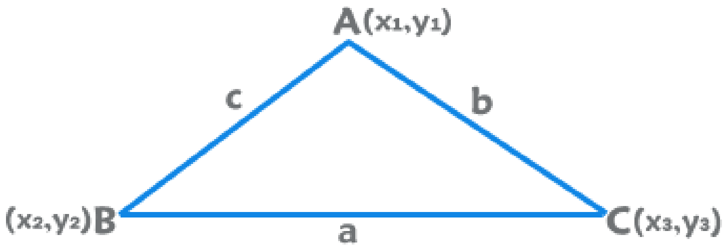
Triangle



The vertices are P (x_1,y_1), Q(x_2,y_2) and R(x_3,y_3)

Incenter = $\{\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\}$

Centroid = $\{\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{a+b+c}\}$

Area = ½ [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

Circle

General Equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

⇨ Centre is (-g, -f) and radius = $\sqrt{g^2 + f^2 - c}$

Centre is (h, k) and radius is r

⇨ $\sqrt{(x-h)^2 + (y-k)^2} = r$

Centre is origin and radius is r

⇨ $x^2 + y^2 = r^2$

Trigonometry

$\sin\theta = \frac{Perpendicular}{Hypotenuse} = \frac{AB}{AC}$

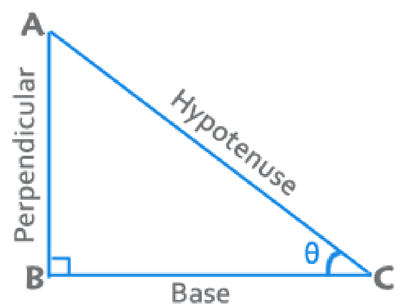
$\cos\theta = \frac{Base}{Hypotenuse} = \frac{BC}{AC}$

$\tan\theta = \frac{Perpendicular}{Base} = \frac{AB}{BC}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

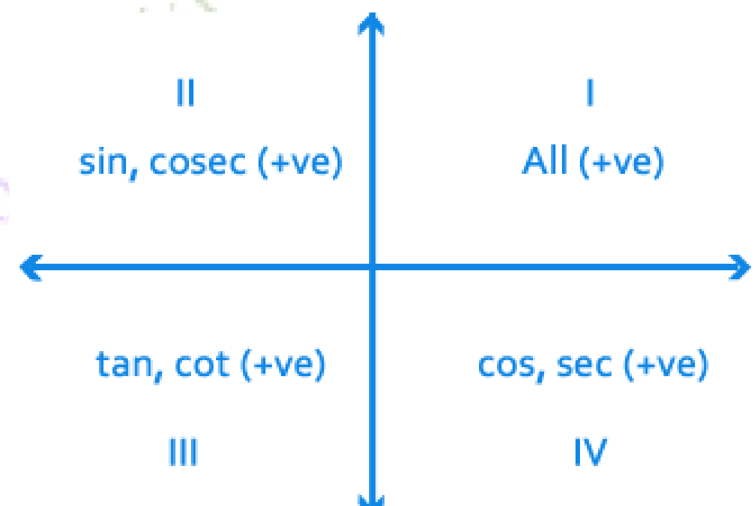
$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$



θ	Sin θ	Cos θ	Tan θ
0^0	0	1	0
30^0	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45^0	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60^0	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90^0	1	0	∞

Signs of T-ratios in Different Quadrants:



Some Basic Identities:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\,\theta$

Continued >>

Addition Formulae

Sin (A + B) = sinA cos B + cosA sinB

Cos (A + B) = cos A cos B - sin A sin B

Tan (A + B) - $\frac{tanA+tanB}{1-tanAtanB}$

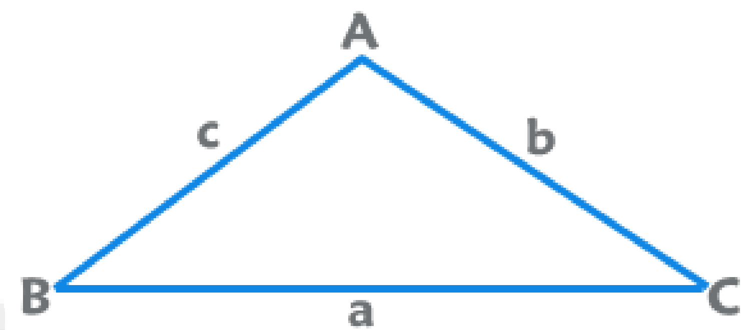
Subtraction Formulae

sin (A - B) = sin A cos B - cos A sin B

cos (A - B) = cos A cos B + sin A sin B

tan (A - B) = $\frac{tanA-tanB}{1+tanAtanB}$

Trigonometric Rules



Sine Rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$

Cosine Rule: Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$

Cos B = $\frac{a^2 + c^2 - b^2}{2ac}$

Cos C = $\frac{a^2 + b^2 - c^2}{2ab}$