FREE EBOOK

oliveboard

Tamil Nadu Textbook

Class 12 - English Medium

Physics Vol-1



For all TN Exams

GOVERNMENT OF TAMIL NADU

HIGHER SECONDARY SECOND YEAR

PHYSICS

VOLUME - I

A publication under Free Textbook Programme of Government of Tamil Nadu

Department of School Education

Untouchability is Inhuman and a Crime

XII_Physics_First 8 pages Higher.indd 1 14-02-2020 19:30:08

Government of Tamil Nadu

First Edition - 2019

Revised Edition - 2020

(Published Under New Syllabus)

NOT FOR SALE

Content Creation

State Council of Educational
Research and Training

© SCERT 2019

Printing & Publishing

Tamil NaduTextbook and Educational
Services Corporation

www.textbooksonline.tn.nic.in

XII_Physics_First 8 pages Higher.indd 2 14-02-2020 19:30:10

CONTENTS

PHYSICS

UNIT	TITLE	PAGE No.	MONTH
1	Electrostatics	01	June
2	Current Electricity	80	June
3	Magnetism and magnetic effects of electric current	125	July
4	Electromagnetic Induction And Alternating Current	195	July
5	Electromagnetic waves	266	August
	Practical	286	
	Appendix	319	
	Glossary	328	

GPTWUZ
E-book

4T8364
Assessment

B263_12_PHYSICS_EM
DIGI links

2020 QR GUIDE

Let's use the QR code in the text books!

- Download DIKSHA app from the Google Play Store.
- Tap the QR code icon to scan QR codes in the textbook.
- Point the device and focus on the QR code.
- On successful scan, content linked to the QR code gets listed.

Note: For ICT corner, Digi Links QR codes use any other QR scanner.

XII_Physics_First 8 pages Higher.indd 3 14-02-2020 19:30:10

HOW TO USE THE BOOK ?

Scope of Physics
- Awareness on higher learning - courses, institutions and required competitive exams
- Financial assistance possible to help students to climb academic ladder

Learning Objectives:
- Overview of the unit
- Gives clarity on the goals and objective of the topics



- Additional facts related to the topics covered to facilitate curiosity driven learning

Example problems
- To ensure understanding, problems/illustrations are given at every stage before advancing to next level



- Visual representation of concepts with illustrations
- Videos, animations, and tutorials



ICT
- To harness the digital skills to class room learning and experimenting

Summary
- Recap of salient points of the lesson

Concept Map
- Schematic outline of salient learning of the unit

Evaluation
- Evaluate students' understanding and get them acquainted with the application of physical concepts to numerical and conceptual questions

Books for Reference
- List of relevant books for further reading

Practical
- List of practical and the description of each is appended for easy access.

Glossary
- Scientific terms frequently used with their Tamil equivalents

Content Focus
- Sequential understanding of the stationary charges, moving charges, electric current, magnetism and the interlink between electric and magnetic phenomena.

Back Wrapper: NIKOLA TESLA a Serbian-American Engineer (10 July 1856 – 7 January 1943)

Nikola Tesla made breakthroughs in the production, transmission and application of electric power. He invented the first alternating current (AC) motor and developed AC generation and transmission technology. In 1884, he was hired by Edison (discoverer of DC dynamos) later on Tesla became his competitor in this field.

Tesla conducted a range of experiments with mechanical oscillators/generators, electrical discharge tubes, and early X – ray imaging. Tesla in 1890 itself conducted research for wireless lighting and worldwide wireless electric power distribution in his high – voltage, high-frequency power experiments in New York. Unfortunately Tesla could not put his ideas in practical use due to lack of funds.

XII_Physics_First 8 pages Higher.indd 4 14-02-2020 19:30:10

XII_Physics_First 8 pages Higher.indd 5

Scope of Physics - Higher Education



Entrance Examinations After +2

- Physics Olympiad Exam
- NEET-National Eligibility cum Entrance Test
- IIT JEE-Joint Entrance Examination(Mains & Advanced)
- NEST- National Entrance Screening Test
- KVPY-Kishore Vaigyanik Protsahan Yojana
- JEE Mains Paper II for B.Arch
- AIIMS - All Indian Institute of Medical Science's Examination
- Chennai Mathematical Institute Entrance Examination
- BITSAT- Birla Institute of Science And Technology Admission Test
- AIEEE – All India Engineering Entrance Exam
- CUCET – Central Universities Common Entrance Test
- JIPMER - Jawaharlal Institute of Postgraduate Medical Education & Research
- CLAT – Common Law Admission Test
- HSEE- Humanities and Social Sciences Entrance Examination
- AIPVT -All India Pre-Veterinary Test
- NDA – National Defence Academy Examination

After Graduation

- JAM- Joint Admission Test
- JEST – Joint Entrance Screening Test
- GATE- Graduate Aptitude Test in Engineering
- CAT – Common Admission Test(for MBA)
- Exams conducted by Respective Universities

After Post Graduation



- CSIR - National Eligibility Test for JRF and Lectureship



Education Career

After Completing +2

- Integrated Msc. Physics
- Central Uiversities through CUCET
- Central Research Institutes like IISER using KVPY, JEE Advanced , IISER

  Aptitude Test
- Top 1% students in State board are eligible for IISER Aptitude Test
- Admission in NISER through NEST
- B.Sc Photonics
- B.Sc Hons in Mathematics and Physics in CMI
- B.Sc Hons in Mathematics and Computer Science in CMI
- Five-Year Dual degree In IIST (B.Tech + Master of Science)
- Master of Science (Astronomy and Astrophysics, Solid State Physics)

Note

- Students admitted to IISc, IIT's, NIT's IISER's, IIST, will get a Scholorship equivalent to INSPIRE
- Assured placement in ISRO and other divisions for the students of IIST
- Institutes and their ranking can be found in www.nirfindia.org

After B.Sc Physics

- M.Sc Physics in IIT's and NIT's through JAM
- Integrated Phd in IISER's and IISc through JAM and JEST
- M.Sc Physics in Central Universities throught CUCET
- M.Sc in Energy Physics, Applied Physics in IIT's through JAM
- Integrated Phd in IMSc, TIFR, JNCASR through JEST score
- Integrated Phd in TIFR through JEST and TIFR exam
- M.Sc Photonics, Reactor physics, Nuclear Engineering ,
- M.Sc Medical Physics
- M.Sc Biophyiscs
- Research Institutes in abroad like CERN, NASA, LIGO offer Summer internship programmes for motivated Indian students pursuing Undergraduate course in physics
- Indian Academy of Science & various other research institutes offer paid Summer Internship for science students to get an hands on experience in research.

14-02-2020 19:30:14

XII_Physics_First 8 pages Higher.indd 6

Opportunities after B.Sc. Physics



Jobs in Government Sector

- Scientific Officer and Scientific Assistant Jobs
- CSIR Labs
- DRDO – Defence Research and Development Organisation
- DAE -Department of Atomic Energy
- DoS - Department of Science
- IMD- Indian Meteorological Department
- ONGC -Oil and Natural Gas Corporation
- ATC – Air Traffic Controller
- Teaching faculty in schools and colleges through SET, NET,TET
- Scientist post in various research institutes in India



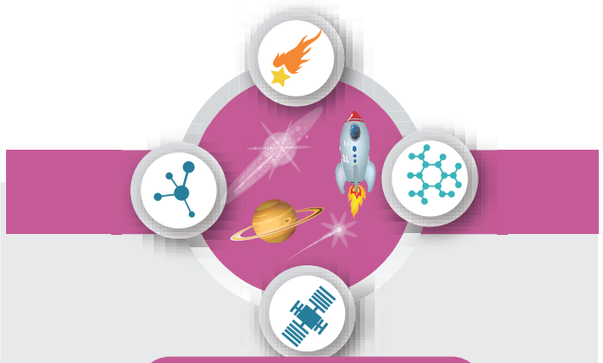
Scholarships

- INSPIRE Scholarship - Scholarship for Higher Education(SHE) - 80000 per annum, for B.Sc/B.S/Int M.Sc/Int M.S
  Eligibility Criteria:- Top 1% students in their plus 2 board exam
  Top 10000 rank holders in JEE or NEET
- Students studying at NIISER, IISER, Department of Atomic Energy Centre for Basic Science NTSE, KVPY, JBNSTS Scholars
- International Olympiad Medalists
- Indira Gandhi Scholarship for single girl child for full time regular Master's Degree
- Post Graduate Merit Scholarship for University rank holders in UG level
- Women Scientist Scheme (WOS -A)
  Eligibility Criteria:- Women who are pursuing M.Sc or Ph.D
- Mathematics Training and Talent Search(MTTS) Programme
  Eligibility Criteria:- Students who studied Maths at UG or PG level
- Dr. K S Krishnan Research Associateship (KSKRA)
  Eligibility Criteria:- Students who posses Master's Degree or Ph.D in science or engineering
- IGCAR JRF
  Eligibility Criteria:- Passing JEST, GATE, NET Exams
- Promotion of Science Education (POSE) Scholarship Scheme
- Dhirubhai Ambani Scholarship Programme
- Foundation for Academic Excellence and Access Scholarship(FAEA)
- Central Sector Scheme of National Fellowship and Scholarship for Higher Education of ST students
- Pre - Matric and Post - Matric Scholarship for students belonging to minority communities to pursue their School and Collegiate education by the Ministry of Minority affairs, Government of India.
- Pre Matric and Post Matric Scholarship for students with Disabilities to pursue their School and Collegiate Education by the Department of Empowerment of Persons with Disabilities, Government of India.

14-02-2020 19:30:15

XII_Physics_First 8 pages Higher.indd 7

Institutes in india to pursue research in physics



Research Areas

General Relativity and Cosmology
Astronomy and Astrophysics
Quantum Optics and Information theory
Plasma physics
Meteorology and Atmospheric science
String Theory, Quantum Gravity
Optics and Photonics
Condensed Matter Theory, Material Science and Spintronics
Cryptography
Mathematical Physics,Statistical Physics
Crystal Growth and Crystallography
Atomic and Molecular Physics
Biophysics, Medical Physics
Nuclear and High energy Particle Physics
Energy and Environmental Studies
Geophysics
Quantum Biology and Quantum Thermodynamics and Cymatics

Famous Research Institutes for Physics in India

Name of the Institution	Website
Institute of Mathematical Sciences,Chennai(IMSc)	www.imsc.res.in
Saha Institute of Nuclear Physics, Kolkata	www.saha.ac.in
International Centre for Theoretical Sciences, Bangalore	www.icts.res.in
Harish chandra Research Institute,Allahabad	www.hri.res.in
Aryabhatta Research Institute of Observational Sciences, Nainital	www.aries.res.in
Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR)	www.jncasr.ac.in
Institute of Physics (IOP), Bhubaneshwar	www.iopb.res.in
Indian Association for the Cultivation of Sciences (IACS), Kolkata	www.iacs.res.in
Vikram Sarabhai Space Centre (VSSC), Thiruvananthapuram	www.vssc.gov.in
National Physical Laboratory (NPL),Delhi	www.nplindia.in
National Institute of Science Education and Research (NISER), Bhubaneshwar	www.niser.ac.in
Indian Institute of Science(IISc),Bangalore	www.iisc.ac.in
Raman Research Institute(RRI),Bangalore	www.rri.res.in
Tata Institute of Fundamental Research (TIFR)	www.tifr.res.in
Bhaba Atomic Research Centre(BARC)	www.barc.gov.in
Indira Gandhi Centre for Atomic Research(IGCAR)	www.igcar.gov.in
Inter University Centre for Astronomy and Astrophysics(IUCAA),Pune	www.iucaa.in
Indian Institute of Space Science and Technology(IIST), Trivandrum	www.iist.ac.in
Institute of Plasma Research(IPR),Gujarat	www.ipr.res.in
Physical Research Laboratory(PRL),Ahmedabad	www.prl.res.in
Inter-University Accelerator Center (IUAC)	www.iuac.res.in
Indian Institute of Astrophysics(IIA),Bangalore	www.iiap.res.in
Chennai Mathematical Institute(CMI),Chennai	www.cmi.ac.in
Liquid Propulsion Systems Centre	www.lpsc.gov.in
S.N.Bose Centre for Basic Sciences	www.bose.res.in
CSIR National laboratories	
Indian Institute of Technology(IIT) in various places	
IISER's in various places	
National Institute of Technology(NIT) in various places	
Indian Institute of Information Technology (IIITs) at various places	
Central and State Universities	

14-02-2020 19:30:16

XII_Physics_First 8 pages Higher.indd 8 14-02-2020 19:30:16

UNIT 1 ELECTROSTATICS

Electricity is really just organized lightning
– George Carlin

Learning Objectives

In this unit, student is exposed to

- Historical background of electricity and magnetism
- The role of electrostatic force in day – to-day life
- Coulomb's law and superposition principle
- The concept of electric field
- Calculation of electric field for various charge configurations
- Electrostatic potential and electrostatic potential energy
- Electric dipole and dipole moment
- Electric field and electrostatic potential for a dipole
- Electric flux
- Gauss law and its various applications
- Electrostatic properties of conductors and dielectrics
- Polarisation
- Capacitors in series and parallel combinations
- Effect of a dielectric in a capacitor
- Distribution of charges in conductors, corona discharge
- Working of a Van de Graaff generator

TLFRJV

1.1 INTRODUCTION

Electromagnetism is one of the most important branches of physics. The technological developments of the modern 21st century are primarily due to our understanding of electromagnetism. The forces we experience in everyday life are electromagnetic in nature except gravity.

In standard XI, we studied about the gravitational force, tension, friction, normal force etc. Newton treated them to be independent of each other with each force being a separate natural force. But what is the origin of all these forces? It is now understood that except gravity, all forces which we experience in every day life (tension in the string, normal force from the surface, friction etc.) arise from electromagnetic forces within the atoms. Some examples are

UNIT-1(XII-Physics_Vol-1).indd 1 14-02-2020 19:31:46

(i) When an object is pushed, the atoms in our hand interact with the atoms in the object and this interaction is basically electromagnetic in nature.

(ii) When we stand on Earth's surface, the gravitational force on us acts downwards and the normal force acts upward to counter balance the gravitational force. What is the origin of this normal force?

It arises due to the electromagnetic interaction of atoms on the surface of the Earth with the atoms present in the feet of the person. Though, we are attracted by the gravitational force of the Earth, we stand on Earth only because of electromagnetic force of atoms.

(iii) When an object is moved on a surface, static friction resists the motion of the object. This static friction arises due to electromagnetic interaction between the atoms present in the object and atoms on the surface. Kinetic friction also has similar origin.

From these examples, it is clear that understanding electromagnetism is very essential to understand the universe in a holistic manner. The basic principles of electromagnetism are dealt with in volume 1 at XII standard physics. This unit deals with the behaviour and other related phenomena of charges at rest. This **branch of electricity which deals with stationary charges is called Electrostatics.**

1.1.1 Historical background of electric charges

Two millenniums ago, Greeks noticed that amber (a solid, translucent material formed from the resin of a fossilized tree) after rubbing with animal fur attracted small pieces of leaves and dust. The amber possessing this property is said to be 'charged'. It was initially thought that amber has this special property. Later people found that not only amber but even a glass rod rubbed with silk cloth, attracts pieces of papers. So glass rod also becomes 'charged' when rubbed with a suitable material.

Consider a charged rubber rod hanging from a thread as shown in Figure 1.1. Suppose another charged rubber rod is brought near the first rubber rod; the rods repel each other. Now if we bring a charged glass rod close to the charged rubber rod, they attract each other. At the same time, if a charged glass rod is brought near another charged glass rod, both the rods repel each other.

From these observations, the following inferences are made

(i) The charging of rubber rod and that of glass rod are different from one another.

(ii) The charged rubber rod repels another charged rubber rod, which implies that 'like charges repel each other'. We can also arrive at the same inference by observing that a charged glass rod repels another charged glass rod.

(iii) The charged rubber rod attracts the charged glass rod, implying that the charge in the glass rod is not the same kind of charge present in the rubber. Thus unlike charges attract each other.

Therefore, two kinds of charges exist in the universe. In the 18th century, Benjamin Franklin called one type of charge as positive (+) and another type of charge as negative (–). Based on Franklin's convention, rubber and amber rods are negatively charged while the glass rod is positively charged. **If the net charge is zero in the object, it is said to be electrically neutral.**

UNIT-1(XII-Physics_Vol-1).indd 2 14-02-2020 19:31:46

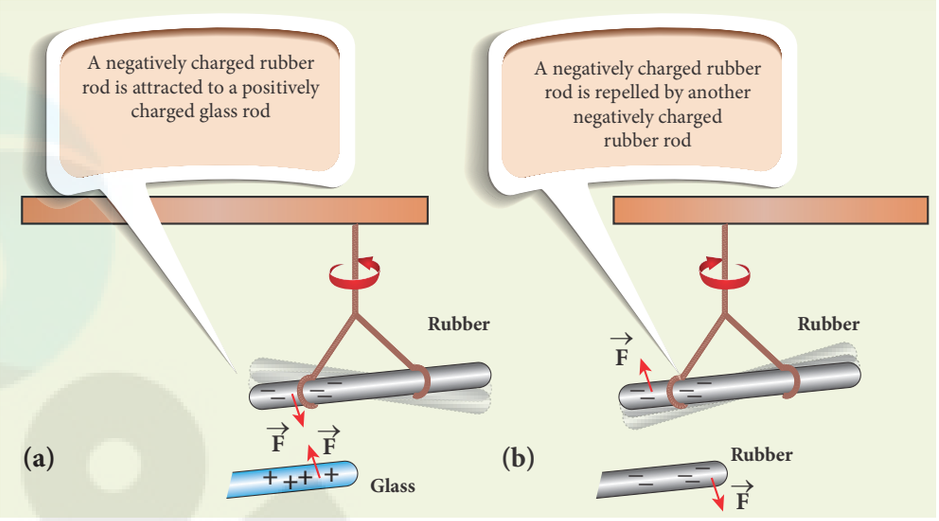


Figure 1.1 (a) Unlike charges attract each other (b) Like charges repel each other

Following the pioneering work of J. J. Thomson and E. Rutherford, in the late 19th century and in the beginning of 20th century, we now understand that the atom is electrically neutral and is made up of the negatively charged electrons, positively charged protons, and neutrons which have zero charge. The material objects made up of atoms are neutral in general. When an object is rubbed with another object (for example rubber with silk cloth), some amount of charge is transferred from one object to another due to the friction between them and the object is then said to be electrically charged. **Charging the objects through rubbing is called triboelectric charging.**

1.1.2 Basic properties of charges

(i) Electric charge

Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric charge is another intrinsic and fundamental property of particles. The nature of charges is understood through various experiments performed in the 19th and 20th century. The SI unit of charge is coulomb.

(ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged).

From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical

UNIT-1(XII-Physics_Vol-1).indd 3 14-02-2020 19:31:46

process, the net change in charge will always be zero.

(iii) Quantisation of charges

What is the smallest amount of charge that can be found in nature? Experiments show that the charge on an electron is $-e$ and the charge on the proton is $+e$. Here, e denotes the fundamental unit of charge. The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

$$q = ne \qquad (1.1)$$

Here n is any integer (0, ±1, ±2, ±3, ±4……..). This is called quantisation of electric charge.

Robert Millikan in his famous experiment found that the value of $e = 1.6 \times 10^{-19}$ C. The charge of an electron is -1.6×10^{-19} C and the charge of the proton is $+1.6 \times 10^{-19}$ C. When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of 10^{10}. So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

EXAMPLE 1.1

Calculate the number of electrons in one coulomb of negative charge.

Solution

According to the quantisation of charge,

$$q = ne$$

Here $q = 1$C. So the number of electrons in 1 coulomb of charge is

$$n = \frac{q}{e} = \frac{1C}{1.6\times10^{-19}} = 6.25\times10^{18} \text{ electrons}$$

1.2 COULOMB'S LAW

In the year 1786, Coulomb deduced the expression for the force between two stationary point charges in vacuum or free space. Consider two point charges q_1 and q_2 at rest in vacuum, and separated by a distance of r as shown in Figure 1.2. According to Coulomb, the force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad (1.2)$$

where $\hat{r}_{12}$ is the unit vector directed from charge q_1 to charge q_2 and k is the proportionality constant.

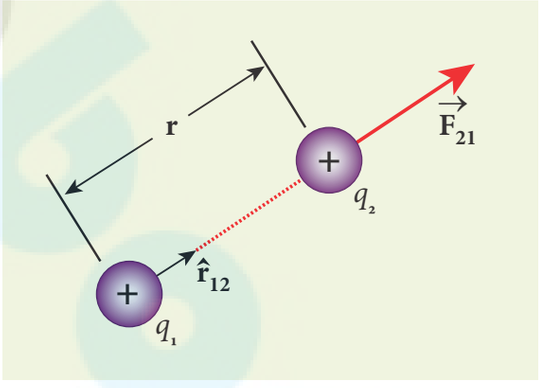


Figure 1.2 Coulomb force between two positive point charges

Important aspects of Coulomb's law

(i) Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

(ii) The force on the charge q_2 exerted by the charge q_1 always lies along the line

UNIT-1(XII-Physics_Vol-1).indd 4 14-02-2020 19:31:48

joining the two charges. $\hat{r}_{12}$ is the unit vector pointing from charge q_1 to q_2 .It is shown in the Figure 1.2. Likewise, the force on the charge q_1 exerted by q_2 is along $-\hat{r}_{12}$ (i.e., in the direction opposite to $\hat{r}_{12}$).

(iii) In SI units, $k = \frac{1}{4\pi\epsilon_o}$ and its value is $9\times10^9\,\text{N m}^2\,\text{C}^{-2}$. Here ϵ_o is the permittivity of free space or vacuum and its value is

$$\epsilon_o = \frac{1}{4\pi k} = 8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}.$$

(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$$|F| = \frac{9\times10^9\times1\times1}{1^2} = 9\times10^9\,\text{N}.$$

This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of μC (micro coulomb) or nC (nano coulomb).

(v) In SI units, Coulomb's law in vacuum takes the form $\vec{F}_{21} = \frac{1}{4\pi\epsilon_\circ}\frac{q_1q_2}{r^2}\hat{r}_{12}$. In a medium of permittivity ϵ, the force between two point charges is given by $\vec{F}_{21} = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}\hat{r}_{12}$. Since $\epsilon>\epsilon_o$, the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as $\epsilon_r = \frac{\epsilon}{\epsilon_\circ}$. For vacuum or air, $\epsilon_r = 1$ and for all other media $\epsilon_r > 1$.

(vi) Coulomb's law has same structure as Newton's law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses. But there are some important differences between these two laws.

- The gravitational force between two masses is always attractive but Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.
- The value of the gravitational constant $G = 6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2}$. The value of the constant k in Coulomb law is $k = 9 \times 10^9\ \text{N m}^2\,\text{C}^{-2}$. Since k is much more greater than G, the electrostatic force is always greater in magnitude than gravitational force for smaller size objects.
- The gravitational force between two masses is independent of the medium. For example, if 1 kg of two masses are kept in air or inside water, the gravitational force between two masses remains the same. But the electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.

(vii) The force on a charge q_1 exerted by a point charge q_2 is given by

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_\circ}\frac{q_1q_2}{r^2}\hat{r}_{21}$$

Here $\hat{r}_{21}$ is the unit vector from charge q_2 to q_1.

But $\hat{r}_{21} = -\hat{r}_{12}$,

UNIT-1(XII-Physics_Vol-1).indd 5 14-02-2020 19:31:51

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}(\hat{r}_{12})$$

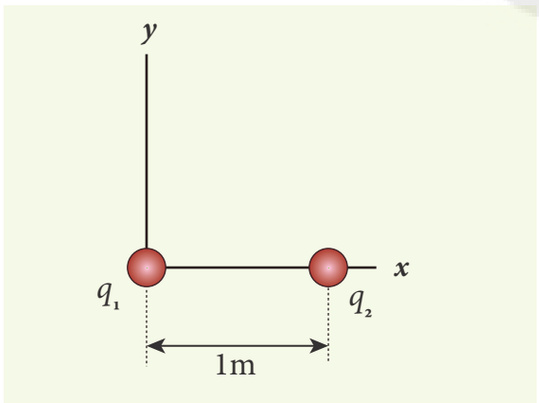
(or) $\vec{F}_{12} = -\vec{F}_{21}$

Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

EXAMPLE 1.2

Consider two point charges q_1 and q_2 at rest as shown in the figure.



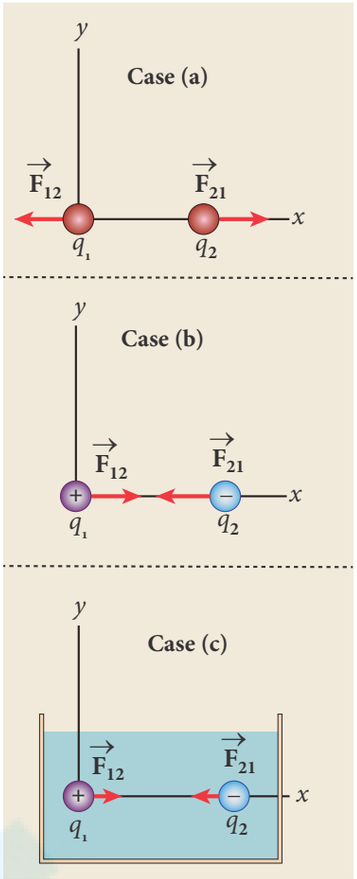
They are separated by a distance of 1m. Calculate the force experienced by the two charges for the following cases:

(a) $q_1 = +2\ \mu C$ and $q_2 = +3\ \mu C$

(b) $q_1 = +2\ \mu C$ and $q_2 = -3\ \mu C$

(c) $q_1 = +2\ \mu C$ and $q_2 = -3\ \mu C$ kept in water ($\epsilon_r = 80$)

Solution



(a) $q_1 = +2\ \mu C$, $q_2 = +3\ \mu C$, and $r = 1$m. Both are positive charges. so the force will be repulsive.

Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Here $\hat{r}_{12}$ is the unit vector from q_1 to q_2. Since q_2 is located on the right of q_1, we have

$\hat{r}_{12} = \hat{i}$ and $\frac{1}{4\pi\epsilon_0} = 9\times10^9$ so that

$$\vec{F}_{21} = \frac{9\times10^9 \times 2\times10^{-6} \times 3\times10^{-6}}{1^2}\hat{i}$$
$$= 54\times10^{-3}\hat{i}\ \text{N}$$

UNIT-1(XII-Physics_Vol-1).indd 6 14-02-2020 19:31:53

According to Newton's third law, the force experienced by the charge q_1 due to q_2 is $\vec{F}_{12} = -\vec{F}_{21}$. Therefore,

$$\vec{F}_{12} = -54\times10^{-3}\,\hat{i}\ \text{N}.$$

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the above figure in case (a)

(b) $q_1 = +2\ \mu\text{C}$, $q_2 = -3\ \mu\text{C}$, and $r = 1\text{m}$. They are unlike charges. So the force will be attractive.

Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{9\times10^{9}\times\left(2\times10^{-6}\right)\times\left(-3\times10^{-6}\right)}{1^{2}}\hat{r}_{12}$$
$$= -54\times10^{-3}\,\text{N}\hat{i}\quad (\text{Using } \hat{r}_{12} = \hat{i})$$

The charge q_2 will experience an attractive force towards q_1 which is in the negative x direction.

According to Newton's third law, the force experienced by the charge q_1 due to q_2 is $\vec{F}_{12} = -\vec{F}_{21}$. Therefore,

$$\vec{F}_{12} = 54\times10^{-3}\,\hat{i}\ \text{N}$$

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the figure (case (b)).

(c) If these two charges are kept inside the water, then the force experienced by q_2 due to q_1

$$\vec{F}_{21}^{W} = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

since $\epsilon = \epsilon_r\epsilon_o$

we have $\vec{F}_{21}^{W} = \frac{1}{4\pi\epsilon_r\epsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{12} = \frac{\vec{F}_{21}}{\epsilon_r}$

Therefore,

$$\vec{F}_{21}^{W} = -\frac{54\times10^{-3}\,\text{N}}{80}\hat{i} = -0.675\times10^{-3}\,\text{N}\hat{i}$$

Note

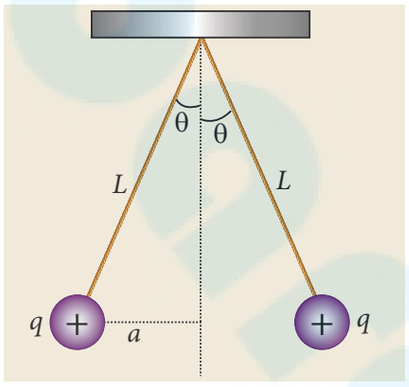
Note that the strength of the force between the two charges in water is reduced by 80 times compared to the force between the same two charges in vacuum.

When common salt (*NaCl*) is taken in water, the electrostatic force between *Na* and *Cl* ions is reduced due to the high relative permittivity of water (ϵ_r = 80). This is the reason water acts as a good solvent.

EXAMPLE 1.3

Two small-sized identical equally charged spheres, each having mass 1 g are hanging in equilibrium as shown in the figure. The length of each string is 10 cm and the angle θ is 30° with the vertical. Calculate the magnitude of the charge in each sphere.

(Take $g = 10\ \text{ms}^{-2}$)



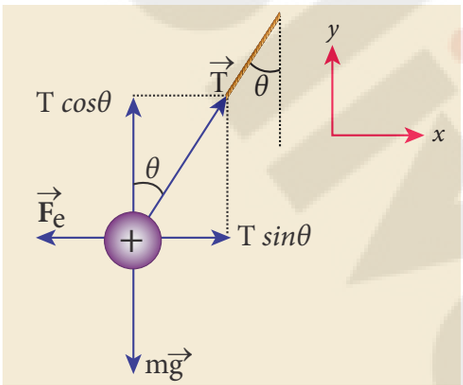
Solution

If the two spheres are neutral, the angle between them will be 0° when

UNIT-1(XII-Physics_Vol-1).indd 7 14-02-2020 19:31:57

hanged vertically. Since they are positively charged spheres, there will be a repulsive force between them and they will be at equilibrium with each other at an angle of 30° with the vertical. At equilibrium, each charge experiences zero net force in each direction. We can draw a free body diagram for one of the charged spheres and apply Newton's second law for both vertical and horizontal directions.

The free body diagram is shown below.



In the x-direction, the acceleration of the charged sphere is zero.

Using Newton's second law $\left(\vec{F}_{tot} = m\vec{a}\right)$, we have

$$T\sin\theta\,\hat{i} - F_e\hat{i} = 0$$

$$T\sin\theta = F_e \qquad (1)$$

Here T is the tension acting on the charge due to the string and F_e is the electrostatic force between the two charges.

In the y-direction also, the net acceleration experienced by the charge is zero.

$$T\cos\theta\,\hat{j} - mg\,\hat{j} = 0$$

$$T\cos\theta = mg\ . \qquad (2)$$

By dividing equation (1) by equation (2),

$$\tan\theta = \frac{F_e}{mg} \qquad (3)$$

Since they are equally charged, the magnitude of the electrostatic force is

$$F_e = k\frac{q^2}{r^2} \quad \text{where } k = \frac{1}{4\pi\epsilon_0}$$

Here $r = 2a = 2L\sin\theta$. By substituting these values in equation (3),

$$\tan\theta = k\frac{q^2}{mg\left(2L\sin\theta\right)^2} \qquad (4)$$

Rearranging the equation (4) to get q

$$q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\times0.1\times\sin30^\circ\times\sqrt{\frac{10^{-3}\times10\times\tan30^\circ}{9\times10^9}}$$

$$q = 8.01\times10^{-8}\ \text{C} = 80.1\ \text{nC}$$

EXAMPLE 1.4

Calculate the electrostatic force and gravitational force between the proton and the electron in a hydrogen atom. They are separated by a distance of 5.3×10^{-11} m. The magnitude of charges on the electron and proton are 1.6×10^{-19} C. Mass of the electron is $m_e = 9.1\times10^{-31}$ kg and mass of proton is $m_p = 1.6\times10^{-27}$ kg.

Solution

The proton and the electron attract each other. The magnitude of the electrostatic force between these two particles is given by

$$F_e = \frac{ke^2}{r^2} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(5.3\times10^{-11}\right)^2}$$

$$= \frac{9\times2.56}{28.09}\times10^{-7} = 8.2\times10^{-8}\ \text{N}$$

UNIT-1(XII-Physics_Vol-1).indd 8 14-02-2020 19:32:01

The gravitational force between the proton and the electron is attractive. The magnitude of the gravitational force between these particles is

$$F_G = \frac{Gm_e m_p}{r^2}$$
$$= \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{\left(5.3\times10^{-11}\right)^2}$$
$$= \frac{97.11}{28.09}\times10^{-47} = 3.4\times10^{-47}\ \text{N}$$

The ratio of the two forces

$$\frac{F_e}{F_G} = \frac{8.2\times10^{-8}}{3.4\times10^{-47}} = 2.41\times10^{39}$$

Note that $F_e \approx 10^{39} F_G$

The electrostatic force between a proton and an electron is enormously greater than the gravitational force between them. Thus the gravitational force is negligible when compared with the electrostatic force in many situations such as for small size objects and in the atomic domain. This is the reason why a charged comb attracts an uncharged piece of paper with greater force even though the piece of paper is attracted downward by the Earth. This is shown in Figure 1.3

6I2L9R

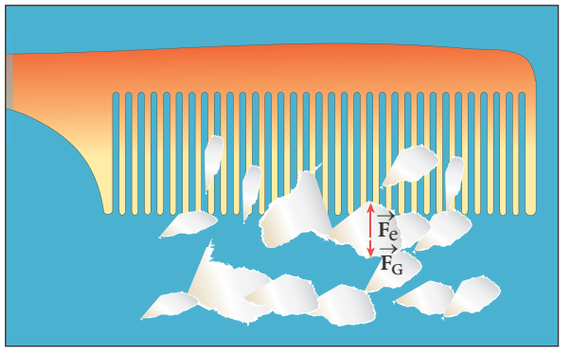


Figure 1.3 Electrostatic attraction between a comb and pieces of papers

1.2.1 Superposition principle

Coulomb's law explains the interaction between two point charges. If there are more than two charges, the force on one charge due to all the other charges needs to be calculated. Coulomb's law alone does not give the answer. The superposition principle explains the interaction between multiple charges.

According to this superposition principle, **the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.**

Consider a system of n charges, namely $q_1, q_2, q_3 \ldots . q_n$. The force on q_1 exerted by the charge q_2

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector from q_2 to q_1 along the line joining the two charges and r_{21} is the distance between the charges q_1 and q_2. The electrostatic force between two charges is not affected by the presence of other charges in the neighbourhood.

The force on q_1 exerted by the charge q_3 is

$$\vec{F}_{13} = k\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31}$$

By continuing this, the total force acting on the charge q_1 due to all other charges is given by

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots\ldots\ldots\vec{F}_{1n}$$

$$\vec{F}_1^{tot} = k\left\{\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \frac{q_1 q_4}{r_{41}^2}\hat{r}_{41} + \ldots \right.$$
$$\left. \ldots. + \frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right\} \qquad (1.3)$$

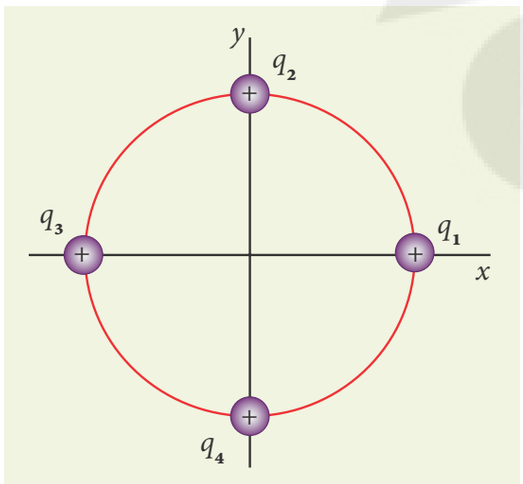
UNIT-1(XII-Physics_Vol-1).indd 9 14-02-2020 19:32:04

Note

Without the superposition principle, Coulomb's law will be incomplete when applied to more than two charges. Both the superposition principle and Coulomb's law form fundamental principles of electrostatics and explain all the phenomena in electrostatics. But they are not derivable from each other.

EXAMPLE 1.5

Consider four equal charges q_1, q_2, q_3 and q_4 = q = +1 μC located at four different points on a circle of radius 1m, as shown in the figure. Calculate the total force acting on the charge q_1 due to all the other charges.

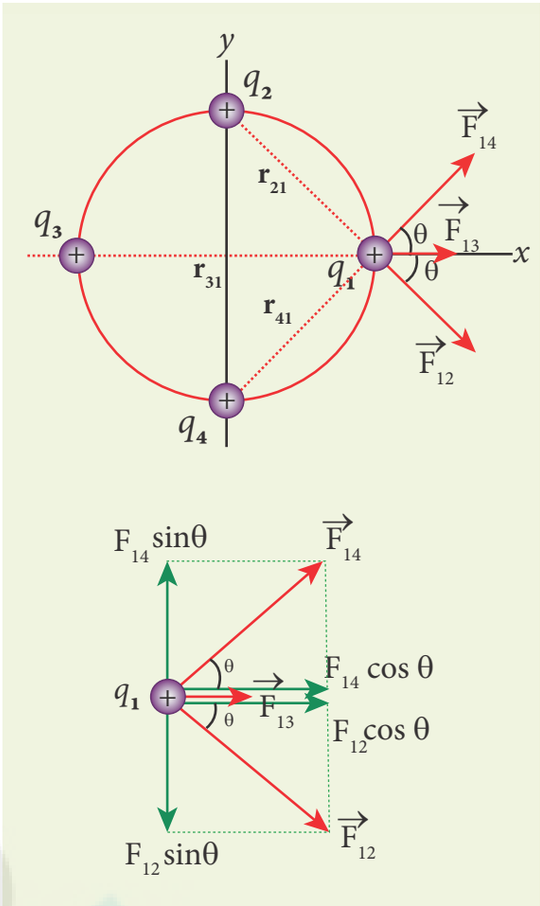


Solution

According to the superposition principle, the total electrostatic force on charge q_1 is the vector sum of the forces due to the other charges,

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

The following diagram shows the direction of each force on the charge q_1.



The charges q_2 and q_4 are equidistant from q_1. As a result the strengths (magnitude) of the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge q_3 is located farther compared to q_2 and q_4. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F}_{13}$ is lesser than that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$. Hence the vector representing the force $\vec{F}_{13}$ is drawn with smaller length compared to that for forces $\vec{F}_{12}$ and $\vec{F}_{14}$.

From the figure, $r_{21} = \sqrt{2}\,\text{m} = r_{41}$ and $r_{31} = 2\,\text{m}$

The magnitudes of the forces are given by

$$F_{13} = \frac{kq^2}{r_{31}^2} = \frac{9\times10^9\times10^{-12}}{4}$$

UNIT-1(XII-Physics_Vol-1).indd 10 14-02-2020 19:32:07

$F_{13} = 2.25 \times 10^{-3}$ N

$$F_{12} = \frac{kq^2}{r_{21}^2} = F_{14} = \frac{9\times10^{9}\times10^{-12}}{2}$$

$$= 4.5 \times 10^{-3} \text{ N}$$

From the figure, the angle $\theta = 45^{o}$. In terms of the components, we have

$$\vec{F}_{12} = F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\,\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} - 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{F}_{13} = F_{13}\hat{i} = 2.25\times10^{-3}\text{N}\hat{i}$$

$$\vec{F}_{14} = F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\,\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} + 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

Then the total force on q_1 is

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\,\hat{j}\right) + F_{13}\hat{i} + \left(F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\,\hat{j}\right)$$

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i} + \left(-F_{12}\sin\theta + F_{14}\sin\theta\right)\hat{j}$$

Since $F_{12} = F_{14}$, the jth component is zero. Hence we have

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i}$$

substituting the values in the above equation,

$$= \left(\frac{4.5}{\sqrt{2}} + 2.25 + \frac{4.5}{\sqrt{2}}\right)\times10^{-3}\hat{i}$$

$$= \left(4.5\sqrt{2} + 2.25\right)\times10^{-3}\hat{i}$$

$$\vec{F}_1^{tot} = 8.61\times10^{-3}\hat{i}\text{ N}$$

The resultant force is along the positive x axis.

1.3 ELECTRIC FIELD AND ELECTRIC FIELD LINES

1.3.1 Electric Field

The interaction between two charges is determined by Coulomb's law. How does the interaction itself occur? Consider a point charge kept at a point in space. If another point charge is placed at some distance from the first point charge, it experiences either an attractive force or repulsive force. This is called 'action at a distance'. But how does the second charge know about existence of the first charge which is located at some distance away from it? To answer this question, Michael Faraday introduced the **concept of field**.

According to Faraday, every charge in the universe creates an electric field in the surrounding space, and if another charge is brought into its field, it will interact with the electric field at that point and will experience a force. It may be recalled that the interaction of two masses is similarly explained using the concept of gravitational field (Refer unit 6, volume 2, XI physics). Both the electric and gravitational forces are non-contact forces, hence the field concept is required to explain action at a distance.

Consider a source point charge q located at a point in space. Another point charge q_o (test charge) is placed at some point P which is at a distance r from the charge q. The electrostatic force experienced by the charge q_o due to q is given by Coulomb's law.

$$\vec{F} = \frac{kqq_0}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r} \text{ where } k = \frac{1}{4\pi\epsilon_0}$$

The charge q creates an electric field in the surrounding space within which its effect can be felt by another charge. It is measured in

UNIT-1(XII-Physics_Vol-1).indd 11 14-02-2020 19:32:11

terms of a quantity called electric field intensity or simply called electric field $\vec{E}$. **The electric field at the point P at a distance *r* from the point charge *q* is defined as the force that would be experienced by a unit positive charge placed at that point P** and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad (1.4)$$

Here $\hat{r}$ is the unit vector pointing from q to the point of interest P. The electric field is a vector quantity and its SI unit is newton per coulomb (NC^{-1}).

Important aspects of Electric field

(i) If the charge q is positive then the electric field points away from the source charge and if q is negative, the electric field points towards the source charge q. This is shown in the Figure 1.4.

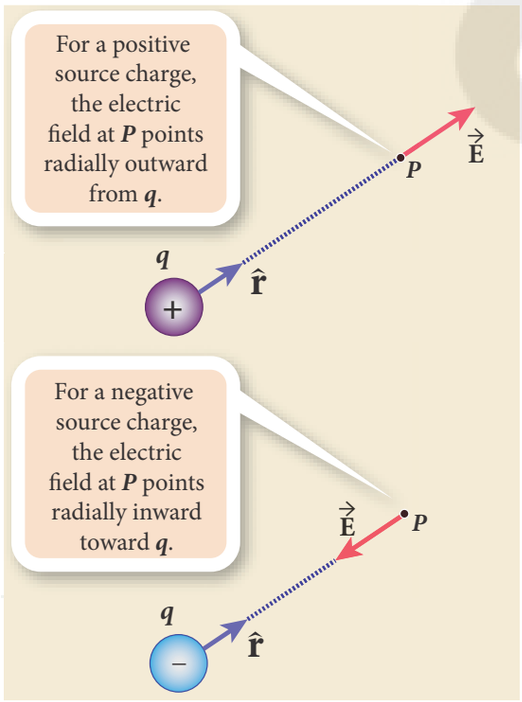


Figure 1.4 Electric field of positive and negative charges

(ii) If the electric field at a point P is $\vec{E}$, then the force experienced by the test charge q_o placed at the point P is

$$\vec{F} = q_0\vec{E} \qquad (1.5)$$

This is Coulomb's law in terms of electric field. This is shown in Figure 1.5

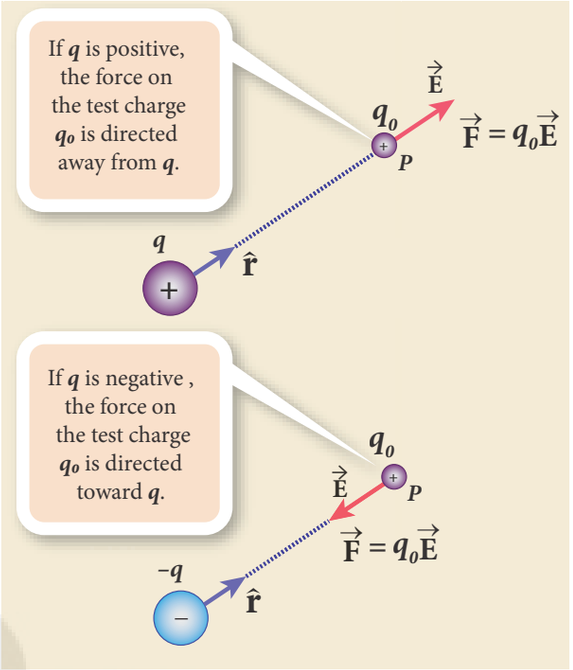


Figure 1.5 Coulomb's law in terms of electric field

(iii) The equation (1.4) implies that the electric field is independent of the test charge q_o and it depends only on the source charge q.

(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures 1.6(a) and (b). From equation (1.4), we can infer that as distance increases, the electric field decreases in magnitude.

Note that in Figures 1.6 (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the points Q and R because the point P is closer to the source charge.

UNIT-1(XII-Physics_Vol-1).indd 12 14-02-2020 19:32:13

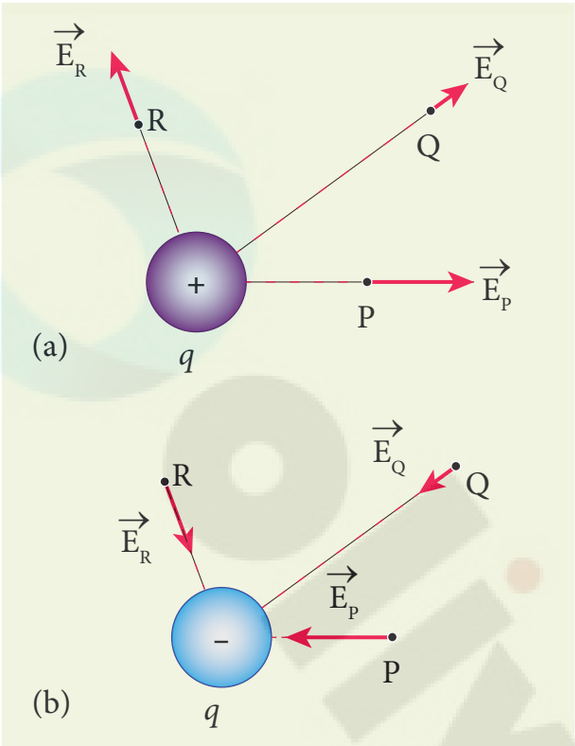


Figure 1.6 (a) Electric field due to positive charge (b) Electric field due to negative charge

(v) In the definition of electric field, it is assumed that the test charge q_0 is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

(vi) The expression (1.4) is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used (Refer Appendix A1.1). However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge. Note that we similarly treat the Earth as a point mass when we calculate the gravitational field of the Sun on the Earth (Refer unit 6, volume 2, XI physics).

(vii) There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases. These are shown in Figure 1.7.

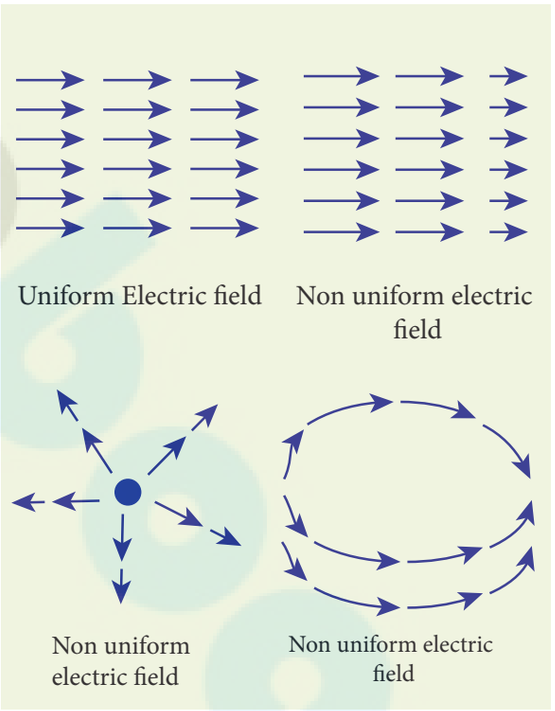


Figure 1.7 Uniform and non-uniform electric field

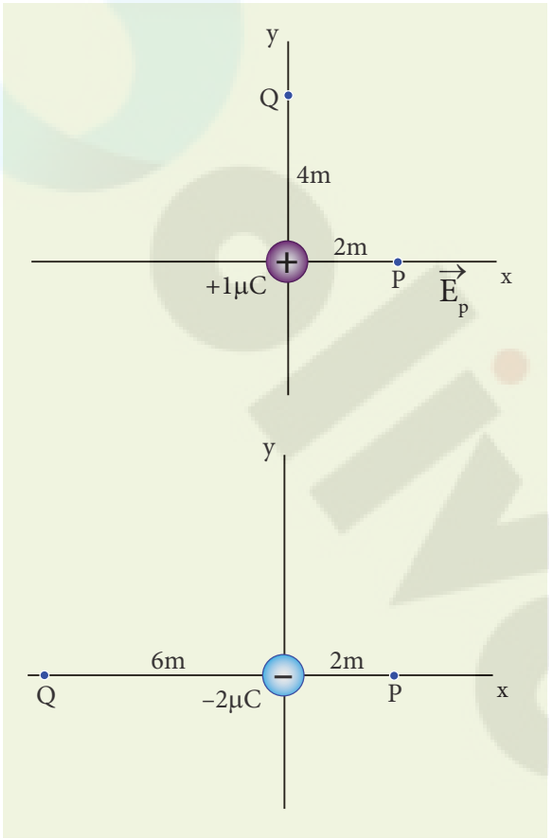
EXAMPLE 1.6

Calculate the electric field at points P, Q for the following two cases, as shown in the figure.

UNIT-1(XII-Physics_Vol-1).indd 13 14-02-2020 19:32:13

(a) A positive point charge +1 μC is placed at the origin

(b) A negative point charge −2 μC is placed at the origin



Solution

Case (a)

The magnitude of the electric field at point P is

$$E_P = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times1\times10^{-6}}{4}$$
$$= 2.25\times10^3\,\text{NC}^{-1}$$

Since the source charge is positive, the electric field points away from the charge. So the electric field at the point P is given by

$$\vec{E}_P = 2.25\times10^3\,\hat{i}\,\text{NC}^{-1}$$

For the point Q

$$|\vec{E}_Q| = \frac{9\times10^9\times1\times10^{-6}}{16} = 0.56\times10^3\,\text{NC}^{-1}$$

Hence $\vec{E}_Q = 0.56\times10^3\,\hat{j}\ \text{NC}^{-1}$

Case (b)

The magnitude of the electric field at point P

$$|\vec{E}_P| = \frac{kq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times2\times10^{-6}}{4}$$
$$= 4.5\times10^3\,\text{NC}^{-1}$$

Since the source charge is negative, the electric field points towards the charge. So the electric field at the point P is given by

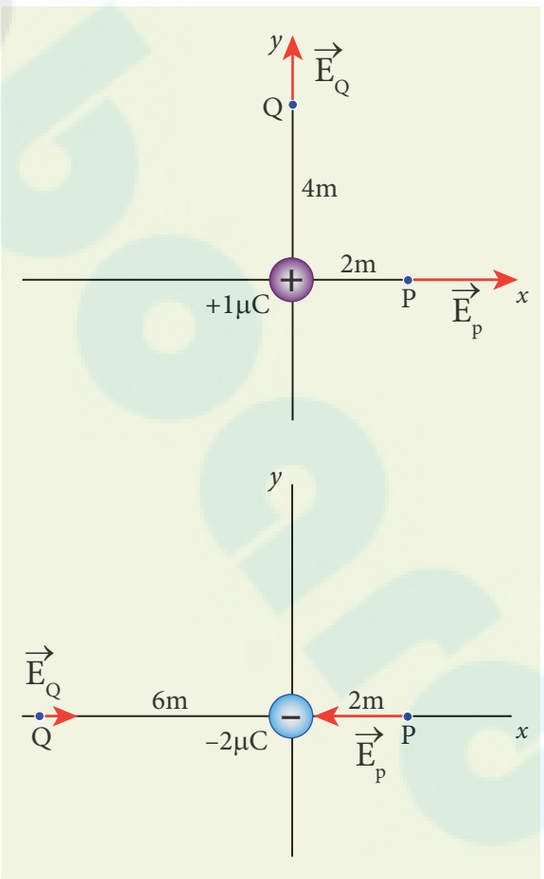
$$\vec{E}_P = -4.5\times10^3\,\hat{i}\ \text{NC}^{-1}$$

For the point Q, $|\vec{E}_Q| = \frac{9\times10^9\times2\times10^{-6}}{36}$

$$= 0.5\times10^3\,\text{NC}^{-1}$$

$$\vec{E}_Q = 0.5\times10^3\,\hat{i}\ \text{NC}^{-1}$$

At the point Q the electric field is directed along the positive *x*-axis.



UNIT-1(XII-Physics_Vol-1).indd 14 14-02-2020 19:32:16

1.3.2 Electric field due to the system of point charges

Suppose a number of point charges are distributed in space. To find the electric field at some point P due to this collection of point charges, superposition principle is used. **The electric field at an arbitrary point due to a collection of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields**.

Consider a collection of point charges $q_1, q_2, q_3 \ldots\ldots\ldots q_n$ located at various points in space. The total electric field at some point P due to all these *n* charges is given by

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots + \vec{E}_n \qquad (1.6)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \frac{q_3}{r_{3P}^2}\hat{r}_{3P} + \ldots \ldots + \frac{q_n}{r_{nP}^2}\hat{r}_{nP}\right\} \qquad (1.7)$$

where $r_{1P}, r_{2P}, r_{3P} \ldots\ldots\ldots r_{nP}$ are the distance of the the charges $q_1, q_2, q_3 \ldots\ldots\ldots q_n$ from the point P respectively. Also $\hat{r}_{1P}, \hat{r}_{2P}, \hat{r}_{3P} \ldots\ldots\ldots \hat{r}_{nP}$ are the corresponding unit vectors directed from $q_1, q_2, q_3 \ldots\ldots\ldots q_n$ to P. Equation (1.7) can be re-written as,

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_\circ}\sum_{i=1}^{n}\left(\frac{q_i}{r_{iP}^2}\hat{r}_{iP}\right) \qquad (1.8)$$

For example in Figure 1.8, the resultant electric field due to three point charges q_1, q_2, q_3 at point P is shown.

Note that the relative lengths of the electric field vectors for the charges depend on relative distances of the charges to the point P.

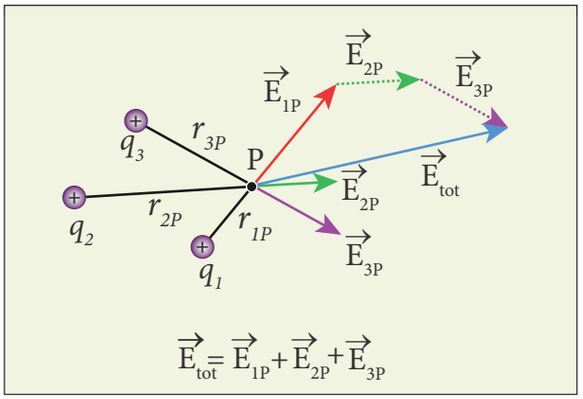
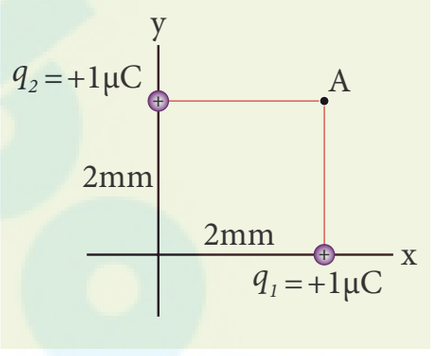


Figure 1.8 Superposition of Electric field

EXAMPLE 1.7

Consider the charge configuration as shown in the figure. Calculate the electric field at point A. If an electron is placed at points A, what is the acceleration experienced by this electron? (mass of the electron = 9.1×10^{-31} kg and charge of electron = -1.6×10^{-19} C)



Solution

By using superposition principle, the net electric field at point A is

$$\vec{E}_A = \frac{1}{4\pi\epsilon_\circ}\frac{q_1}{r_{1A}^2}\hat{r}_{1A} + \frac{1}{4\pi\epsilon_\circ}\frac{q_2}{r_{2A}^2}\hat{r}_{2A}$$

where r_{1A} and r_{2A} are the distances of point A from the two charges respectively.

$$\vec{E}_A = \frac{9\times10^9\times1\times10^{-6}}{\left(2\times10^{-3}\right)^2}\left(\hat{j}\right) + \frac{9\times10^9\times1\times10^{-6}}{\left(2\times10^{-3}\right)^2}\left(\hat{i}\right)$$

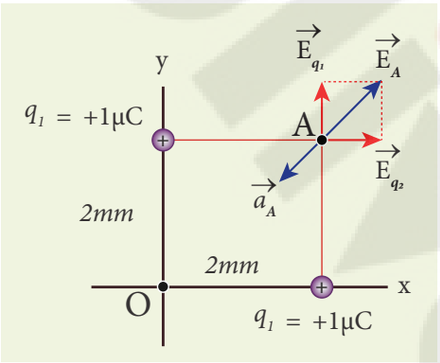
UNIT-1(XII-Physics_Vol-1).indd 15 14-02-2020 19:32:19

$$= 2.25\times10^{9}\ \hat{j} + 2.25\times10^{9}\hat{i} = 2.25\times10^{9}\left(\hat{i}+\hat{j}\right)$$

The magnitude of electric field

$$\left|\vec{E}_A\right| = \sqrt{\left(2.25\times10^{9}\right)^2 + \left(2.25\times10^{9}\right)^2}$$
$$= 2.25\times\sqrt{2}\times10^{9}\ \mathrm{NC}^{-1}$$

The direction of $\vec{E}_A$ is given by $\frac{\vec{E}_A}{\left|\vec{E}_A\right|} = \frac{2.25\times10^{9}\left(\hat{i}+\hat{j}\right)}{2.25\times\sqrt{2}\times10^{9}} = \frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$ which is the unit vector along OA as shown in the figure.



The acceleration experienced by an electron placed at point A is

$$\vec{a}_A = \frac{\vec{F}}{m} = \frac{q\vec{E}_A}{m}$$
$$= \frac{\left(-1.6\times10^{-19}\right)\times\left(2.25\times10^{9}\right)\left(\hat{i}+\hat{j}\right)}{9.1\times10^{-31}}$$
$$= -3.95\times10^{20}\left(\hat{i}+\hat{j}\right)\mathrm{N\ kg}^{-1}$$

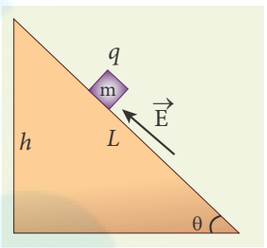
The electron is accelerated in a direction exactly opposite to $\vec{E}_A$.

1.3.3 Electric field due to continuous charge distribution

The electric charge is quantized microscopically. The expressions (1.2), (1.3), (1.4) are applicable to only point charges. While dealing with the electric field due to a charged sphere or a charged wire etc., it is very difficult to look at individual charges in these charged bodies. Therefore, it is assumed that charge is distributed continuously on the charged bodies and the discrete nature of charges is not considered here. The electric field due to such continuous charge distributions is found by invoking the method of calculus. (For further reading, refer Appendix A1.1).

EXAMPLE 1.8

A block of mass m carrying a positive charge q is placed on an insulated frictionless inclined plane as shown in the figure. A uniform electric field E is applied parallel to the inclined surface such that the block is at rest. Calculate the magnitude of the electric field E.



Solution

Note: A similar problem is solved in XI[th] Physics volume I, unit 3 section 3.3.2.
There are three forces that acts on the mass m:

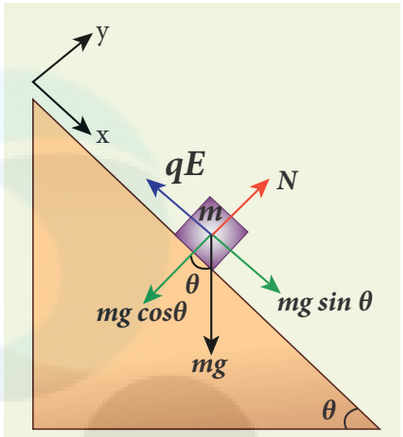
(i) The downward gravitational force exerted by the Earth (mg)

(ii) The normal force exerted by the inclined surface (N)

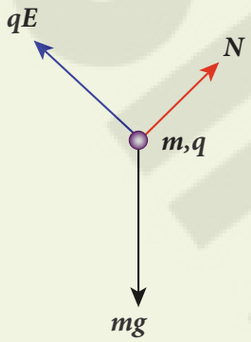
(iii) The Coulomb force given by uniform electric field (qE)

The free body diagram for the mass m is drawn below.

UNIT-1(XII-Physics_Vol-1).indd 16 14-02-2020 19:32:21



Forces acting on the mass m



Free body diagram

A convenient inertial coordinate system is located in the inclined surface as shown in the figure. The mass m has zero net acceleration both in x and y-direction.

Along x-direction, applying Newton's second law, we have

$$mg\sin\theta\hat{i} - qE\hat{i} = 0$$

$$mg\sin\theta - qE = 0$$

$$E = \frac{mg\sin\theta}{q}$$

Note that the magnitude of the electric field is directly proportional to the mass m and inversely proportional to the charge q. It implies that, if the mass is increased by keeping the charge constant, then a strong electric field is required to stop the object from sliding. If the charge is increased by keeping the mass constant, then a weak electric field is sufficient to stop the mass from sliding down the plane.

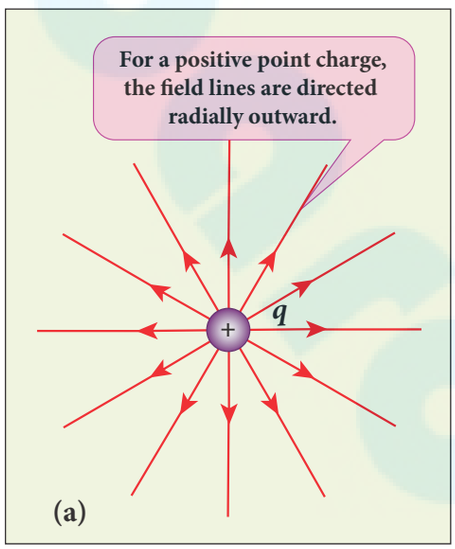
The electric field also can be expressed in terms of height and the length of the inclined surface of the plane.

$$E = \frac{mg\,h}{qL}$$

1.3.4 Electric field lines

Electric field vectors are visualized by the concept of electric field lines. They form a set of continuous lines which are the visual representation of the electric field in some region of space. The following rules are followed while drawing electric field lines for charges.

- The electric field lines start from a positive charge and end at negative charges or at infinity. For a positive point charge the electric field lines point radially outward and for a negative point charge, the electric field lines point radially inward. These are shown in Figure 1.9 (a) and (b).



UNIT-1(XII-Physics_Vol-1).indd 17 14-02-2020 19:32:23

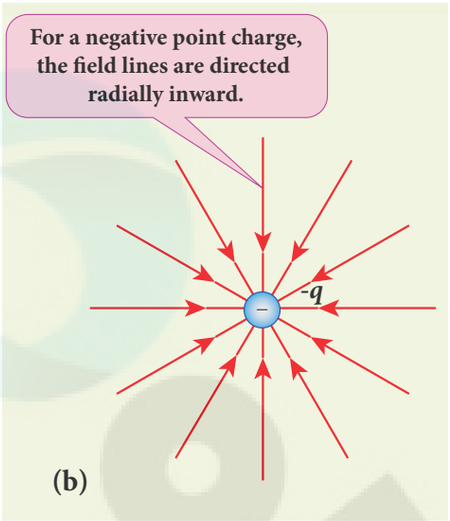


Figure 1.9 Electric field lines for isolated positive and negative charges

Note that for an isolated positive point charge the electric field line starts from the charge and ends only at infinity. For an isolated negative point charge the electric field lines start at infinity and end at the negative charge.

- The electric field vector at a point in space is tangential to the electric field line at that point. This is shown in Figure 1.10

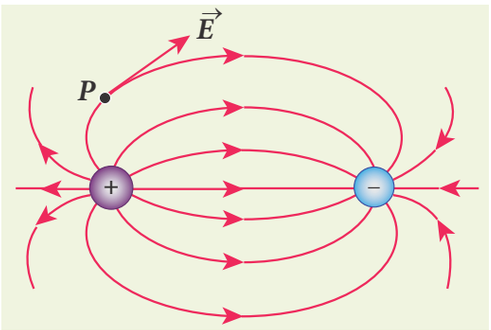


Figure 1.10 Electric field at a point P

- The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region. This is shown in Figure1.11

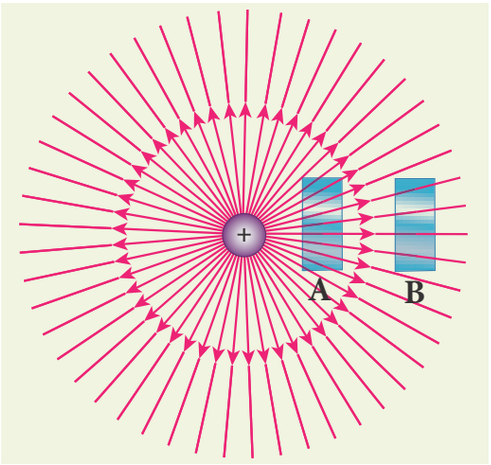


Figure 1.11 Electric field has larger magnitude at surface A than B

Figure 1.11 shows electric field lines from a positive point charge. The magnitude of the electric field for a point charge decreases as the distance increases $\left(|\vec{E}| \propto \frac{1}{r^2}\right)$. So the electric field has greater magnitude at the surface A than at B. Therefore, the number of lines crossing the surface A is greater than the number of lines crossing the surface B. Note that at surface B the electric field lines are farther apart compared to the electric field lines at the surface A.

- No two electric field lines intersect each other. If two lines cross at a point, then there will be two different electric field vectors at the same point, as shown in Figure 1.12.

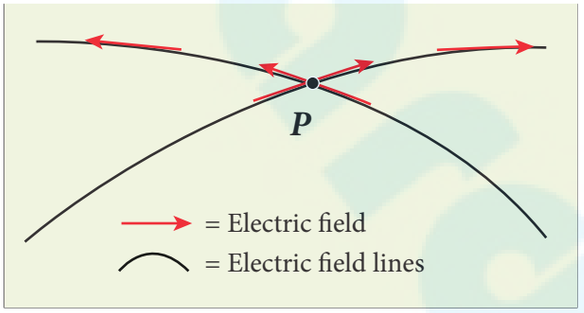


Figure 1.12 Two electric field lines never intersect each other

UNIT-1(XII-Physics_Vol-1).indd 18 14-02-2020 19:32:24

As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

- The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

For example in the Figure 1.13, the electric field lines are drawn for charges $+q$ and $-2q$. Note that the number of field lines emanating from $+q$ is 8 and the number of field lines ending at $-2q$ is 16. Since the magnitude of the second charge is twice that of the first charge, the number of field lines drawn for $-2q$ is twice in number than that for charge $+q$.

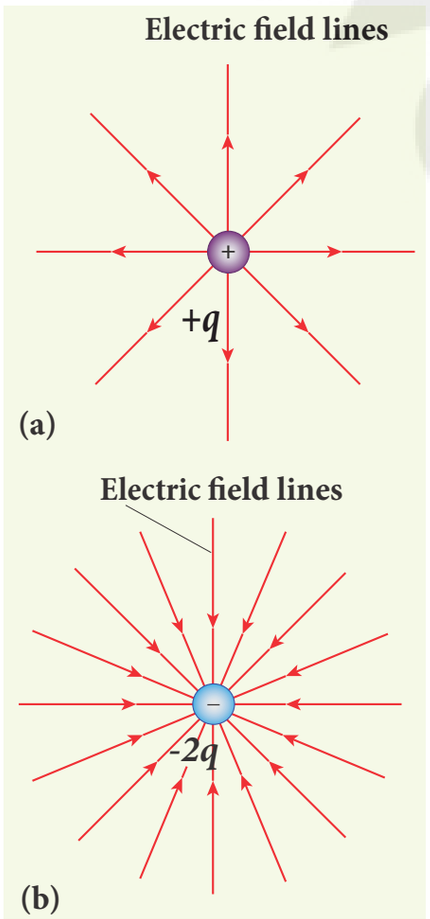
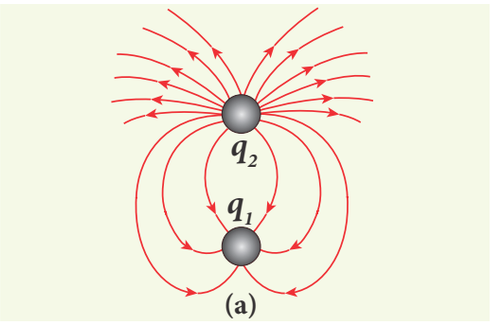
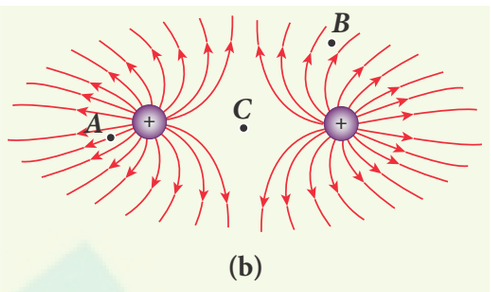


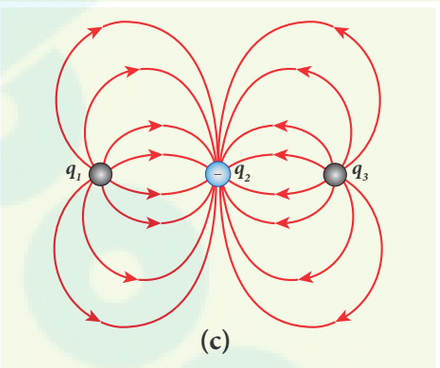
Figure 1.13 Electric field lines and magnitude of the charge

EXAMPLE 1.9

The following pictures depict electric field lines for various charge configurations.







(i) In figure (a) identify the signs of two charges and find the ratio $\left|\frac{q_1}{q_2}\right|$

(ii) In figure (b), calculate the ratio of two positive charges and identify the strength of the electric field at three points A, B, and C

(iii) Figure (c) represents the electric field lines for three charges. If $q_2 = -20$ nC, then calculate the values of q_1 and q_3

UNIT-1(XII-Physics_Vol-1).indd 19 14-02-2020 19:32:24

Solution

(i) The electric field lines start at q_2 and end at q_1. In figure (a), q_2 is positive and q_1 is negative. The number of lines starting from q_2 is 18 and number of the lines ending at q_1 is 6. So q_2 has greater magnitude. The ratio of $\left|\frac{q_1}{q_2}\right| = \frac{N_1}{N_2} = \frac{6}{18} = \frac{1}{3}$. It implies that $|q_2| = 3|q_1|$

(ii) In figure (b), the number of field lines emanating from both positive charges are equal (N=18). So the charges are equal. At point A, the electric field lines are denser compared to the lines at point B. So the electric field at point A is greater in magnitude compared to the field at point B. Further, no electric field line passes through C, which implies that the resultant electric field at C due to these two charges is zero.

(iii) In the figure (c), the electric field lines start at q_1 and q_3 and end at q_2. This implies that q_1 and q_3 are positive charges. The ratio of the number of field lines is $\left|\frac{q_1}{q_2}\right| = \frac{8}{16} = \left|\frac{q_3}{q_2}\right| = \frac{1}{2}$, implying that q_1 and q_3 are half of the magnitude of q_2. So $q_1 = q_3 = +10$ nC.

1.4 ELECTRIC DIPOLE AND ITS PROPERTIES

1.4.1 Electric dipole

Two equal and opposite charges separated by a small distance constitute an electric dipole. In many molecules, the centres of positive and negative charge do not coincide. Such molecules behave as permanent dipoles. Examples: CO, water, ammonia, HCl etc.

Consider two equal and opposite point charges $(+q, -q)$ that are separated by a distance $2a$ as shown in Figure 1.14(a).
The electric dipole moment is defined as

$$\vec{p} = q\vec{r}_+ + (-q)\vec{r}_- \qquad (1.9)$$

where $\vec{r}_+$ is the position vector of $+q$ from the origin and $\vec{r}_-$ is the position vector of $-q$ from the origin. Then, from Figure 1.14 (a),

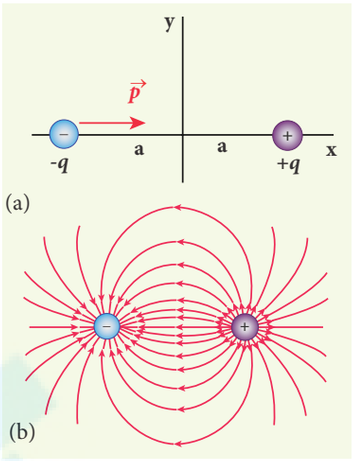


Figure 1.14 (a) Electric dipole (b) Electric field lines for the electric dipole

$$\vec{p} = qa\hat{i} - qa(-\hat{i}) = 2qa\hat{i} \qquad (1.10)$$

The electric dipole moment vector lies along the line joining two charges and is directed from $-q$ to $+q$. The SI unit of dipole moment is coulomb metre (Cm). The electric field lines for an electric dipole are shown in Figure 1.14 (b).

- For simplicity, the two charges are placed on the x-axis. Even if the two charges are placed on y or z-axis, dipole moment will point from $-q$ to $+q$.
- The magnitude of the electric dipole moment is equal to the product of the magnitude of one of the charges and the distance between them,

UNIT-1(XII-Physics_Vol-1).indd 20 14-02-2020 19:32:26

$$|\vec{p}| = 2qa \qquad (1.11)$$

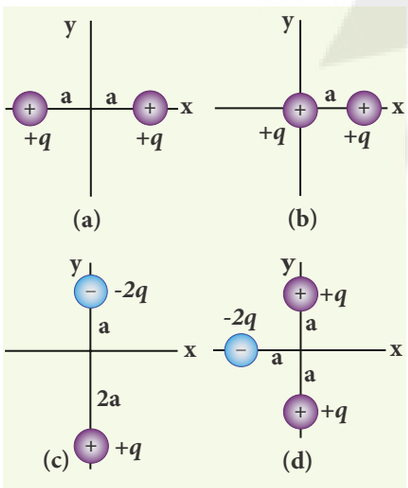
• Though the electric dipole moment for two equal and opposite charges is defined, it is possible to define and calculate the electric dipole moment for a collection of point charges. The electric dipole moment for a collection of n point charges is given by

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i \qquad (1.12)$$

where $\vec{r}_i$ is the position vector of charge q_i from the origin.

EXAMPLE 1.10

Calculate the electric dipole moment for the following charge configurations.



Solution

Case (a) The position vector for the $+q$ on the positive x-axis is $a\hat{i}$ and position vector for the $+q$ charge the negative x axis is $-a\hat{i}$. So the dipole moment is,

$$\vec{p} = (+q)(a\hat{i}) + (+q)(-a\hat{i}) = 0$$

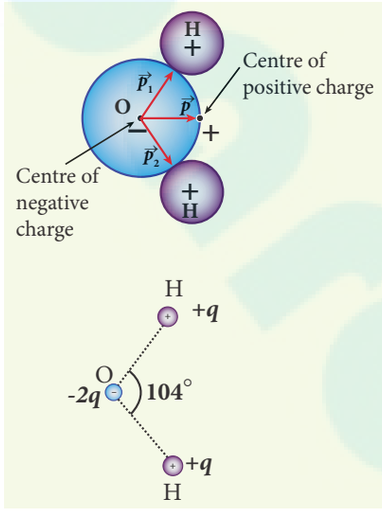
Case (b) In this case one charge is placed at the origin, so its position vector is zero. Hence only the second charge $+q$ with position vector $a\hat{i}$ contributes to the dipole moment, which is $\vec{p} = qa\hat{i}$.

From both cases (a) and (b), we can infer that in general the electric dipole moment depends on the choice of the origin and charge configuration. But for one special case, the electric dipole moment is independent of the origin. If the total charge is zero, then the electric dipole moment will be the same irrespective of the choice of the origin. It is because of this reason that the electric dipole moment of an electric dipole (total charge is zero) is always directed from $-q$ to $+q$, independent of the choice of the origin.

Case (c) $\vec{p} = (-2q)a\hat{j} + q(2a)(-\hat{j}) = -4qa\hat{j}$. Note that in this case $\vec{p}$ is directed from $-2q$ to $+q$.

Case (d) $\vec{p} = -2qa(-\hat{i}) + qa\hat{j} + qa(-\hat{j})$
$= 2qa\hat{i}$

The water molecule (H_2O) has this charge configuration. The water molecule has three atoms (two H atom and one O atom). The centres of positive (H) and negative (O) charges of a water molecule lie at different points, hence it possess permanent dipole moment. The electric dipole moment $\vec{p}$ is directed from centre of negative charge to the centre of positive charge, as shown in the figure.



UNIT-1(XII-Physics_Vol-1).indd 21 14-02-2020 19:32:28

1.4.2 Electric field due to a dipole

Case (i) Electric field due to an electric dipole at points on the axial line

Consider an electric dipole placed on the x-axis as shown in Figure 1.15. A point C is located at a distance of r from the midpoint O of the dipole on the axial line.

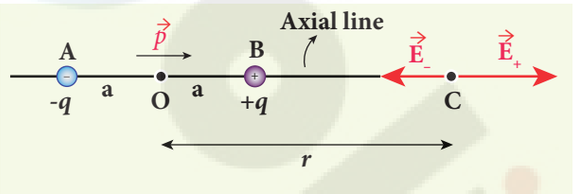


Figure 1.15 Electric field of the dipole along the axial line

The electric field at a point C due to $+q$ is

$$\vec{E}_{+} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} \text{ along BC}$$

Since the electric dipole moment vector $\vec{p}$ is from $-q$ to $+q$ and is directed along BC, the above equation is rewritten as

$$\vec{E}_{+} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\hat{p} \qquad (1.13)$$

where $\hat{p}$ is the electric dipole moment unit vector from $-q$ to $+q$.

The electric field at a point C due to $-q$ is

$$\vec{E}_{-} = -\frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\hat{p} \qquad (1.14)$$

Since $+q$ is located closer to the point C than $-q$, $\vec{E}_{+}$ is stronger than $\vec{E}_{-}$. Therefore, the length of the $\vec{E}_{+}$ vector is drawn larger than that of $\vec{E}_{-}$ vector.

The total electric field at point C is calculated using the superposition principle of the electric field.

$$\vec{E}_{tot} = \vec{E}_{+} + \vec{E}_{-}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p} \qquad (1.15)$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p} \qquad (1.16)$$

Note that the total electric field is along $\vec{E}_{+}$, since $+q$ is closer to C than $-q$. The direction of $\vec{E}_{tot}$ is shown in Figure 1.16.

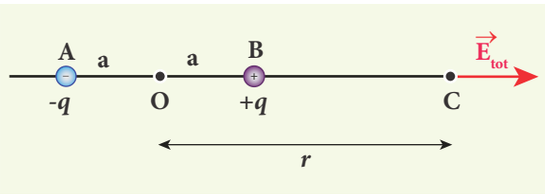


Figure 1.16 Total electric field of the dipole on the axial line

If the point C is very far away from the dipole ($r >> a$). Then under this limit the term $(r^2-a^2)^2 \approx r^4$. Substituting this into equation (1.16), we get

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p} \quad (r >> a)$$

since $2aq\,\hat{p} = \vec{p}$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad (r >> a) \qquad (1.17)$$

If the point C is chosen on the left side of the dipole, the total electric field is still in the direction of $\vec{p}$. We infer this result by examining the electric field lines of the dipole shown in Figure 1.14(b).

Case (ii) Electric field due to an electric dipole at a point on the equatorial plane

Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in Figure 1.17. Since the point C is equi-distant from $+q$ and $-q$, the magnitude of the electric fields at C due to $+q$ and $-q$ are the same. The direction

UNIT-1(XII-Physics_Vol-1).indd 22 14-02-2020 19:32:33

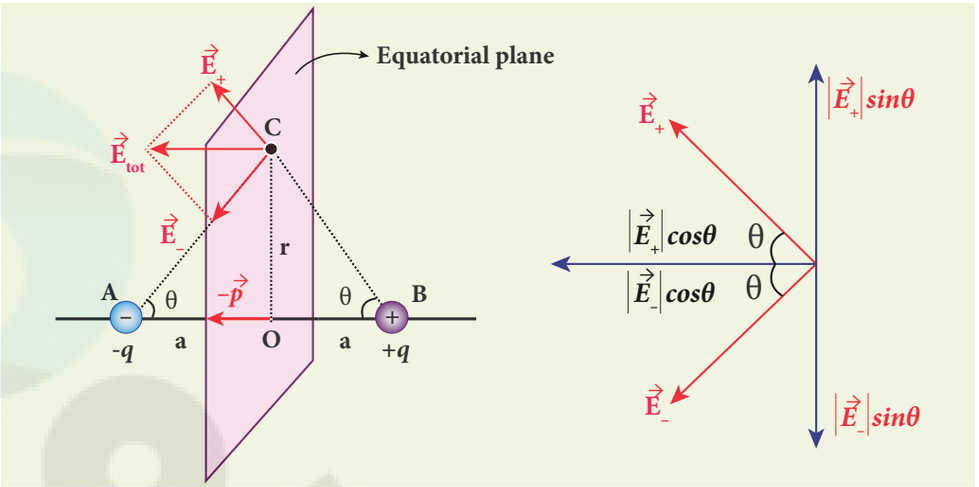


Figure 1.17 Electric field due to a dipole at a point on the equatorial plane

of $\vec{E}_+$ is along BC and the direction of $\vec{E}_-$ is along CA. $\vec{E}_+$ and $\vec{E}_-$ can be resolved into two components; one component parallel to the dipole axis and the other perpendicular to it. Since perpendicular components $|\vec{E}_+|\sin\theta$ and $|\vec{E}_-|\sin\theta$ are eqaul in magnitude and oppositely directed, they cancel each other. The magnitude of the total electric field at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$ as shown in the Figure 1.17.

$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\,\hat{p} - |\vec{E}_-|\cos\theta\,\hat{p} \qquad (1.18)$$

The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the same and are given by

$$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\epsilon_0}\frac{q}{\left(r^2+a^2\right)} \qquad (1.19)$$

By substituting equation (1.19) into equation (1.18), we get

$$\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_0}\frac{2q\cos\theta}{\left(r^2+a^2\right)}\hat{p}$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{2qa}{\left(r^2+a^2\right)^{\frac{3}{2}}}\hat{p}$$

since $\cos\theta = \dfrac{a}{\sqrt{r^2+a^2}}$

$$\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{\left(r^2+a^2\right)^{\frac{3}{2}}}$$

since $\vec{p} = 2qa\hat{p}$ (1.20)

At very large distances ($r >> a$), the equation (1.20) becomes

$$\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \qquad (r >> a) \quad (1.21)$$

Important inferences

(i) From equations (1.17) and (1.21), it is inferred that for very large distances, the magnitude of the electric field at point on the dipole axis is twice the magnitude of the electric field at the point at the same distance on the equatorial plane. The direction of the electric field at points on the dipole axis is directed along the direction of dipole moment vector $\vec{p}$ but at points on the equatorial plane it is directed opposite to the dipole moment vector, that is along $-\vec{p}$.

(ii) At very large distances, the electric field due to a dipole varies as $\frac{1}{r^3}$. Note that for a point charge, the electric field

UNIT-1(XII-Physics_Vol-1).indd 23 14-02-2020 19:32:37

varies as $\frac{1}{r^2}$. This implies that the electric field due to a dipole at very large distances goes to zero faster than the electric field due to a point charge. The reason for this behavior is that at very large distance, the two charges appear to be close to each other and neutralize each other.

(iii) The equations (1.17) and (1.21) are valid only at very large distances ($r>>a$). Suppose the distance $2a$ approaches zero and q approaches infinity such that the product of $2aq = p$ is finite, then the dipole is called a point dipole. For such point dipoles, equations (1.17) and (1.21) are exact and hold true for any r.

1.4.3 Torque experienced by an electric dipole in the uniform electric field

Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ whose field lines are equally spaced and point in the same direction. The charge $+q$ will experience a force $q\vec{E}$ in the direction of the field and charge $-q$ will experience a force $-q\vec{E}$ in a direction opposite to the field. Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque as shown in Figure 1.18. This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction).

The total torque on the dipole about the point O

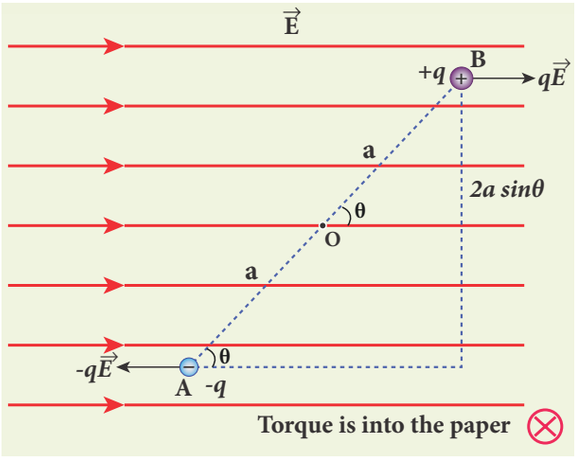


Figure 1.18 Torque on dipole

$$\vec{\tau} = \overrightarrow{OA} \times \left(-q\vec{E}\right) + \overrightarrow{OB} \times q\vec{E} \qquad (1.22)$$

Using right-hand corkscrew rule (Refer XI, volume 1, unit 2), it is found that total torque is perpendicular to the plane of the paper and is directed into it.

The magnitude of the total torque $\tau = \left|\overrightarrow{OA}\right|\left|\left(-q\vec{E}\right)\right|\sin\theta + \left|\overrightarrow{OB}\right|\left|q\vec{E}\right|\sin\theta$

$$\tau = qE \cdot 2a\sin\theta \qquad (1.23)$$

where θ is the angle made by $\vec{p}$ with $\vec{E}$. Since $p = 2aq$, the torque is written in terms of the vector product as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad (1.24)$$

The magnitude of this torque is $\tau = pE\sin\theta$ and is maximum when $\theta = 90^\circ$.

This torque tends to rotate the dipole and align it with the electric field $\vec{E}$. Once $\vec{p}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero.

If the electric field is not uniform, then the force experienced by $+q$ is different from that experienced by $-q$. In addition to the torque, there will be net force acting on the dipole. This is shown in Figure 1.19.

UNIT-1(XII-Physics_Vol-1).indd 24 14-02-2020 19:32:39

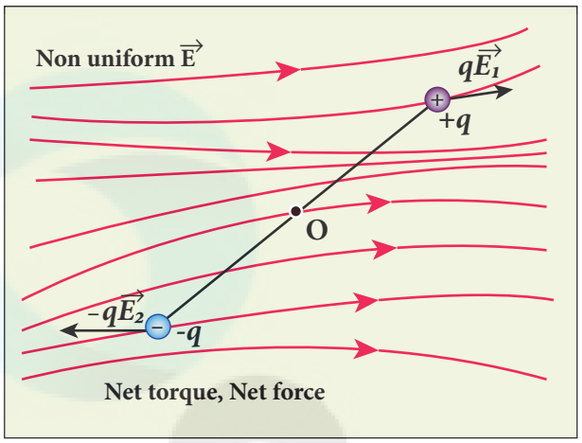


Figure 1.19 The dipole in a non-uniform electric field

EXAMPLE 1.11

A sample of HCl gas is placed in a uniform electric field of magnitude 3×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Calculate the maximum torque experienced by each HCl molecule.

Solution

The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

$$\tau_{max} = pE\sin 90^\circ = 3.4\times10^{-30}\times3\times10^4$$
$$\tau_{max} = 10.2\times10^{-26}\,\text{N m}$$

1.5 ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Introduction

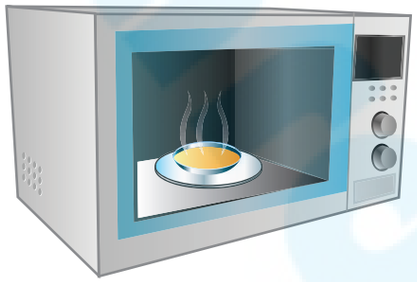
In mechanics, potential energy is defined for conservative forces. Since gravitational force is a conservative force, its gravitational potential energy is defined in XI standard physics (Unit 6). Since Coulomb force is an inverse-square-law force, its also a conservative force like gravitational force. Therefore, we can define potential energy for charge configurations.

1.5.1 Electrostatic Potential energy and Electrostatic potential

Consider a positive charge q kept fixed at the origin which produces an electric field $\vec{E}$ around it. A positive test charge q' is brought from point R to point P against the repulsive force between q and q' as shown in Figure 1.20. Work must be done to overcome the repulsion between the charges and this work done is stored as potential energy of the system.



Microwave oven works on the principle of torque acting on an electric dipole. The food we consume has water molecules which are permanent electric dipoles. Oven produces microwaves that are oscillating electromagnetic fields and produce torque on the water molecules. Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy. Thus, heat generated is used to heat the food.



UNIT-1(XII-Physics_Vol-1).indd 25 14-02-2020 19:32:40

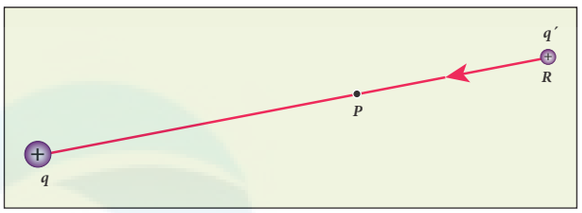


Figure 1.20 Work done is equal to potential energy

The test charge q' is brought from R to P with constant velocity which means that external force used to bring the test charge q' from R to P must be equal and opposite to the coulomb force $\left(\vec{F}_{ext} = -\vec{F}_{coulomb}\right)$. The work done is

$$W = \int_R^P \vec{F}_{ext} \cdot d\vec{r} \qquad (1.25)$$

Since coulomb force is conservative, work done is independent of the path and it depends only on the initial and final positions of the test charge. If potential energy associated with q' at P is U_P and that at R is U_R, then difference in potential energy is defined as the work done to bring a test charge q' from point R to P and is given as $U_P - U_R = W = \Delta U$

$$\Delta U = \int_R^P \vec{F}_{ext} \cdot d\vec{r} \qquad (1.26)$$

$$\text{Since } \vec{F}_{ext} = -\vec{F}_{coulomb} = -q'\vec{E} \qquad (1.27)$$

$$\Delta U = \int_R^P -\left(q'\vec{E}\right) \cdot d\vec{r} = q' \int_R^P \left(-\vec{E}\right) \cdot d\vec{r} \qquad (1.28)$$

The potential energy difference per unit charge is given by

$$\frac{\Delta U}{q'} = \frac{q' \int_R^P \left(-\vec{E}\right) \cdot d\vec{r}}{q'} = -\int_R^P \vec{E} \cdot d\vec{r} \qquad (1.29)$$

The above equation (1.29) is independent of q'. The quantity $\frac{\Delta U}{q'} = -\int_R^P \vec{E} \cdot d\vec{r}$ is called electric potential difference between P and R and is denoted as $V_P - V_R = \Delta V$.

In otherwords, the electric potential difference is defined as the work done by an external force to bring unit positive charge from point R to point P.

$$V_P - V_R = \Delta V = \int_R^P -\vec{E} \cdot d\vec{r} \qquad (1.30)$$

The electric potential energy difference can be written as $\Delta U = q'\ \Delta V$. Physically potential difference between two points is a meaningful quantity. The value of the potential itself at one point is not meaningful. Therefore the point R is taken to infinity and the potential at infinity is considered as zero $\left(V_\infty = 0\right)$.

Then **the electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$.** Mathematically this is written as

$$V_P = -\int_\infty^P \vec{E} \cdot d\vec{r} \qquad (1.31)$$

Important points

1. Electric potential at point P depends only on the electric field which is due to the source charge q and not on the test charge q'. Unit positive charge is brought from infinity to the point P with constant velocity because external agency should not impart any kinetic energy to the test charge.

UNIT-1(XII-Physics_Vol-1).indd 26 14-02-2020 19:32:42

2. From equation (1.29), the unit of electric potential is Joule per coulomb. The practical unit is volt (V) named after Alessandro Volta (1745-1827) who invented the electrical battery. The potential difference between two points is expressed in terms of volt.

Note

The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.

1.5.2 Electric potential due to a point charge

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q. This is shown in Figure 1.21.

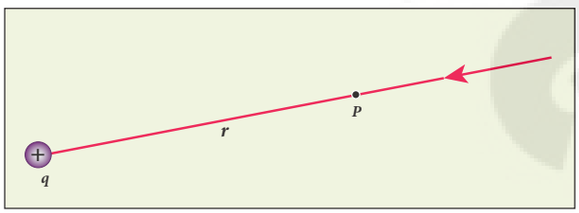


Figure 1.21 Electrostatic potential at a point P

The electric potential at the point P is

$$V = \int_{\infty}^{r} \left(-\vec{E}\right) \cdot d\vec{r} = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r} \qquad (1.32)$$

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = \frac{-1}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{q}{r^2} \hat{r} . d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr\hat{r}$ and using $\hat{r} . \hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{q}{r^2} \hat{r} \cdot dr\hat{r} = -\frac{1}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{q}{r^2} dr$$

After the integration,

$$V = -\frac{1}{4\pi\epsilon_0} q \left\{ -\frac{1}{r} \right\}_{\infty}^{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \qquad (1.33)$$

Important points

(i) If the source charge q is positive, $V > 0$. If q is negative, then V is negative and equal to $V = -\frac{1}{4\pi\epsilon_0} \frac{q}{r}$

(ii) From expression (1.33), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity $(r = \infty)$ electrostatic potential is zero (V = 0).

In the case of gravitational force, mass moves from a point of higher gravitational potential to a point of lower

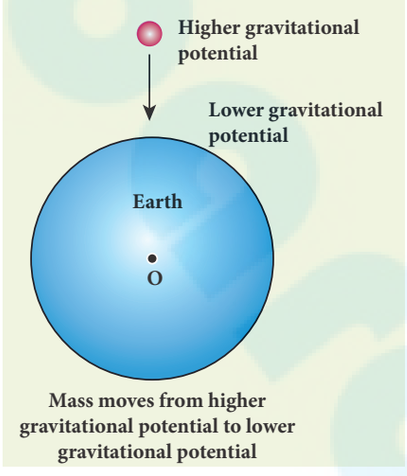


Figure 1.22 Motion of masses in terms of gravitational potential

UNIT-1(XII-Physics_Vol-1).indd 27 14-02-2020 19:32:45

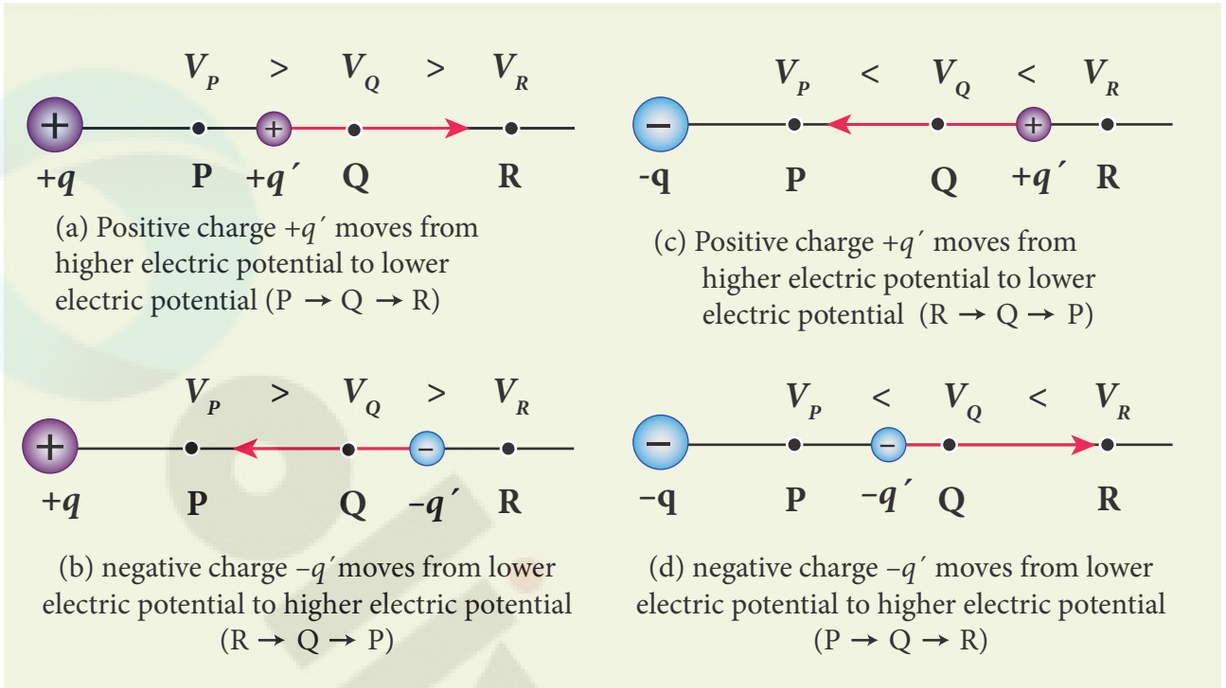


Figure 1.23 Motion of charges in terms of electric potential

gravitational potential (Figure 1.22). Similarly a positive charge moves from a point of higher electrostatic potential to a point of lower electrostatic potential. However a negative charge moves from lower electrostatic potential to higher electrostatic potential. This comparison is shown in Figure 1.23.

(iii) The electric potential at a point P due to a collection of charges $q_1, q_2, q_3 \ldots\ldots q_n$ is equal to sum of the electric potentials due to individual charges.

$$V_{tot} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \ldots$$
$$\ldots. + \frac{kq_n}{r_n} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad (1.34)$$

where $r_1, r_2, r_3 \ldots\ldots r_n$ are the distances of q_1, q_2, q_3 ….. q_n respectively from P (Figure 1.24).

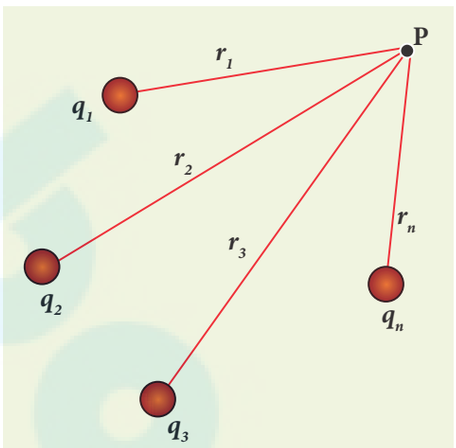


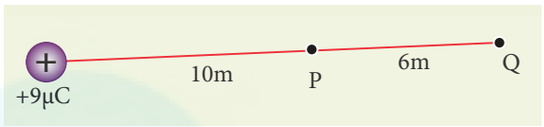
Figure 1.24 Electrostatic potential due to collection of charges

EXAMPLE 1.12

(a) Calculate the electric potential at points P and Q as shown in the figure below.

(b) Suppose the charge +9 μC is replaced by −9 μC find the electrostatic potentials at points P and Q

UNIT-1(XII-Physics_Vol-1).indd 28 14-02-2020 19:32:46



(c) Calculate the work done to bring a test charge +2 μC from infinity to the point Q. Assume the charge +9 μC is held fixed at origin and +2 μC is brought from infinity to P.

Solution

(a) Electric potential at point P is given by

$$V_P = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r_P} = \frac{9\times10^9\times9\times10^{-6}}{10} = 8.1\times10^3\ \text{V}$$

Electric potential at point Q is given by

$$V_Q = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r_Q} = \frac{9\times10^9\times9\times10^{-6}}{16} = 5.06\times10^3\ \text{V}$$

Note that the electric potential at point Q is less than the electric potential at point P. If we put a positive charge at P, it moves from P to Q. However if we place a negative charge at P it will move towards the charge +9 μC.

The potential difference between the points P and Q is given by

$$\Delta V = V_P - V_Q = +3.04\times10^3\ \text{V}$$

(b) Suppose we replace the charge +9 μC by −9 μC, then the corresponding potentials at the points P and Q are,

$$V_P = -8.1\times10^3\ \text{V},\ V_Q = -5.06\times10^3\ \text{V}$$

Note that in this case electric potential at the point Q is higher than at point P.

The potential difference between the points P and Q is given by

$$\Delta V = V_P - V_Q = -3.04\times10^3\ \text{V}$$

(c) The electric potential V at a point Q due to some charge is defined as the work done by an external force to bring a unit positive charge from infinity to Q. So to bring the q amount of charge from infinity to the point Q, work done is given as follows.

$$W = qV$$

$$W_Q = 2\times10^{-6}\times5.06\times10^3 = 10.12\times10^{-3}\ \text{J}.$$

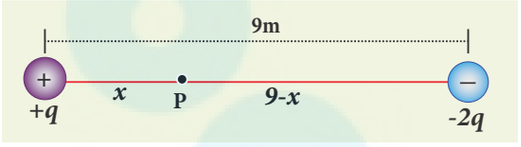
EXAMPLE 1.13

Consider a point charge $+q$ placed at the origin and another point charge $-2q$ placed at a distance of 9 m from the charge $+q$. Determine the point between the two charges at which electric potential is zero.

Solution

According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point.

Consider the point at which the total potential zero is located at a distance x from the charge $+q$ as shown in the figure.



Since the total electric potential at P is zero,

$$V_{tot} = \frac{1}{4\pi\epsilon_\circ}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0 \text{ (or)}$$

$$\frac{q}{x} = \frac{2q}{(9-x)} \text{ (or)}$$

$$\frac{1}{x} = \frac{2}{(9-x)}$$

Hence, x = 3 m

UNIT-1(XII-Physics_Vol-1).indd 29 14-02-2020 19:32:49

1.5.3 Electrostatic potential at a point due to an electric dipole

Consider two equal and opposite charges separated by a small distance $2a$ as shown in Figure 1.25. The point P is located at a distance r from the midpoint of the dipole. Let θ be the angle between the line OP and dipole axis AB.

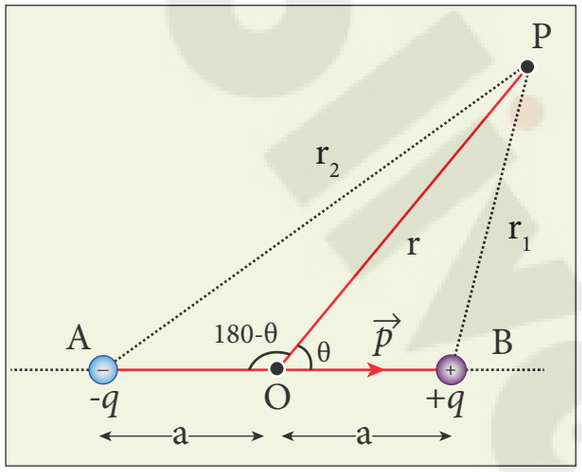


Figure 1.25 Potential due to electric dipole

Let r_1 be the distance of point P from $+q$ and r_2 be the distance of point P from $-q$.

Potential at P due to charge $+q = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$

Potential at P due to charge $-q = -\frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$

Total potential at the point P

$$V = \frac{1}{4\pi\epsilon_0} q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (1.35)$$

Suppose if the point P is far away from the dipole, such that $r>>a$, then equation (1.35) can be expressed in terms of r.

By the cosine law for triangle BOP

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$$

Since the point P is very far from the dipole ($r>>a$). As a result the term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore

$$r_1^2 = r^2\left(1 - 2a\frac{\cos\theta}{r}\right)$$

$$\text{(or) } r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$

Since $\frac{a}{r} << 1$, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) \qquad (1.36)$$

Similarly applying the cosine law for triangle AOP,

$$r_2^2 = r^2 + a^2 - 2ra\cos(180 - \theta)$$

since $\cos(180 - \theta) = -\cos\theta$ we get

$$r_2^2 = r^2 + a^2 + 2ra\cos\theta$$

Neglecting the term $\frac{a^2}{r^2}$ (because $r>>a$)

$$r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

UNIT-1(XII-Physics_Vol-1).indd 30 14-02-2020 19:32:53

$$r_2 = r\left(1+\frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

Using Binomial theorem, we get

$$\frac{1}{r_2} = \frac{1}{r}\left(1-a\frac{\cos\theta}{r}\right) \qquad (1.37)$$

Substituting equation (1.37) and (1.36) in equation (1.35),

$$V = \frac{1}{4\pi\epsilon_\circ} q\left(\frac{1}{r}\left(1+a\frac{\cos\theta}{r}\right)-\frac{1}{r}\left(1-a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{q}{4\pi\epsilon_\circ}\left(\frac{1}{r}\left(1+a\frac{\cos\theta}{r}-1+a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{1}{4\pi\epsilon_\circ}\frac{2aq}{r^2}\cos\theta$$

But the electric dipole moment $p = 2qa$ and we get,

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write $p\cos\theta = \vec{p}\cdot\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

$$V = \frac{1}{4\pi\epsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2} \quad (r>>a) \qquad (1.38)$$

Equation (1.38) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (1.38) is valid for any distance.

Special cases

Case (i) If the point P lies on the axial line of the dipole on the side of $+q$, then $\theta = 0$. Then the electric potential becomes

$$V = \frac{1}{4\pi\epsilon_\circ}\frac{p}{r^2} \qquad (1.39)$$

Case (ii) If the point P lies on the axial line of the dipole on the side of $-q$, then $\theta = 180^\circ$. Then

$$V = -\frac{1}{4\pi\epsilon_\circ}\frac{p}{r^2} \qquad (1.40)$$

Case (iii) If the point P lies on the equatorial line of the dipole, then $\theta = 90^\circ$. Hence

$$V = 0 \qquad (1.41)$$

Note The potential due to an electric dipole falls as $\frac{1}{r^2}$ and the potential due to a single point charge falls as $\frac{1}{r}$. Thus the potential due to the dipole falls faster than that due to a monopole (point charge). As the distance increases from electric dipole, the effects of positive and negative charges nullify each other

1.5.4 Equi-potential Surface

Consider a point charge q located at some point in space and an imaginary sphere of radius r is chosen by keeping the charge q at its centre (Figure 1.26(a)). The electric potential at all points on the surface of the given sphere is the same. Such a surface is called an equipotential surface.

An equipotential surface is a surface on which all the points are at the same electric potential. For a point charge the equipotential surfaces are concentric spherical surfaces as shown in Figure 1.26(b). Each spherical surface is an equipotential surface but the

UNIT-1(XII-Physics_Vol-1).indd 31 14-02-2020 19:32:57

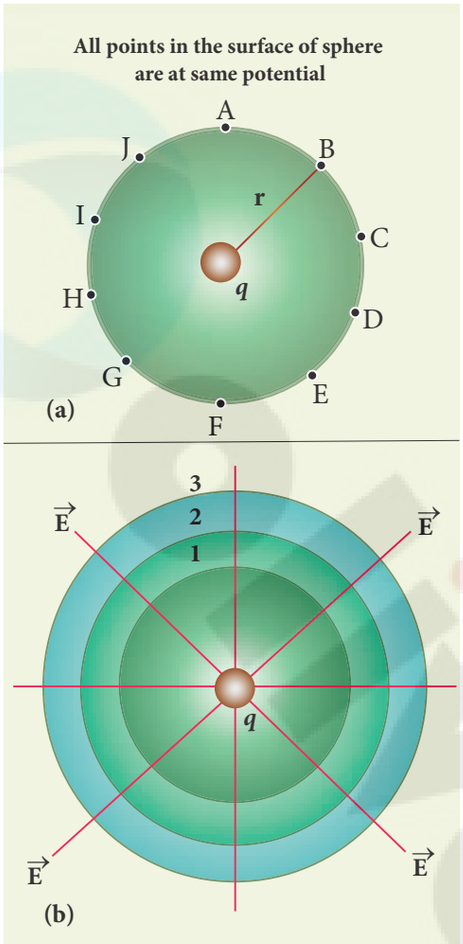


Figure 1.26 Equipotential surface of point Charge

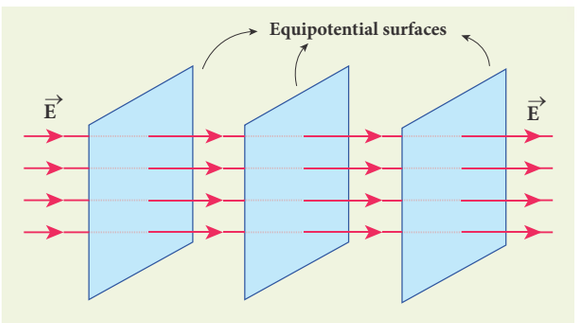


Figure 1.27 Equipotential surface for uniform electric field

value of the potential is different for different spherical surfaces.

For a uniform electric field, the equipotential surfaces form a set of planes normal to the electric field $\vec{E}$. This is shown in the Figure 1.27.

Properties of equipotential surfaces

(i) The work done to move a charge q between any two points A and B, $W = q\,(V_B - V_A)$. If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$.

(ii) The electric field is normal to an equipotential surface. If it is not normal, then there is a component of the field parallel to the surface. Then work must be done to move a charge between two points on the same surface. This is a contradiction. Therefore the electric field must always be normal to equipotential surface.

1.5.5 Relation between electric field and potential

Consider a positive charge q kept fixed at the origin. To move a unit positive charge by a small distance dx towards q in the electric field E, the work done is given by $dW = -E\,dx$. The minus sign implies that work is done against the electric field. This work done is equal to electric potential difference. Therefore,

$$dW = dV.$$

$$\text{(or) } dV = -E\,dx \qquad (1.42)$$

$$\text{Hence } E = -\frac{dV}{dx} \qquad (1.43)$$

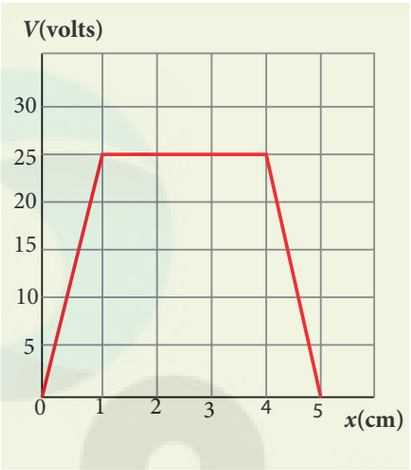
The electric field is the negative gradient of the electric potential. In vector form,

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \qquad (1.44)$$

EXAMPLE 1.14

The following figure represents the electric potential as a function of x – coordinate. Plot the corresponding electric field as a function of x.

UNIT-1(XII-Physics_Vol-1).indd 32 14-02-2020 19:32:58



Solution

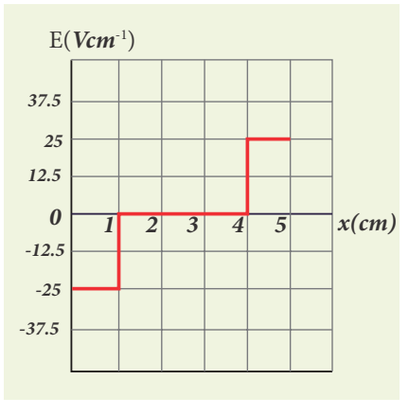
In the given problem, since the potential depends only on x, we can use $\vec{E} = -\frac{dV}{dx}\hat{i}$ (the other two terms $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$ are zero)

From 0 to 1 cm, the slope is constant and so $\frac{dV}{dx} = 25\,\text{V cm}^{-1}$. So $\vec{E} = -25\,\text{V cm}^{-1}\hat{i}$

From 1 to 4 cm, the potential is constant, V = 25 V. It implies that $\frac{dV}{dx} = 0$. So $\vec{E} = 0$

From 4 to 5 cm, the slope $\frac{dV}{dx} = -25\,\text{V cm}^{-1}$. So $\vec{E} = +25\,\text{V cm}^{-1}\hat{i}$.

The plot of electric field for the various points along the x axis is given below.



1.5.6 Electrostatic potential energy for collection of point charges

The electric potential at a point at a distance r from point charge q_1 is given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r}$$

This potential V is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance r from q_1, the work done is the product of q_2 and the electric potential at that point. Thus we have

$$W = q_2 V$$

This work done is stored as the electrostatic potential energy U of a system of charges q_1 and q_2 separated by a distance r. Thus we have

$$U = q_2 V = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \qquad (1.45)$$

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression (1.45) is derived by assuming that q_1 is fixed and q_2 is brought from infinity. The equation (1.45) holds true when q_2 is fixed and q_1 is brought from infinity or both q_1 and q_2 are simultaneously brought from infinity to a distance r between them.

Three charges are arranged in the following configuration as shown in Figure 1.28.

To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown in Figure 1.28.

UNIT-1(XII-Physics_Vol-1).indd 33 14-02-2020 19:33:01

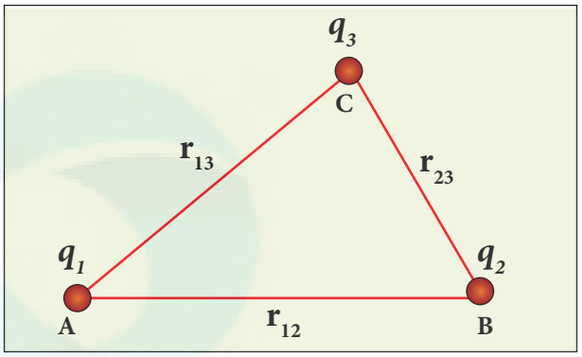


Figure 1.28 Electrostatic potential energy for collection of point charges

(i) Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1.

(ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1. So the work done on the charge q_2 is $W = q_2 V_{1B.}$ Here V_{1B} is the electrostatic potential due to the charge q_1 at point *B*.

$$U_I = \frac{1}{4\pi\epsilon_\circ}\frac{q_1 q_2}{r_{12}} \qquad (1.46)$$

Note that the expression is same when q_2 is brought first and then q_1 later.

(iii) Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is = q_3 (V_{1C} + V_{2C}). Here V_{1C} is the electrostatic potential due to charge q_1 at point C and V_{2C} is the electrostatic potential due to charge q_2 at point C.

The electrostatic potential energy is

$$U_{II} = \frac{1}{4\pi\epsilon_\circ}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad (1.47)$$

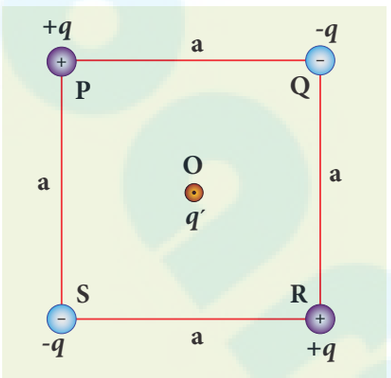
(iv) Adding equations (1.46) and (1.47), the total electrostatic potential energy for the system of three charges q_1, q_2 and q_3 is $U = U_I + U_{II}$

$$U = \frac{1}{4\pi\epsilon_\circ}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad (1.48)$$

Note that this stored potential energy U is equal to the total external work done to assemble the three charges at the given locations. The expression (1.48) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

EXAMPLE 1.15

Four charges are arranged at the corners of the square *PQRS* of side *a* as shown in the figure.(a) Find the work required to assemble these charges in the given configuration. (b) Suppose a charge q' is brought to the centre of the square, by keeping the four charges fixed at the corners, how much extra work is required for this?



Solution

(a) The work done to arrange the charges in the corners of the square is independent

UNIT-1(XII-Physics_Vol-1).indd 34 14-02-2020 19:33:02

of the way they are arranged. We can follow any order.

(i) First, the charge $+q$ is brought to the corner P. This requires no work since no charge is already present, $W_P = 0$

(ii) Work required to bring the charge $-q$ to the corner Q = $(-q) \times$ potential at a point Q due to $+q$ located at a point P.

$$W_Q = -q \times \frac{1}{4\pi\epsilon_\circ}\frac{q}{a} = -\frac{1}{4\pi\epsilon_\circ}\frac{q^2}{a}$$

(iii) Work required to bring the charge $+q$ to the corner R = $q \times$ potential at the point R due to charges at the point P and Q.

$$W_R = q \times \frac{1}{4\pi\epsilon_\circ}\left(-\frac{q}{a} + \frac{q}{\sqrt{2}a}\right)$$
$$= \frac{1}{4\pi\epsilon_\circ}\frac{q^2}{a}\left(-1 + \frac{1}{\sqrt{2}}\right)$$

(iv) Work required to bring the fourth charge $-q$ at the position S = $q \times$ potential at the point S due the all the three charges at the point P, Q and R

$$W_S = -q \times \frac{1}{4\pi\epsilon_\circ}\left(\frac{q}{a} + \frac{q}{a} - \frac{q}{\sqrt{2}a}\right)$$
$$W_S = -\frac{1}{4\pi\epsilon_\circ}\frac{q^2}{a}\left(2 - \frac{1}{\sqrt{2}}\right)$$

(b) Work required to bring the charge q' to the centre of the square = $q' \times$ potential at the centre point O due to all the four charges in the four corners

The potential created by the two $+q$ charges are canceled by the potential created by the $-q$ charges which are located in the opposite corners. Therefore the net electric potential at the centre O due to all the charges in the corners is zero.

Hence no work is required to bring any charge to the point O. Physically this implies that if any charge q' when brought close to O, then it moves to the point O without any external force.

1.5.7 Electrostatic potential energy of a dipole in a uniform electric field

Consider a dipole placed in the uniform electric field $\vec{E}$ as shown in the Figure 1.29. A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle θ' to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

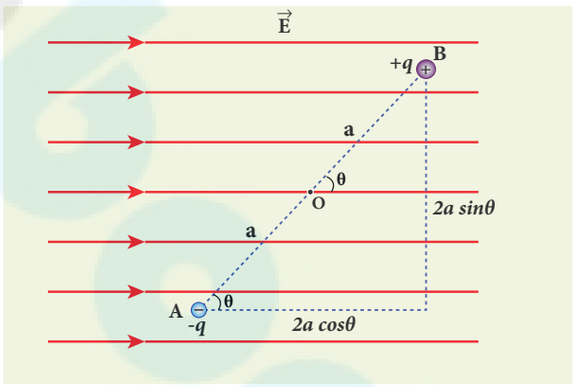


Figure 1.29 The dipole in a uniform electric field

The work done by the external torque to rotate the dipole from angle θ' to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext}\, d\theta \qquad (1.49)$$

Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have

UNIT-1(XII-Physics_Vol-1).indd 35 14-02-2020 19:33:04

$$|\vec{\tau}_{ext}| = |\vec{\tau}_E| = |\vec{p} \times \vec{E}| \qquad (1.50)$$

Substituting equation (1.50) in equation (1.49), we get

$$W = \int_{\theta'}^{\theta} pE \sin\theta \, d\theta$$
$$W = pE(\cos\theta' - \cos\theta)$$

This work done is equal to the potential energy difference between the angular positions θ and θ'.

$$U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$$

If the initial angle is $\theta' = 90^\circ$ and is taken as reference point, then $U(\theta') = pE\cos 90^\circ = 0$.

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E} \qquad (1.51)$$

In addition to p and E, the potential energy also depends on the orientation θ of the electric dipole with respect to the external electric field.

The potential energy is maximum when the dipole is aligned anti-parallel ($\theta = \pi$) to the external electric field and minimum when the dipole is aligned parallel ($\theta = 0$) to the external electric field.

EXAMPLE 1.16

A water molecule has an electric dipole moment of 6.3×10^{-30} Cm. A sample contains 10^{22} water molecules, with all the dipole moments aligned parallel to the external electric field of magnitude 3×10^5 N C^{-1}. How much work is required to rotate all the water molecules from $\theta = 0^o$ to 90°?

Solution

When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from $\theta = 0^o$ to 90° is equal to the potential energy difference between these two configurations.

$$W = \Delta U = U(90^\circ) - U(0^\circ)$$

From the equation (1.51), we write $U = -pE\cos\theta$, Next we calculate the work done to rotate one water molecule from $\theta = 0^o$ to 90°.

For one water molecule

$$W = -pE\cos 90^\circ + pE\cos 0^\circ = pE$$
$$W = 6.3\times10^{-30}\times3\times10^5 = 18.9\times10^{-25}\,\text{J}$$

For 10^{22} water molecules, the total work done is

$$W_{tot} = 18.9\times10^{-25}\times10^{22} = 18.9\times10^{-3}\,\text{J}$$

1.6 GAUSS LAW AND ITS APPLICATIONS

1.6.1 Electric Flux

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter Φ_E and its unit is N m^2 C^{-1}. Electric flux is a scalar quantity and it can be positive or negative. For a simpler understanding of electric flux, the following Figure 1.30 is useful.

UNIT-1(XII-Physics_Vol-1).indd 36 14-02-2020 19:33:07

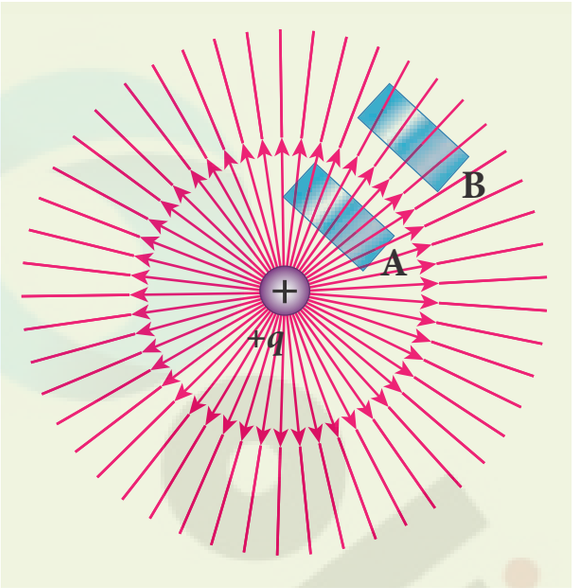


Figure 1.30 Electric flux

The electric field of a point charge is drawn in this figure. Consider two small rectangular area elements placed normal to the field at regions A and B. Even though these elements have the same area, the number of electric field lines crossing the element in region A is more than that crossing the element in region B. Therfore the electric flux in region A is more than that in region B. Since electric field strength for a point charge decreases as the distance increases, electric flux also decreases as the distance increases. The above discussion gives a qualitative idea of electric flux. However a precise definition of electric flux is needed.

Electric flux for uniform Electric field

Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown in Figure 1.31 (a). The electric flux for this case is

$$\Phi_E = EA \qquad (1.52)$$

Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pass through the area A , as shown in Figure 1.31(b). The electric flux for this case is zero.

$$\Phi_E = 0 \qquad (1.53)$$

If the area is inclined at an angle θ with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. This is shown in Figure 1.31 (c). For this case, the electric flux

$$\Phi_E = (E\cos\theta)\, A \qquad (1.54)$$

Further, θ is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is defined as

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta \qquad (1.55)$$

Here, note that $\vec{A}$ is the area vector $\vec{A} = A\hat{n}$. Its magnitude is simply the area A and its direction is along the unit vector $\hat{n}$ perpendicular to the area as shown in Figure 1.31. Using this definition for flux, $\Phi_E = \vec{E}\cdot\vec{A}$, equations (1.53) and (1.54) can be obtained as special cases.

In Figure 1.31 (a), $\theta = 0^o$. Therefore,

$$\Phi_E = \vec{E}\cdot\vec{A} = EA$$

In Figure 1.31 (b), $\theta = 90^o$. Therefore,

$$\Phi_E = \vec{E}\cdot\vec{A} = 0$$

UNIT-1(XII-Physics_Vol-1).indd 37 14-02-2020 19:33:09

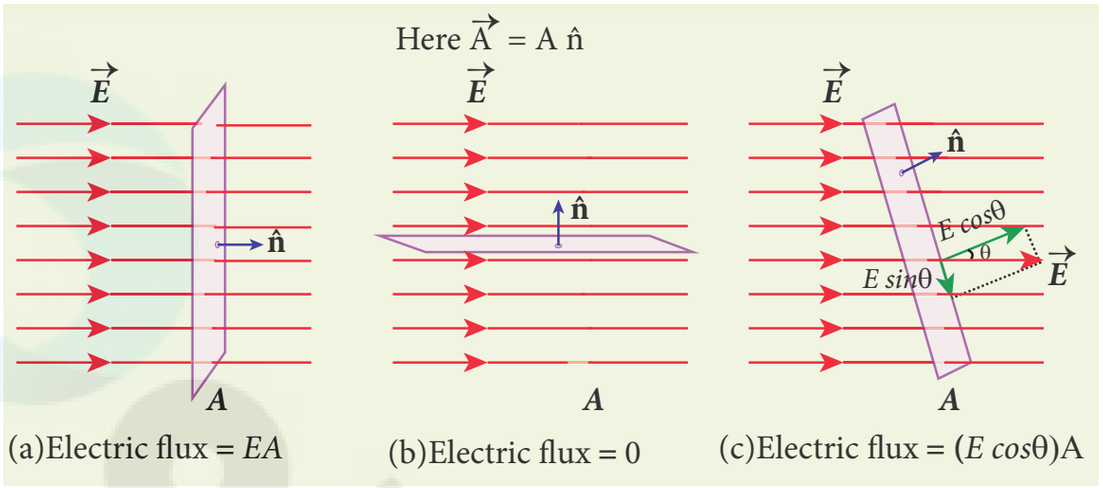
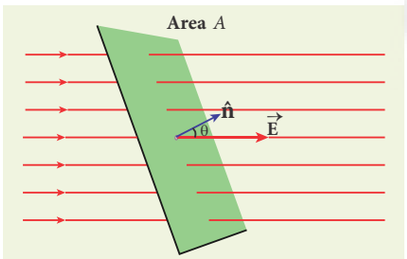


Figure 1.31 The electric flux for Uniform electric field

EXAMPLE 1.17

Calculate the electric flux through the rectangle of sides 5 cm and 10 cm kept in the region of a uniform electric field 100 NC^{-1}. The angle θ is 60°. If θ becomes zero, what is the electric flux?



Solution

The electric flux through the rectangular area

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$
$$= 100\times5\times10\times10^{-4}\times\cos60^{\circ}$$
$$\Phi_E = 0.25\,\mathrm{Nm^2C^{-1}}$$

For $\theta = 0^{o}$,

$$\Phi_E = \vec{E}\cdot\vec{A} = EA$$
$$= 100\times5\times10\times10^{-4}$$
$$= 0.5\,\mathrm{Nm^2C^{-1}}$$

Electric flux through an arbitrary area kept in a non uniform electric field

Suppose the electric field is not uniform and the area *A* is not flat surface (Figure 1.32). Then the entire area can be divided into *n* small area segments $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3 \ldots\ldots \Delta\vec{A}_n$ such that each area element is almost flat and the electric field over such area element can be considered uniform.

The electric flux for the entire area *A* is approximately written as

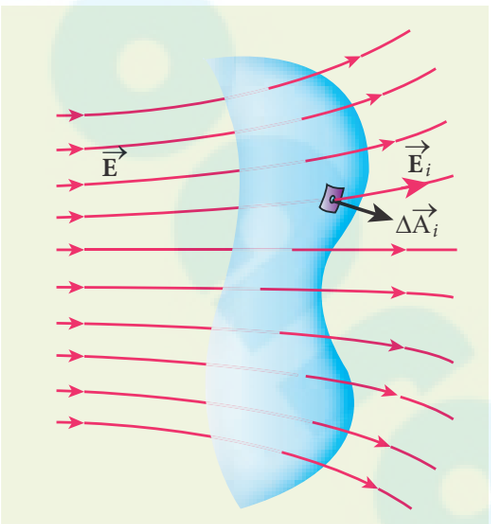


Figure 1.32 Electric flux for non-uniform electric Field

UNIT-1(XII-Physics_Vol-1).indd 38 14-02-2020 19:33:11

$$\Phi_E = \vec{E}_1 \cdot \Delta\vec{A}_1 + \vec{E}_2 \cdot \Delta\vec{A}_2 + \vec{E}_3 \cdot \Delta\vec{A}_3 ..... \vec{E}_n \cdot \Delta\vec{A}_n = \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i \quad (1.56)$$

By taking the limit $\Delta\vec{A}_i \to 0$ (for all i) the summation in equation (1.56) becomes integration. The total electric flux for the entire area is given by

$$\Phi_E = \int \vec{E} \cdot d\vec{A} \quad (1.57)$$

From Equation (1.57), it is clear that the electric flux for a given surface depends on both the electric field pattern on the surface area and orientation of the surface with respect to the electric field.

1.6.2 Electric flux for closed surfaces

In the previous section, the electric flux for any arbitrary curved surface is discussed. Suppose a closed surface is present in the region of the non-uniform electric field as shown in Figure 1.33 (a).

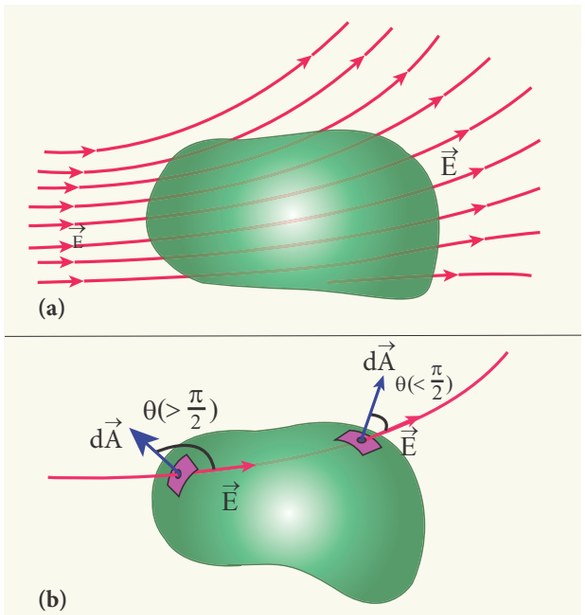


Figure 1.33 Electric flux over a closed surface

The total electric flux over this closed surface is written as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \quad (1.58)$$

Note the difference between equations (1.57) and (1.58). The integration in equation (1.58) is a closed surface integration and for each areal element, the outward normal is the direction of $d\vec{A}$ as shown in the Figure 1.33(b).

The total electric flux over a closed surface can be negative, positive or zero. In the Figure 1.33(b), it is shown that in one area element, the angle between $d\vec{A}$ and $\vec{E}$ is less than 90°, then the electric flux is positive and in another areal element, the angle between $d\vec{A}$ and $\vec{E}$ is greater than 90°, then the electric flux is negative.

In general, the electric flux is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

1.6.3 Gauss law

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure 1.34. We can calculate the total electric flux through the closed surface of the sphere using the equation (1.58).

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\, dA \cos\theta$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and $\theta = 0°$.

$$\Phi_E = \oint E\, dA \qquad \text{since } \cos 0^0 = 1 \quad (1.59)$$

UNIT-1(XII-Physics_Vol-1).indd 39 14-02-2020 19:33:14

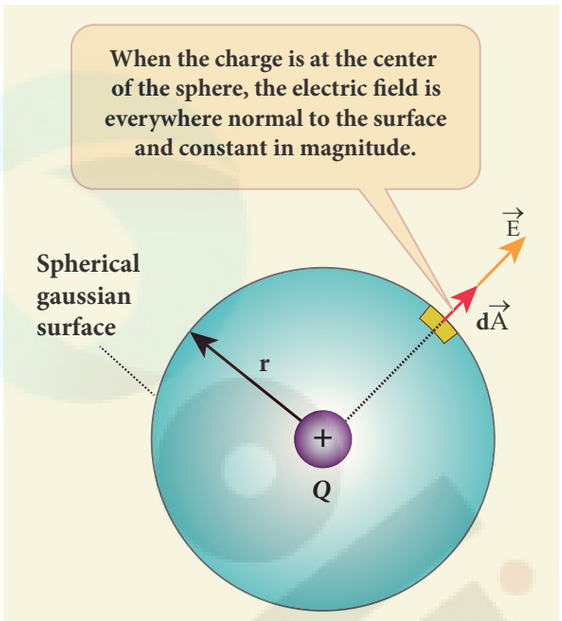


Figure 1.34 Total electric flux of point charge

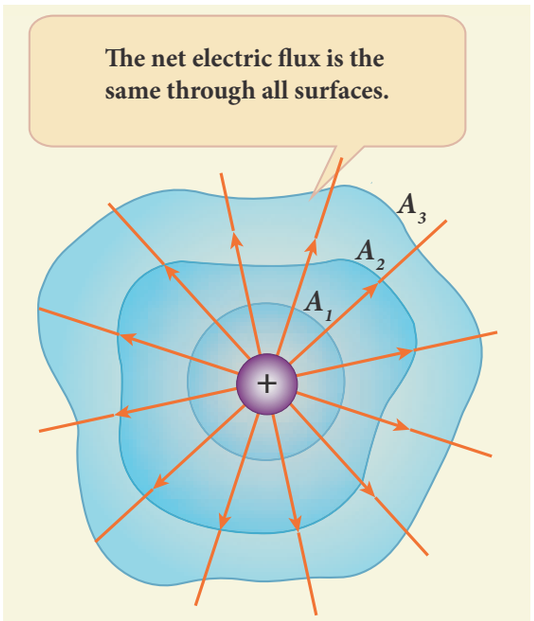


Figure 1.35 Gauss law for arbitrarily shaped surface

E is uniform on the surface of the sphere,

$$\Phi_E = E \oint dA \qquad (1.60)$$

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ in equation (1.60), we get

$$\Phi_E = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi \frac{1}{4\pi\epsilon_\circ} Q$$

$$\Phi_E = \frac{Q}{\epsilon_\circ} \qquad (1.61)$$

The equation (1.61) is called as Gauss's law.

The remarkable point about this result is that the equation (1.61) is equally true for any arbitrary shaped surface which encloses the charge Q and as shown in the Figure 1.35. It is seen that the total electric flux is the same for closed surfaces A_1, A_2 and A_3 as shown in the Figure 1.35.

Gauss's law states that **if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is**

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_\circ} \qquad (1.62)$$

where Q_{encl} denotes the charges within the closed surface.

Discussion of Gauss law

(i) The total electric flux through the closed surface depends only on the charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

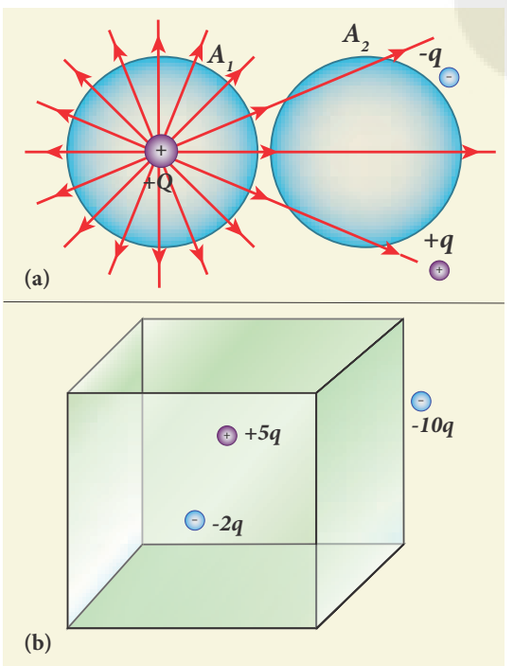
(ii) The total electric flux is independent of the location of the charges inside the closed surface.

UNIT-1(XII-Physics_Vol-1).indd 40 14-02-2020 19:33:16

(iii) To arrive at equation (1.62), we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge, therefore spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation (1.62), the electric field $\vec{E}$ is due to charges present inside and outside the Gaussian surface but the charge Q_{encl} denotes the charges which lie only inside the Gaussian surface.

EXAMPLE 1.18



(i) In figure (a), calculate the electric flux through the closed areas A_1 and A_2.

(ii) In figure (b), calculate the electric flux through the cube

Solution

(i) In figure (a), the area A_1 encloses the charge Q. So electric flux through this closed surface A_1 is $\frac{Q}{\epsilon_0}$. But the closed surface A_2 contains no charges inside, so electric flux through A_2 is zero.

(ii) In figure (b), the net charge inside the cube is $3q$ and the total electric flux in the cube is therefore $\Phi_E = \frac{3q}{\epsilon_0}$. Note that the charge -10 q lies outside the cube and it will not contribute the total flux through the surface of the cube.

1.6.4 Applications of Gauss law

Electric field due to any arbitrary charge configuration can be calculated using Coulomb's law or Gauss law. If the charge configuration possesses some kind of symmetry, then Gauss law is a very efficient way to calculate the electric field. It is illustrated in the following cases.

(i) Electric field due to an infinitely long charged wire

Consider an infinitely long straight wire having uniform linear charge density λ(charge per unit length). Let P be a point located at a perpendicular distance r from the wire (Figure 1.36(a)). The electric field at the point P can be found using Gauss law.

We choose two small charge elements A_1 and A_2 on the wire which are at equal distances from the point P. The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r. This is shown in the Figure 1.36(b). Since the charged wire possesses

UNIT-1(XII-Physics_Vol-1).indd 41 14-02-2020 19:33:17

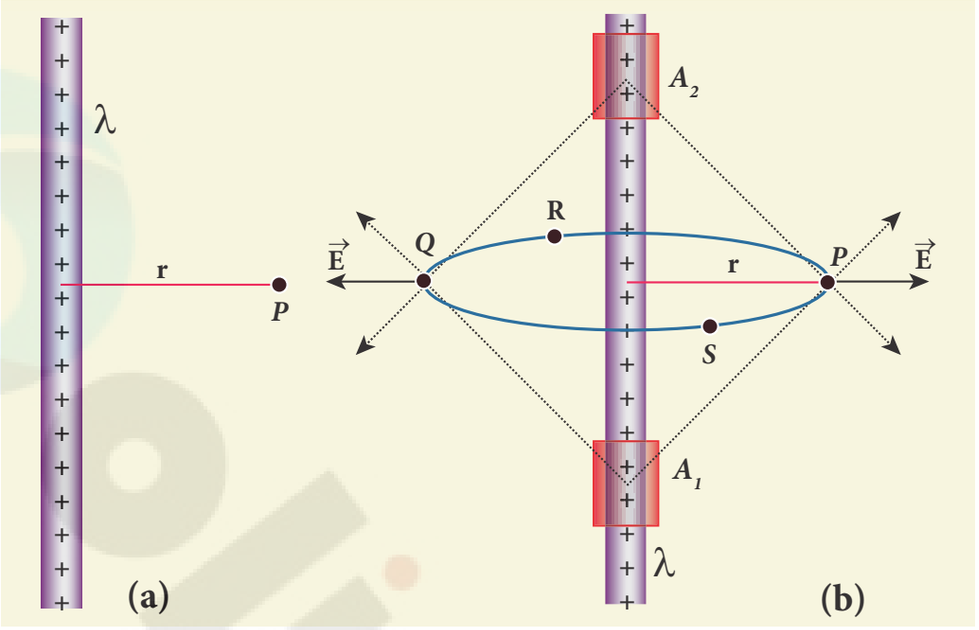


Figure 1.36 Electric field due to infinite long charged wire

a cylindrical symmetry, let us choose a cylindrical Gaussian surface of radius r and length L as shown in the Figure 1.37.

The total electric flux through this closed surface is calculated as follows.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \int_{\substack{Curved\\surface}} \vec{E}\cdot d\vec{A} + \int_{\substack{top\\surface}} \vec{E}\cdot d\vec{A} + \int_{\substack{bottom\\surface}} \vec{E}\cdot d\vec{A} \quad (1.63)$$

It is seen from Figure (1.37) that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = E\,dA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = 0$

Substituting these values in the equation (1.63) and applying Gauss law to the cylindrical surface, we have

$$\Phi_E = \int_{\substack{Curved\\surface}} E\,dA = \frac{Q_{encl}}{\epsilon_\circ} \quad (1.64)$$

Since the magnitude of the electric field for the entire curved surface is constant, E is

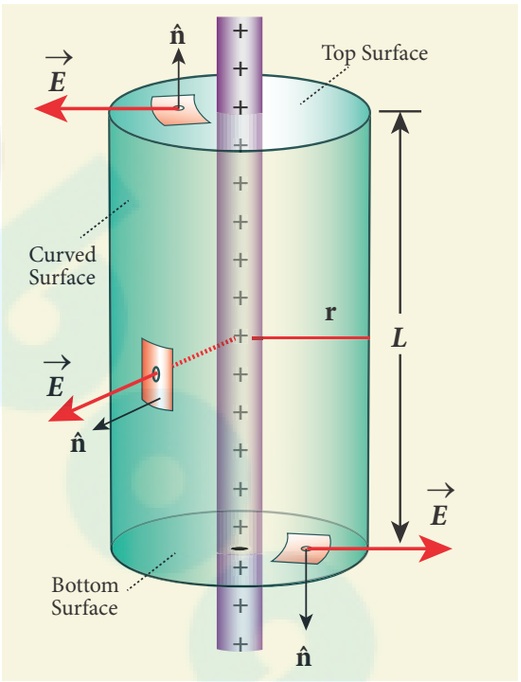


Figure 1.37 Cylindrical Gaussian surface

taken out of the integration and Q_{encl} is given by $Q_{encl} = \lambda L$, where λ is the linear charge density (charge present per unit length).

$$E\int_{\substack{Curved\\surface}} dA = \frac{\lambda L}{\epsilon_\circ} \quad (1.65)$$

UNIT-1(XII-Physics_Vol-1).indd 42 14-02-2020 19:33:19

Here $\int_{\substack{Curved\\surface}} dA =$ total area of the curved surface = $2\pi rL$. Substituting this in equation (1.65), we get

$$E \cdot 2\pi rL = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} \qquad (1.66)$$

In vector form,

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r} \qquad (1.67)$$

The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ which is for a point charge.

Equation (1.67) indicates that the electric field is always along the perpendicular direction ($\hat{r}$) to wire. In fact, if $\lambda > 0$ then $\vec{E}$ points perpendicularly outward ($\hat{r}$) from the wire and if $\lambda < 0$, then $\vec{E}$ points perpendicularly inward ($-\hat{r}$).

The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

(ii) Electric field due to charged infinite plane sheet

Consider an infinite plane sheet of charges with uniform surface charge density σ (charge present per unit area). Let P be a point at a distance of r from the sheet as shown in the Figure 1.38.

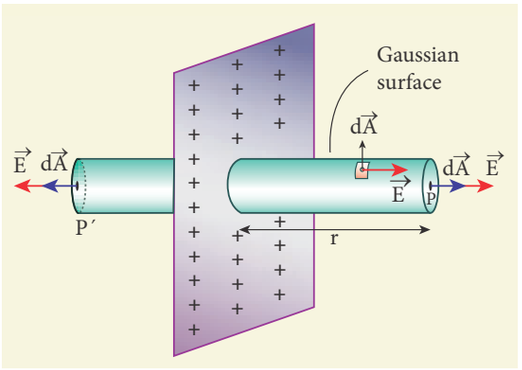


Figure 1.38 Electric field due to charged infinite planar sheet

Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed outward at all points. A cylindrical Gaussian surface of length $2r$ and two flats surfaces each of area A is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.

Total electric flux linked with the cylindrical surface,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\substack{Curved\\surface}} \vec{E} \cdot d\vec{A} + \int_P \vec{E} \cdot d\vec{A} + \int_{P'} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad (1.68)$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P' (Figure 1.38). Then, applying Gauss' law,

$$\Phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\epsilon_0} \qquad (1.69)$$

Since the magnitude of the electric field at these two equal flat surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

UNIT-1(XII-Physics_Vol-1).indd 43 14-02-2020 19:33:22

$$2E\int_P dA = \frac{\sigma A}{\epsilon_0}$$

The total area of surface either at P or P′

$$\int_P dA = A$$

Hence $2EA = \frac{\sigma A}{\epsilon_0}$ or $E = \frac{\sigma}{2\epsilon_0}$ (1.70)

In vector form, $\vec{E} = \frac{\sigma}{2\epsilon_\circ}\hat{n}$ (1.71)

Here $\hat{n}$ is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance *r*.

The electric field will be the same at any point farther away from the charged plane. Equation (1.71) implies that if σ > 0 the electric field at any point P is along outward perpendicular $\hat{n}$ drawn to the plane and if σ < 0, the electric field points inward perpendicularly to the plane ($-\hat{n}$).

For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

(iii) Electric field due to two parallel charged infinite sheets

Consider two infinitely large charged plane sheets with equal and opposite charge densities +σ and -σ which are placed parallel to each other as shown in the Figure 1.39.

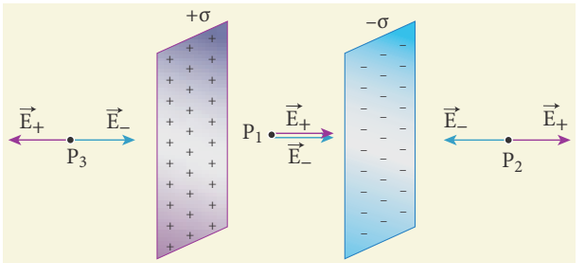


Figure 1.39 Electric field due to two parallel charged sheets

The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\epsilon_\circ}$ and it points perpendicularly outward if σ > 0 and points inward if σ < 0.

At the points P_2 and P_3, the electric field due to both plates are equal in magnitude and opposite in direction (Figure 1.41). As a result, electric field at a point outside the plates is zero. But between the plates, electric fields are in the same direction i.e., towards the right and the total electric field at a point P_1 is

$$E_{inside} = \frac{\sigma}{2\epsilon_\circ} + \frac{\sigma}{2\epsilon_\circ} = \frac{\sigma}{\epsilon_\circ} \quad (1.72)$$

The direction of the electric field between the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere between the plates.

(iv) Electric field due to a uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius *R* carrying total charge *Q* as shown in Figure 1.40. The electric field at points outside and inside the sphere can be found using Gauss law.

Case (a) At a point outside the shell (*r* > *R*)

Let us choose a point P outside the shell at a distance r from the centre as shown in Figure 1.40 (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if *Q* > 0 and point radially inward if *Q* < 0. So a spherical Gaussian surface of radius *r* is chosen and

UNIT-1(XII-Physics_Vol-1).indd 44 14-02-2020 19:33:24

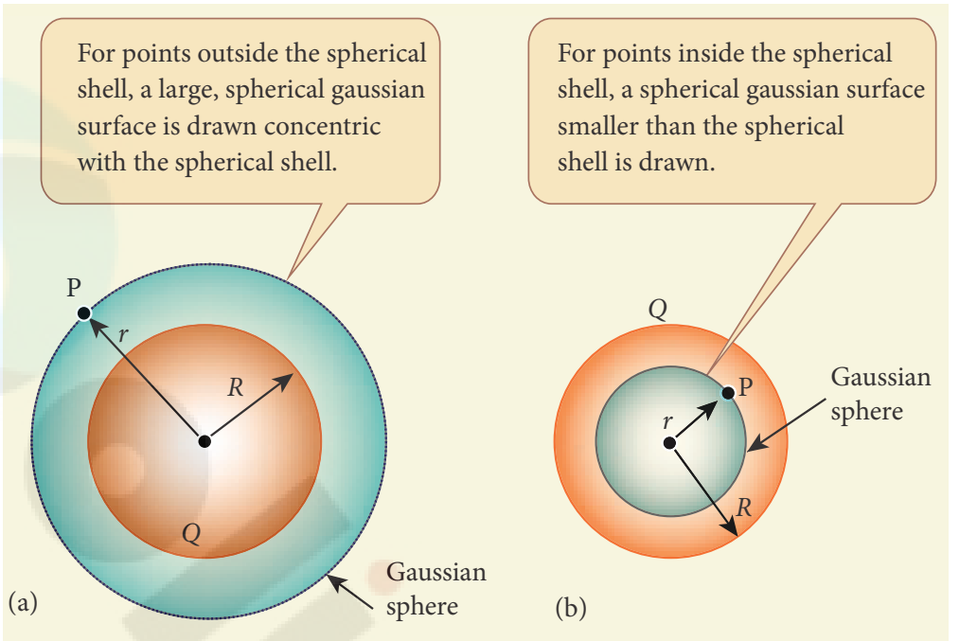


Figure 1.40 The electric field due to a charged spherical shell

the total charge enclosed by this Gaussian surface is Q. Applying Gauss law

$$\oint_{\text{Gaussian surface}} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \qquad (1.73)$$

The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E \oint_{\text{Gaussian surface}} dA = \frac{Q}{\epsilon_0} \qquad (1.74)$$

But $\oint_{\text{Gaussian surface}} dA =$ total area of Gaussian surface $= 4\pi r^2$. Substituting this value in equation (1.74)

$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \quad \text{(or)} \quad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

In vector form,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad (1.75)$$

The electric field is radially outward if $Q > 0$ and radially inward if $Q < 0$. From equation (1.75), we infer that the electric field at a point outside the shell will be the same as if the entire charge Q is concentrated at the centre of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell ($r = R$)

The electrical field at points on the spherical shell ($r = R$) is given by

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\hat{r} \qquad (1.76)$$

Case (c): At a point inside the spherical shell ($r < R$)

Consider a point P inside the shell at a distance r from the centre. A Gaussian

UNIT-1(XII-Physics_Vol-1).indd 45 14-02-2020 19:33:27

sphere of radius r is constructed as shown in the Figure 1.40 (b). Applying Gauss law

$$\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \qquad (1.77)$$

Since Gaussian surface encloses no charge, $Q = 0$. The equation (1.77) becomes

$$E = 0 \qquad (r < R) \qquad (1.78)$$

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.

A graph is plotted between the electric field and radial distance. This is shown in Figure 1.41.

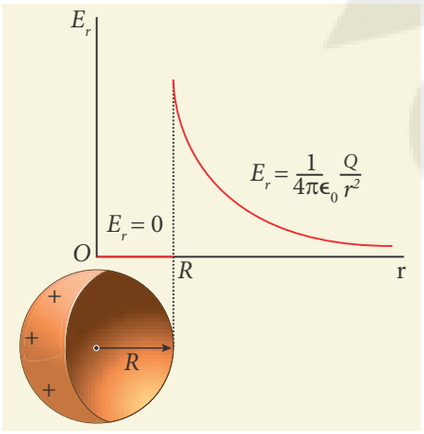


Figure 1.41 Electric field versus distance for a spherical shell of radius R

Note

Gauss law is a powerful technique whenever a given charge configuration possesses spherical, cylindrical or planar symmetry, then the electric field due to such a charge configuration can be easily found. If there is no such symmetry, the direct method (Coulomb's law and calculus) can be used. For example, it is difficult to use Gauss law to find the electric field for a dipole since it has no spherical, cylindrical or planar symmetry.

1.7 ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS

1.7.1 Conductors at electrostatic equilibrium

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown in Figure 1.42.

UNIT-1(XII-Physics_Vol-1).indd 46 14-02-2020 19:33:28

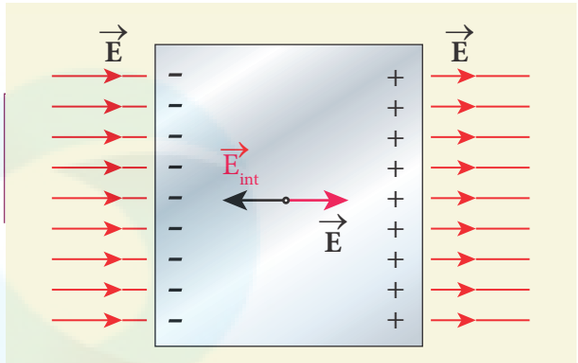


Figure 1.42 Electric field of conductors

Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown in Figure 1.44.

Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of 10^{-16}s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.43.

A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside the conductor, the net electric flux is also zero over this Gaussian surface. From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced

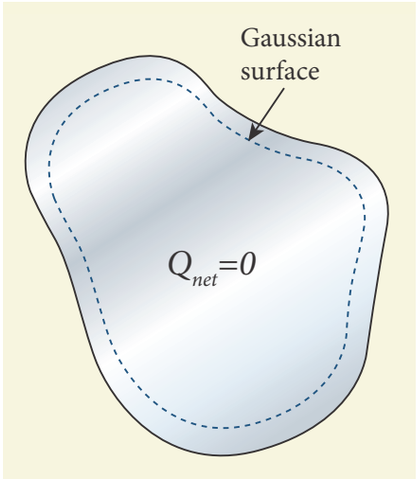


Figure 1.43 No net charge inside the conductor

inside the conductor, it immediately reaches the surface of the conductor.

(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\epsilon_0}$ where σ is the surface charge density at that point.

If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure 1.44(a)). This means that the

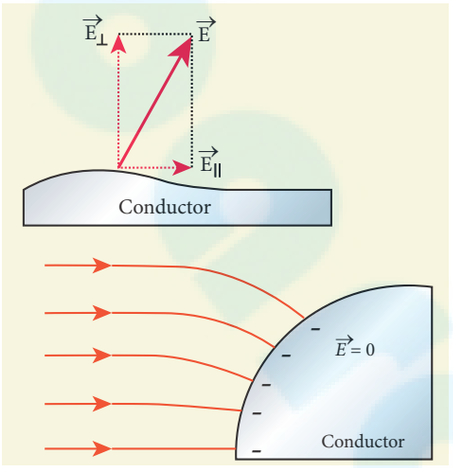


Figure 1.44 (a) Electric field is along the surface (b)Electric field is perpendicular to the surface of the conductor

UNIT-1(XII-Physics_Vol-1).indd 47 14-02-2020 19:33:29

conductor is not in equilibrium. Therefore at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure 1.44 (b).

We now prove that the electric field has magnitude $\frac{\sigma}{\epsilon_0}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the Figure 1.45. One half of this cylinder is embedded inside the conductor.

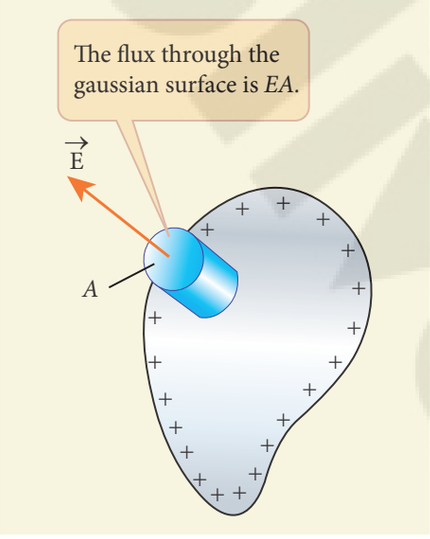


Figure 1.45 The electric field on the surface of the conductor

Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux.

Therefore the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is σA. By applying Gaus's law,

$$EA = \frac{\sigma A}{\epsilon_0}$$

In vector form,

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n} \qquad (1.79)$$

where $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.

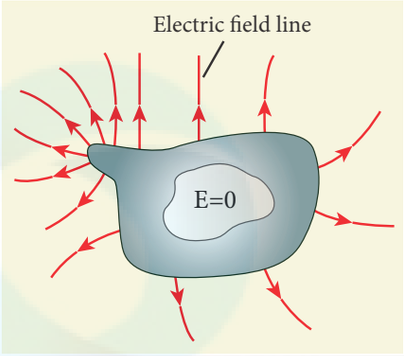
Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

1.7.2 Electrostatic shielding

Using Gauss law, we can prove that the electric field inside the charged spherical shell is zero, Further, we can show that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.

Consider a cavity inside the conductor as shown in Figure 1.46 (a). Whatever be the charges at the surfaces and whatever be the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance can be kept inside this cavity. This is called electrostatic shielding.

UNIT-1(XII-Physics_Vol-1).indd 48 14-02-2020 19:33:29



IENFYM



Figure 1.46 (a) Electric field inside the cavity (b) Faraday cage

Faraday cage is an instrument used to demonstrate this effect. It is made up of metal bars as shown in Figure1.46 (b). If an artificial lightning jolt is created outside, the person inside is not affected.

During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

1.7.3 Electrostatic induction

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of **charging without actual contact is called electrostatic induction.**

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure 1.47(a).

The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side.

Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

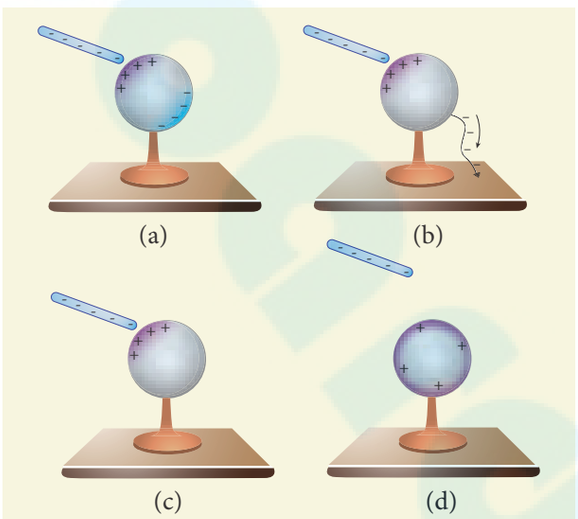


Figure 1.47 Various steps in electrostatic induction

UNIT-1(XII-Physics_Vol-1).indd 49 14-02-2020 19:33:30

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure 1.47(b)).

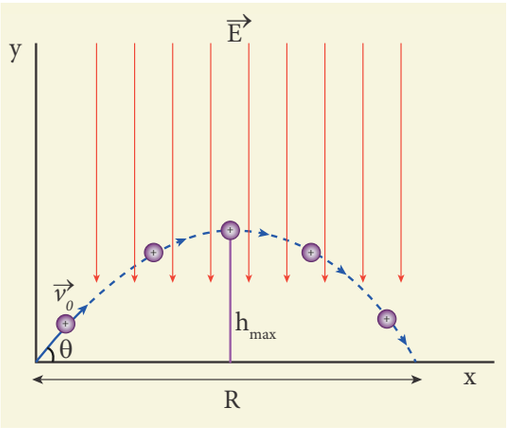
(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure 1.47(c))

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure 1.47 (d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section 1.9

EXAMPLE 1.19

A small ball of conducting material having a charge $+q$ and mass m is thrown upward at an angle θ to horizontal surface with an initial speed v_o as shown in the figure. There exists an uniform electric field E downward along with the gravitational field g. Calculate the range, maximum height and time of flight in the motion of this charged ball. Neglect the effect of air and treat the ball as a point mass.



Solution

If the conductor has no net charge, then its motion is the same as usual projectile motion of a mass m which we studied in Kinematics (unit 2, vol-1 XI physics). Here, in this problem, in addition to downward gravitational force, the charge also will experience a downward uniform electrostatic force.

The acceleration of the charged ball due to gravity = $-g\,\hat{j}$

The acceleration of the charged ball due to uniform electric field = $-\frac{qE}{m}\hat{j}$

The total acceleration of charged ball in downward direction $\vec{a} = -\left(g + \frac{qE}{m}\right)\hat{j}$

It is important here to note that the acceleration depends on the mass of the object. Galileo's conclusion that all objects fall at the same rate towards the Earth is true only in a uniform gravitational field. When a uniform electric field is included, the acceleration of a charged object depends on both mass and charge.

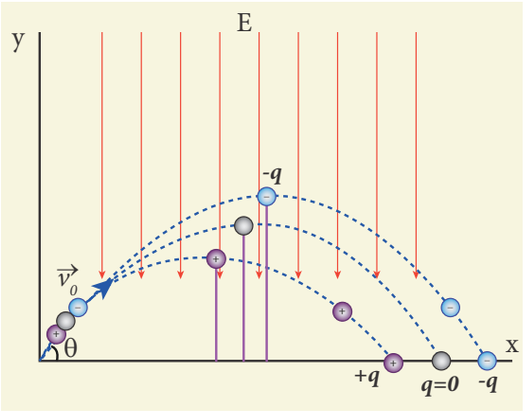
But still the acceleration $a = \left(g + \frac{qE}{m}\right)$ is constant throughout the motion. Hence we use kinematic equations to calculate the

UNIT-1(XII-Physics_Vol-1).indd 50 14-02-2020 19:33:31

range, maximum height and time of flight. In fact we can simply replace g by $g + \frac{qE}{m}$ in the usual expressions of range, maximum height and time of flight of a projectile.

	Without charge	With the charge +q
Time of flight T	$\frac{2v_\circ \sin\theta}{g}$	$\frac{2v_\circ \sin\theta}{\left(g + \frac{qE}{m}\right)}$
Maximum height h_{max}	$\frac{v_\circ^2 \sin^2\theta}{2g}$	$\frac{v_\circ^2 \sin^2\theta}{2\left(g + \frac{qE}{m}\right)}$
Range R	$\frac{v_\circ^2 \sin 2\theta}{g}$	$\frac{v_\circ^2 \sin 2\theta}{\left(g + \frac{qE}{m}\right)}$

Note that the time of flight, maximum height, range are all inversely proportional to the acceleration of the object. Since $\left(g + \frac{qE}{m}\right) > g$ for charge $+q$, the quantities T, h_{max}, and R will decrease when compared to the motion of an object of mass m and zero net charge. Suppose the charge is $-q$, then $\left(g - \frac{qE}{m}\right) < g$, and the quantities T, h_{max} and R will increase. Interestingly the trajectory is still parabolic as shown in the figure.



1.7.4 Dielectrics or insulators

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms. Ebonite, glass and mica are some examples of dielectrics. When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way. A dielectric is made up of either polar molecules or non-polar molecules.

Non-polar molecules

A non-polar molecule is one in which centres of positive and negative charges coincide. As a result, it has no permanent dipole moment. Examples of non-polar molecules are hydrogen (H_2), oxygen (O_2), and carbon dioxide (CO_2) etc.

When an external electric field is applied, the centres of positive and negative charges are separated by a small distance which induces dipole moment in the direction of the external electric field. Then the dielectric is said to be polarized by an external electric field. This is shown in Figure 1.48.

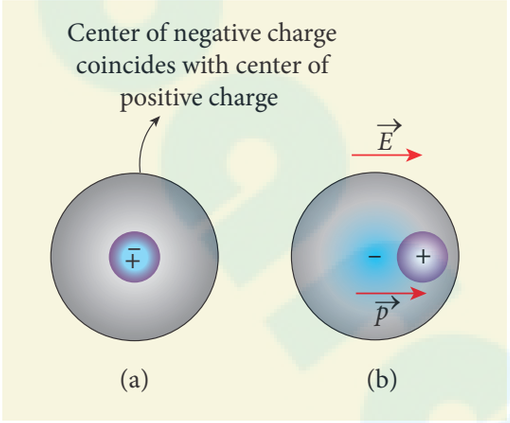


Figure 1.48 Non polar molecules (a) without external field (b) with the external field

UNIT-1(XII-Physics_Vol-1).indd 51 14-02-2020 19:33:34

Polar molecules

In polar molecules, the centres of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment. Due to thermal motion, the direction of each dipole moment is oriented randomly (Figure 1.49(a)). Hence the net dipole moment is zero in the absence of an external electric field. Examples of polar molecules are H_2O, N_2O, HCl, NH_3.

When an external electric field is applied, the dipoles inside the material tend to align in the direction of the electric field. Hence a net dipole moment is induced in it. Then the dielectric is said to be polarized by an external electric field (Figure 1.49(b)).

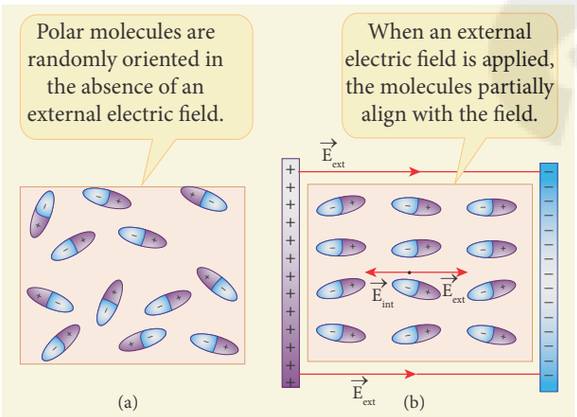


Figure 1.49 (a) Randomly oriented polar molecules (b) Align with the external electric field

Polarisation

In the presence of an external electric field, the dipole moment is induced in the dielectric material. **Polarisation $\vec{P}$ is defined as the total dipole moment per unit volume of the dielectric.** For most dielectrics (linear isotropic), the Polarisation is directly proportional to the strength of the external electric field. This is written as

$$\vec{P} = \chi_e \vec{E}_{ext} \qquad (1.80)$$

where χ_e is a constant called the electric susceptibility which is a characteristic of each dielectric.

1.7.5 Induced Electric field inside the dielectric

When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which tends to cancel the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field. Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the Figure 1.50.

The uniform electric field between the plates acts as an external electric field $\vec{E}_{ext}$ which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface.

But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in

UNIT-1(XII-Physics_Vol-1).indd 52 14-02-2020 19:33:35

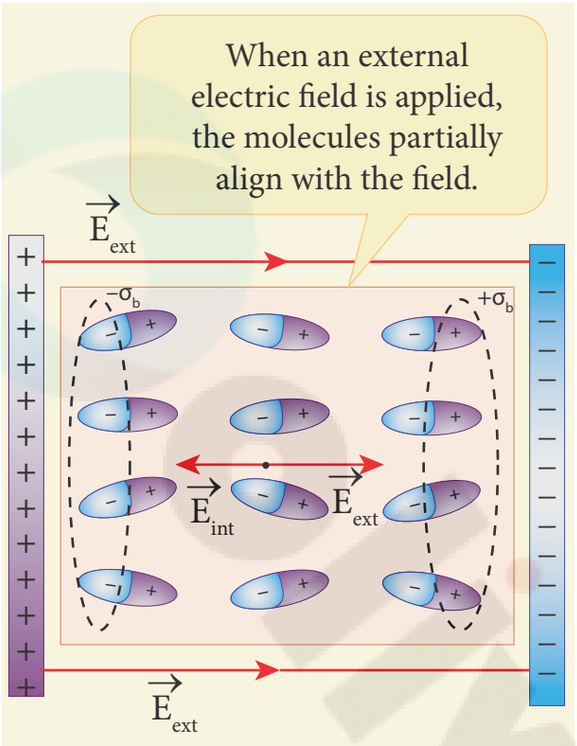


Figure 1.50 Induced electric field lines inside the dielectric

conductors. This is shown in the Figure 1.50.

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown in Figure 1.51.

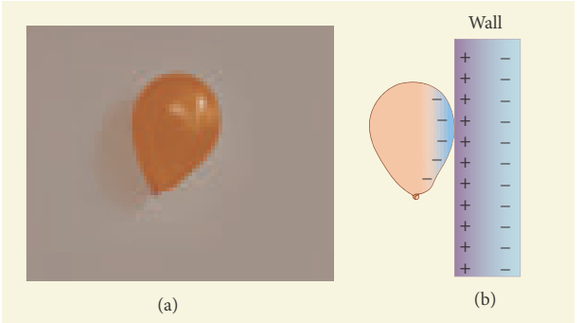


Figure 1.51 (a) Balloon sticks to the wall (b) Polarisation of wall due to the electric field created by the balloon

1.7.6 Dielectric strength

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. The maximum electric field the dielectric can withstand before it breaksdown is called dielectric strength. For example, the dielectric strength of air is 3×10^6 V m^{-1}. If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in the Table 1.1.

Table 1.1 Dielectric strength

Substance	Dielectric strength (Vm^{-1})
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Air	3×10^6
Pyrex glass	14×10^6

1.8 CAPACITORS AND CAPACITANCE

1.8.1 Capacitors

Capacitor is a device used to store electric charge and electrical energy. It consists of two conducting objects (usually plates or sheets) separated by some distance. Capacitors are widely used in many electronic circuits and have applications in many areas of science and technology.

UNIT-1(XII-Physics_Vol-1).indd 53 14-02-2020 19:33:35

A simple capacitor consists of two parallel metal plates separated by a small distance as shown in Figure 1.52 (a).

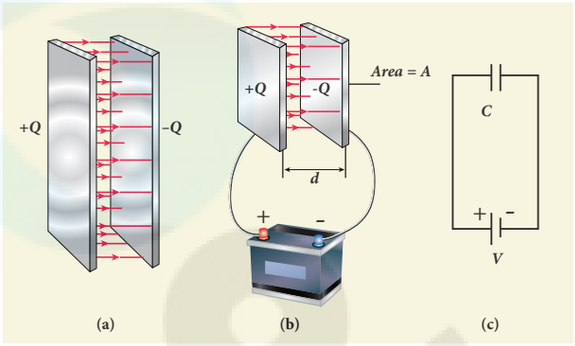


Figure 1.52 (a) Parallel plate capacitor (b) Capacitor connected with a battery (c) Symbolic representation of capacitor.

When a capacitor is connected to a battery of potential difference V, the electrons are transferred from one plate to the other plate by battery so that one plate becomes negatively charged with a charge of –Q and the other plate positively charged with +Q. The potential difference between the plates is equivalent to the battery's terminal voltage. This is shown in Figure 1.52 (b). If the battery voltage is increased, the amount of charges stored in the plates also increase. In general, the charge stored in the capacitor is proportional to the potential difference between the plates.

$$Q \propto V$$

so that $Q = CV$

where the C is the proportionality constant called capacitance. **The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between them.**

$$C = \frac{Q}{V} \qquad (1.81)$$

The SI unit of capacitance is *coulomb per volt* or *farad (*F*)* in honor of Michael Faraday. Farad is a larger unit of capacitance. In practice, capacitors are available in the range of microfarad (1μF = 10^{-6} F) to picofarad (1pF = 10^{-12} F). A capacitor is represented by the symbol ⫫ or ⫫. Note that the total charge stored in the capacitor is zero ($Q - Q = 0$). When we say the capacitor stores charges, it means the amount of charge that can be stored in any one of the plates.

Nowadays there are capacitors available in various shapes (cylindrical, disk) and types (tantalum, ceramic and electrolytic), as shown in Figure 1.53. These capacitors are extensively used in various kinds of electronic circuits.



Figure 1.53 Various types of capacitors

Capacitance of a parallel plate capacitor

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown in Figure 1.54.

The electric field between two infinite parallel plates is uniform and is given by $E = \frac{\sigma}{\epsilon_0}$ where σ is the surface charge density on either plates $\left(\sigma = \frac{Q}{A}\right)$. If the separation distance d is very much smaller than the size of the plate ($d^2 << A$), then the above result can be used even for finite–sized parallel plate capacitor.

UNIT-1(XII-Physics_Vol-1).indd 54 14-02-2020 19:33:36

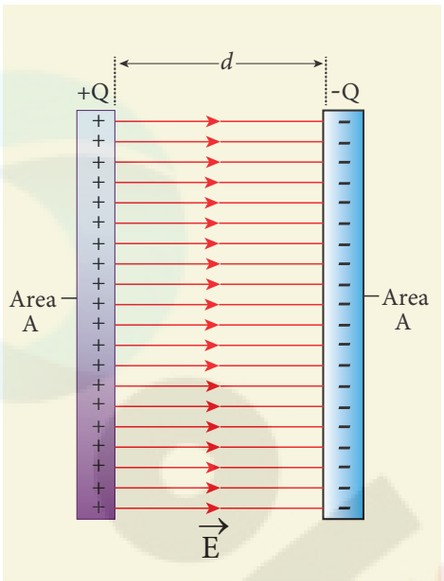


Figure 1.54 Capacitance of a parallel plate capacitor

The electric field between the plates is

$$E = \frac{Q}{A\epsilon_{\circ}} \qquad (1.82)$$

Since the electric field is uniform, the electric potential difference between the plates having separation d is given by

$$V = Ed = \frac{Qd}{A\epsilon_0} \qquad (1.83)$$

Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\epsilon_0}\right)} = \frac{\epsilon_0 A}{d} \qquad (1.84)$$

From equation (1.84), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance d between the two plates is reduced, the potential difference between the plates ($V = Ed$) decreases with E constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery's terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

EXAMPLE 1.20

A parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm. (a) Calculate the capacitance of this capacitor. (b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates? (The value of $\epsilon_{\circ} = 8.85 \times 10^{-12}$ $N^{-1}m^{-2}\ C^2$)

Solution

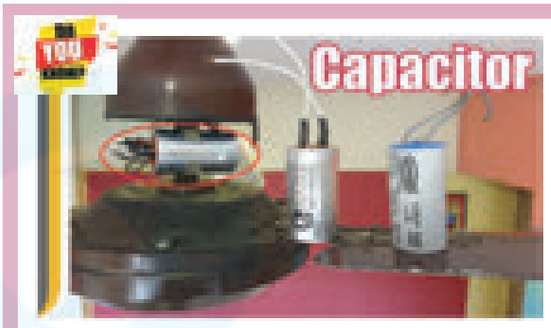
(a) The capacitance of the capacitor is

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$$
$$= 221.2\times10^{-13}\,\text{F}$$
$$C = 22.12\times10^{-12}\,\text{F} = 22.12\,\text{pF}$$

(b) The charge stored in any one of the plates is Q = CV, Then

$$Q = 22.12\times10^{-12}\times10 = 221.2\times10^{-12}\,\text{C} = 221.2\,\text{pC}$$

UNIT-1(XII-Physics_Vol-1).indd 55 14-02-2020 19:33:38



Sometimes we notice that the ceiling fan does not start rotating as soon as it is switched on. But when we rotate the blades, it starts to rotate as usual. Why it is so? We know that to rotate any object, there must be a torque applied on the object. For the ceiling fan, the initial torque is given by the capacitor widely known as a condenser. If the condenser is faulty, it will not give sufficient initial torque to rotate the blades when the fan is switched on.

1.8.2 Energy stored in the capacitor

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge $-Q$ are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = V\,dQ \qquad (1.85)$$

where $V = \frac{Q}{C}$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C} dQ = \frac{Q^2}{2C} \qquad (1.86)$$

This work done is stored as electrostatic potential energy (U_E) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (\because Q = CV) \qquad (1.87)$$

where $Q = CV$ is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor.

But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the results $C = \frac{\epsilon_0 A}{d}$ and $V = Ed$

$$U_E = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 (Ad) E^2 \qquad (1.88)$$

where Ad = volume of the space between the capacitor plates. **The energy stored per unit volume of space is defined as energy density** $u_E = \frac{U}{Volume}$. From equation (1.88), we get

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad (1.89)$$

From equation (1.89), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

UNIT-1(XII-Physics_Vol-1).indd 56 14-02-2020 19:33:40

1.8.3 Applications of capacitors

Capacitors are used in various electronics circuits. A few of the applications.

(a) Flash capacitors are used in digital cameras for taking photographs. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor (Figure 1.55 (a))

(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This is shown in Figure 1.55 (b).

(c) Capacitors are used in the ignition system of automobile engines to eliminate sparking

(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

However, capacitors have disadvantage as well. Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. For example if the TV is switched off, it is always advisable to not touch the back side of the TV panel.

1.8.4 Effect of dielectrics in capacitors

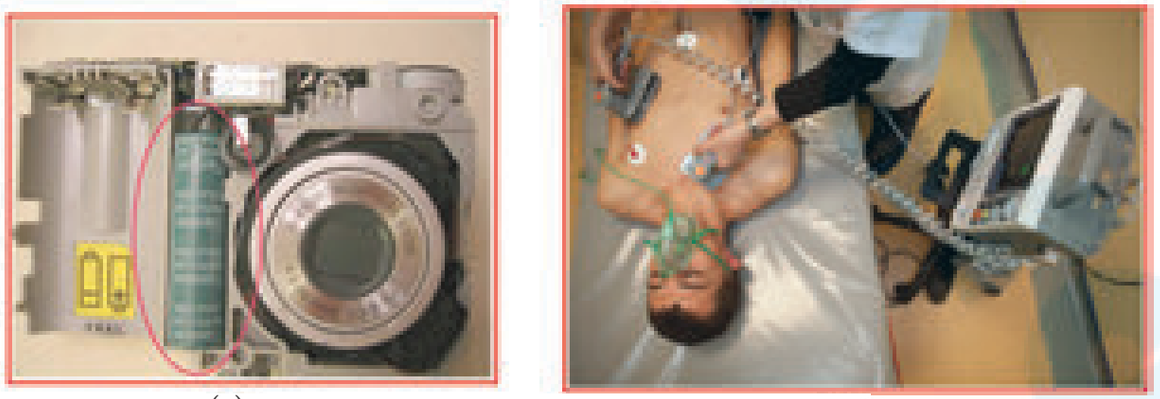
In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) when the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0} \qquad (1.90)$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in Figure 1.56.



(a) (b)

Figure 1.55 (a) Flash capacitor in camera (b) Heart defibrillator

UNIT-1(XII-Physics_Vol-1).indd 57 14-02-2020 19:33:40

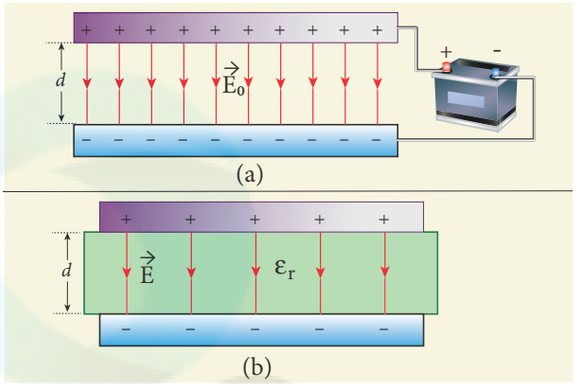


Figure 1.56 (a) Capacitor is charged with a battery (b) Dielectric is inserted after the battery is disconnected

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by

$$E = \frac{E_0}{\epsilon_r} \tag{1.91}$$

where E_o is the electric field inside the capacitors when there is no dielectric and ϵ_r is the relative permittivity of the dielectric or simply known as the dielectric constant. Since $\epsilon_r > 1$, the electric field $E < E_o$.

As a result, the electrostatic potential difference between the plates ($V = Ed$) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected.

Hence the new potential difference is

$$V = Ed = \frac{E_0}{\epsilon_r} d = \frac{V_0}{\epsilon_r} \tag{1.92}$$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases.

Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = \epsilon_r \frac{Q_0}{V_0} = \epsilon_r C_0 \tag{1.93}$$

Since $\epsilon_r > 1$, we have $C > C_o$. Thus insertion of the dielectric increases the capacitance.

Using equation (1.84),

$$C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{\epsilon A}{d} \tag{1.94}$$

where $\epsilon = \epsilon_r \epsilon_o$ is the permittivity of the dielectric medium.

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0} \tag{1.95}$$

After the dielectric is inserted, the charge Q_0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{\epsilon_r C_0} = \frac{U_0}{\epsilon_r} \tag{1.96}$$

Since $\epsilon_r > 1$ we get $U < U_o$. There is a decrease in energy because,when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage V_0 remains connected to the capacitor when the dielectric is inserted into the capacitor. This is shown in Figure1.57.

The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ϵ_r.

UNIT-1(XII-Physics_Vol-1).indd 58 14-02-2020 19:33:42

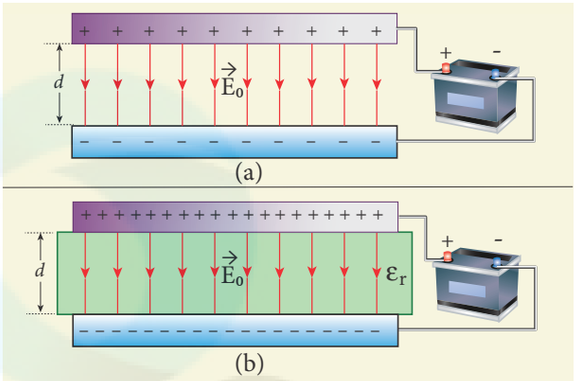


Figure 1.57 (a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

$$Q = \epsilon_r Q_o \qquad (1.97)$$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = \epsilon_r \frac{Q_0}{V_0} = \epsilon_r C_0 \qquad (1.98)$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

Now, $C_0 = \dfrac{\epsilon_0 A}{d}$

and $C = \dfrac{\epsilon A}{d}$ (1.99)

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2 \qquad (1.100)$$

Note that here we have not used the expression $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ because here, both charge and capacitance are changed, whereas in equation (1.100), V_o remains constant.

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} \epsilon_r C_0 V_0^2 = \epsilon_r U_0 \qquad (1.101)$$

Since $\epsilon_r > 1$ we have $U > U_o$.

It may be noted here that since voltage between the capacitor V_0 is constant, the electric field between the plates also remains constant.

The energy density is given by

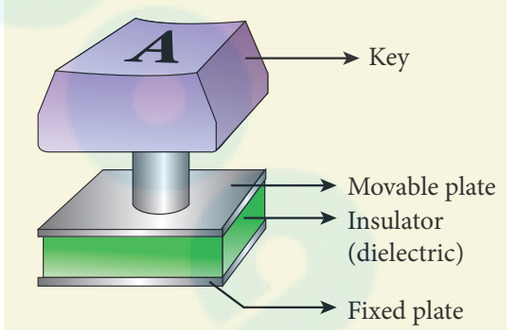
$$u = \frac{1}{2} \epsilon E_0^2 \qquad (1.102)$$

where ϵ is the permittivity of the given dielectric material.

The results of the above discussions are summarised in the following Table 1.2



Computer keyboard keys are constructed using capacitors with a dielectric as shown in the figure.



When the key is pressed, the separation between the plates decreases leading to an increase in the capacitance. This in turn triggers the electronic circuits in the computer to identify which key is pressed.

UNIT-1(XII-Physics_Vol-1).indd 59 14-02-2020 19:33:44

Table 1.2 Effect of dielectrics in capacitors

S. No	Dielectric is inserted	Charge *Q*	Voltage *V*	Electric field *E*	Capacitance *C*	Energy *U*
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

EXAMPLE 1.21

A parallel plate capacitor filled with mica having $\varepsilon_r = 5$ is connected to a 10 V battery. The area of each parallel plate is 6 cm^2 and separation distance is 6 mm. (a) Find the capacitance and stored charge.

(b) After the capacitor is fully charged, the battery is disconnected and the dielectric is removed carefully.

Calculate the new values of capacitance, stored energy and charge.

Solution

(a) The capacitance of the capacitor in the presence of dielectric is

$$C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 6 \times 10^{-4}}{6 \times 10^{-3}}$$
$$= 44.25 \times 10^{-13}\,\text{F} = 4.425\ \text{pF}$$

The stored charge is

$$Q = CV = 44.25 \times 10^{-13} \times 10$$
$$= 442.5 \times 10^{-13}\,\text{C} = 44.25\,\text{pC}$$

The stored energy is

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 44.25C \times 10^{-13} \times 100$$
$$= 2.21 \times 10^{-10}\,\text{J}$$

(b) After the removal of the dielectric, since the battery is already disconnected the total charge will not change. But the potential difference between the plates increases. As a result, the capacitance is decreased.

New capacitance is

$$C_0 = \frac{C}{\varepsilon_r} = \frac{4.425 \times 10^{-12}}{5}$$
$$= 0.885 \times 10^{-12}\,\text{F} = 0.885\,\text{pF}$$

The stored charge remains same and 44.25 pC. Hence newly stored energy is

$$U_0 = \frac{Q^2}{2C_0} = \frac{Q^2 \epsilon_r}{2C} = \epsilon_r U$$
$$= 5 \times 2.21 \times 10^{-10}\,\text{J} = 11.05 \times 10^{-10}\,\text{J}$$

The increased energy is

$$\Delta U = (11.05 - 2.21) \times 10^{-10}\,\text{J} = 8.84 \times 10^{-10}\,\text{J}$$

When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84×10^{-10} J.

1.8.5 Capacitor in series and parallel

(i) Capacitor in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the Figure 1.58 (a).

As soon as the battery is connected to the capacitors in series, the electrons of charge

UNIT-1(XII-Physics_Vol-1).indd 60 14-02-2020 19:33:47

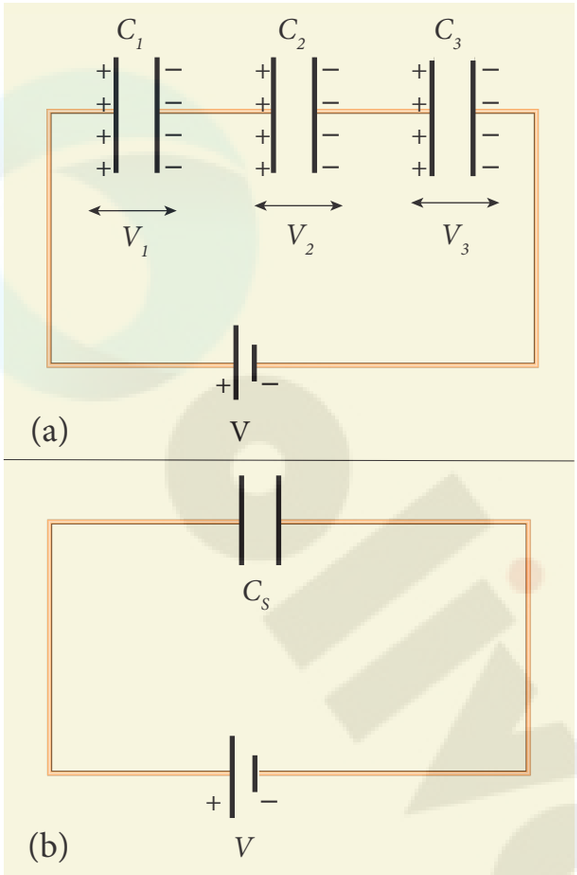


Figure 1.58 (a) Capacitors connected in series (b) Equivalent capacitors C_S

$-Q$ are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount $-Q$ from left plate of C_3 to the right plate of C_2 due to electrostatic induction. Similarly, the left plate of C_2 pushes the charges of $-Q$ to the right plate of C_1 which induces the positive charge $+Q$ on the left plate of C_1. At the same time, electrons of charge $-Q$ are transferred from left plate of C_1 to positive terminal of the battery.

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.

The sum of the voltages across the capacitor must be equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3 \qquad (1.103)$$

Since, $Q = CV$, we have $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad (1.104)$$

If three capacitors in series are considered to form an equivalent single capacitor C_s shown in Figure 1.58(b), then we have $V = \frac{Q}{C_S}$. Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (1.105)$$

Thus, the inverse of the equivalent capacitance C_S of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance C_S is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in Figure 1.59 (a).

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitances of the capacitors are different,

UNIT-1(XII-Physics_Vol-1).indd 61 14-02-2020 19:33:49

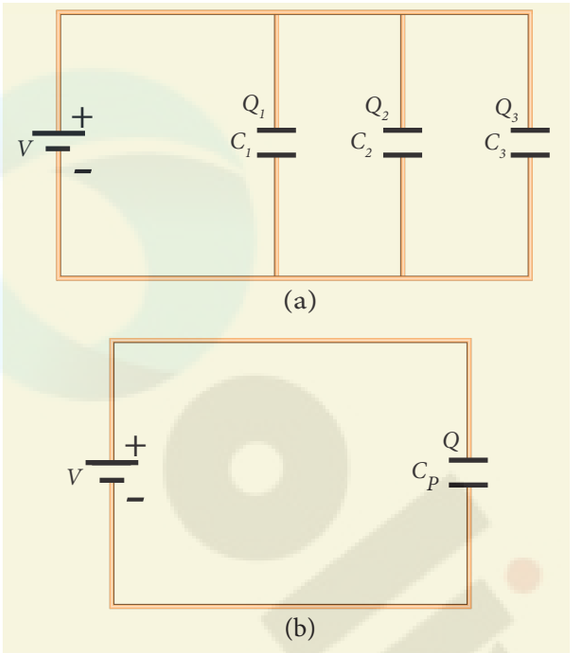


Figure 1.59 (a) capacitors in parallel (b) equivalent capacitance with the same total charge

the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2, and Q_3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3 \qquad (1.106)$$

Since $Q = CV$, we have

$$Q = C_1V + C_2V + C_3V \qquad (1.107)$$

If these three capacitors are considered to form a single equivalent capacitance C_P which stores the total charge Q as shown in the Figure 1.59(b), then we can write $Q = C_PV$. Substituting this in equation (1.107), we get

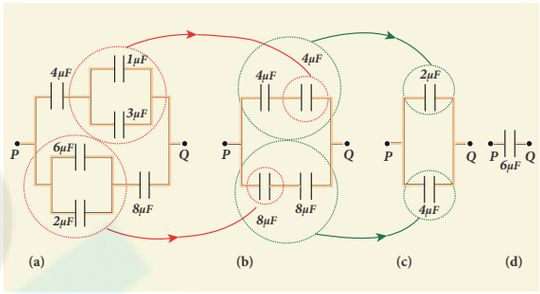
$$C_PV = C_1V + C_2V + C_3V$$

$$C_P = C_1 + C_2 + C_3 \qquad (1.108)$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances. The equivalent capacitance C_P in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

EXAMPLE 1.22

Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).



Solution

The capacitors 1 μF and 3 μF are connected in parallel and 6 μF and 2 μF are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

$$C_{eq} = 1 + 3 = 4\ \mu F$$

$$C_{eq} = 6 + 2 = 8\ \mu F$$

From the figure (b), we infer that the two 4 μF capacitors are connected in series and the two 8 μF capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

UNIT-1(XII-Physics_Vol-1).indd 62 14-02-2020 19:33:49

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \qquad \Rightarrow C_{eq} = 2\ \mu F$$

and

$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \qquad \Rightarrow C_{eq} = 4\ \mu F$$

From the figure (c), we infer that 2 μF and 4 μF are connected in parallel. So the equivalent capacitance is given in the figure (d).

$$C_{eq} = 2 + 4 = 6\ \mu F$$

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 μF.

1.9 DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

1.9.1 Distribution of charges in a conductor

Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in the Figure 1.60. The distance between the spheres is much greater than the radii of either spheres.

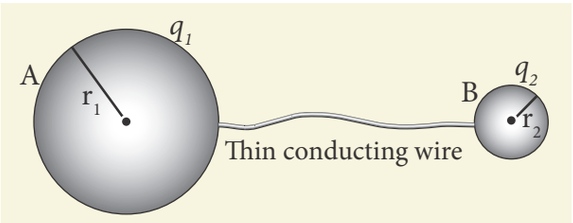


Figure 1.60 Two conductors are connected through conducting wire

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that $Q = q_1 + q_2$. The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\epsilon_\circ}\frac{q_1}{r_1} \qquad (1.110)$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\epsilon_\circ}\frac{q_2}{r_2} \qquad (1.111)$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B$$
$$\text{or} \quad \frac{q_1}{r_1} = \frac{q_2}{r_2} \qquad (1.112)$$

Let the charge density on the surface of sphere A be σ_1 and that on the surface of sphere B be σ_2. This implies that $q_1 = 4\pi r_1^2\sigma_1$ and

$q_2 = 4\pi r_2^2\sigma_2$. Substituting these values into equation (1.112), we get

$$\sigma_1 r_1 = \sigma_2 r_2 \qquad (1.113)$$

from which we conclude that

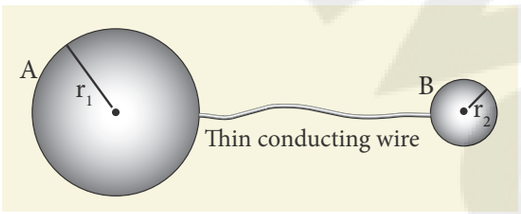
$$\sigma r = \text{constant} \qquad (1.114)$$

UNIT-1(XII-Physics_Vol-1).indd 63 14-02-2020 19:33:51

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

EXAMPLE 1.23

Two conducting spheres of radius r_1 = 8 cm and r_2 = 2 cm are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of $Q = +100$ nC is placed on one of the spheres. After a fraction of a second, the charge Q is redistributed and both the spheres attain electrostatic equilibrium.



(a) Calculate the charge and surface charge density on each sphere.

(b) Calculate the potential at the surface of each sphere.

Solution

(a) The electrostatic potential on the surface of the sphere A is $V_A = \frac{1}{4\pi\epsilon_\circ}\frac{q_1}{r_1}$

The electrostatic potential on the surface of the sphere B is $V_B = \frac{1}{4\pi\epsilon_\circ}\frac{q_2}{r_2}$

Since $V_A = V_B$. We have

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 = \left(\frac{r_1}{r_2}\right)q_2$$

But from the conservation of total charge, $Q = q_1 + q_2$, we get $q_1 = Q - q_2$. By substituting this in the above equation,

$$Q - q_2 = \left(\frac{r_1}{r_2}\right)q_2$$

so that $q_2 = Q\left(\frac{r_2}{r_1 + r_2}\right)$

Therefore,

$$q_2 = 100\times10^{-9}\times\left(\frac{2}{10}\right) = 20 \text{ nC}$$

and $q_1 = Q - q_2 = 80$nC

The electric charge density on sphere A is $\sigma_1 = \frac{q_1}{4\pi r_1^2}$

The electric charge density on sphere B is $\sigma_2 = \frac{q_2}{4\pi r_2^2}$

Therefore,

$$\sigma_1 = \frac{80\times10^{-9}}{4\pi\times64\times10^{-4}} = 0.99\times10^{-6}\,\text{C m}^{-2}$$

and

$$\sigma_2 = \frac{20\times10^{-9}}{4\pi\times4\times10^{-4}} = 3.9\times10^{-6}\,\text{C m}^{-2}$$

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere ($\sigma_2 \approx 4\sigma_1$) which confirms the result $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$.

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

$$V_A = \frac{1}{4\pi\epsilon_\circ}\frac{q_1}{r_1} = \frac{9\times10^9\times80\times10^{-9}}{8\times10^{-2}} = 9\text{kV}$$

1.9.2 Action of points or Corona discharge

Consider a charged conductor of irregular shape as shown in Figure 1.61 (a).

UNIT-1(XII-Physics_Vol-1).indd 64 14-02-2020 19:33:54

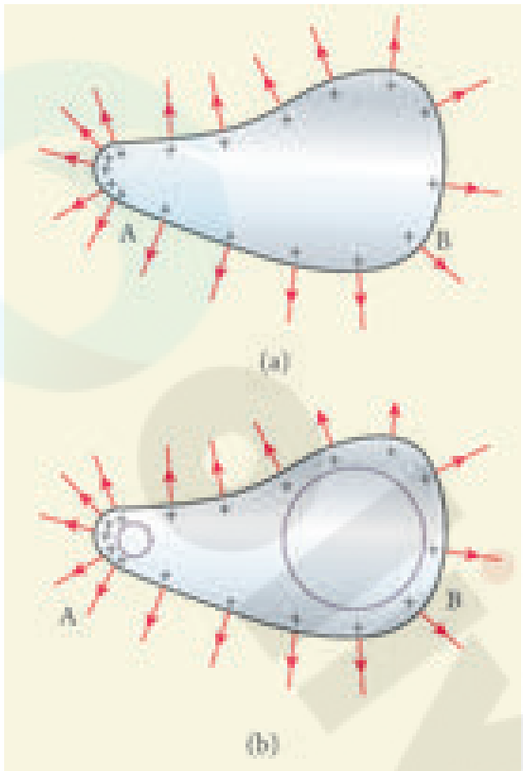


Figure 1.61 Action of points or corona discharge

We know that smaller the radius of curvature, the larger is the charge density. The end of the conductor which has larger curvature (smaller radius) has a large charge accumulation as shown in Figure 1.61 (b).

As a result, the electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action of points or corona discharge.

1.9.3 Lightning arrester or lightning conductor

This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge.

This device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in Figure 1.62 (a) and (b).

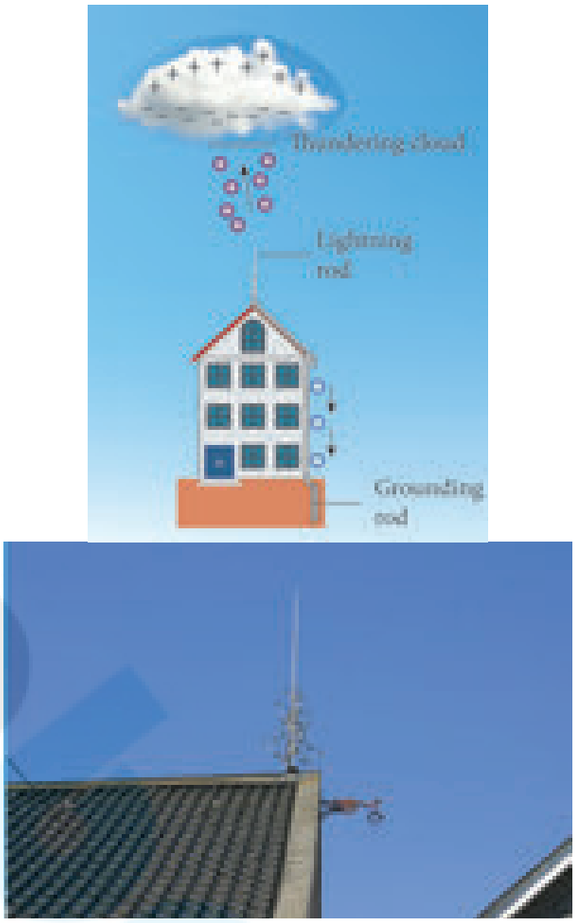


Figure 1.62 (a) Schematic diagram of a lightning arrestor. (b) A house with a lightning arrestor

The lower end of the rod is connected to copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces a positive charge on the spike. Since the induced charge density on thin sharp spike is large, it results in a corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod and

UNIT-1(XII-Physics_Vol-1).indd 65 14-02-2020 19:33:55

is safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely.

1.9.4 Van de Graaff Generator

In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V). This Van de Graff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.63. A pulley B is mounted at the centre of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.

The comb D is maintained at a positive potential of 10^4 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

Due to the high electric field near comb D, air between the belt and comb D gets ionized by the action of points. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges on the belt reach the point near the comb E, the comb E acquires negative charge and the sphere acquires positive charge due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor,

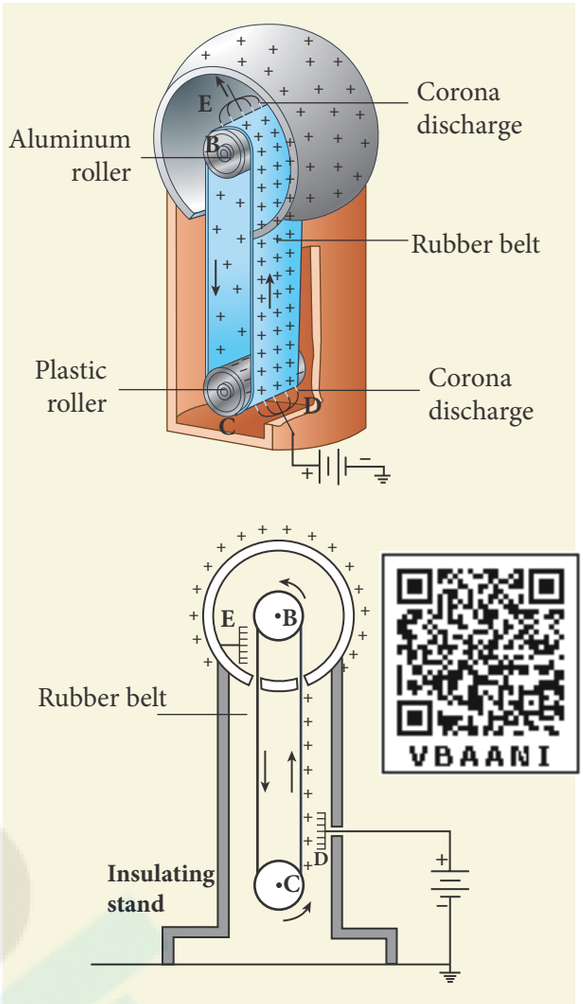


Figure 1.63 Van de Graaff generator

the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can

UNIT-1(XII-Physics_Vol-1).indd 66 14-02-2020 19:33:55

be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

EXAMPLE 1.24

Dielectric strength of air is 3×10^6 V m^{-1}. Suppose the radius of a hollow sphere in the Van de Graff generator is R = 0.5 m, calculate the maximum potential difference created by this Van de Graaff generator.

Solution

The electric field on the surface of the sphere is given by (by Gauss law)

$$E = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{R^2}$$

The potential on the surface of the hollow metallic sphere is given by

$$V = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{R} = ER$$

Since $V_{max} = E_{max}R$

Here $E_{max} = 3\times10^6\ \text{Vm}^{-1}$. So the maximum potential difference created is given by

$$V_{max} = 3 \times 10^6 \times 0.5$$
$$= 1.5 \times 10^6 \text{ V (or) } 1.5 \text{ million volt}$$

UNIT-1(XII-Physics_Vol-1).indd 67 14-02-2020 19:33:56

SUMMARY

- Like charges repel and unlike charges attract
- The total charge in the universe is conserved
- Charge is quantized. Total charge in an object $q = ne$ where $n = 0,1,2,3\ldots$ and e is charge of the electron.
- Coulomb's law in vector form: $\vec{F} = \frac{1}{4\pi\epsilon_\circ}\frac{q_1 q_2}{r^2}\hat{r}$ ($\hat{r}$ is unit vector along joining q_1, q_2)
- Electrostatic force obeys the superposition principle.
- Electric field at a distance r from a point charge: $\vec{E} = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r^2}\hat{r}$
- Electric field lines starts at a positive charge and end at a negative charge or at infinity
- Electric field due to electric dipole at points on the axial line : $\vec{E}_{tot} = \frac{1}{4\pi\epsilon_\circ}\left(\frac{2\vec{p}}{r^3}\right)$
- Electric field due to electric dipole at points on the equatorial line: $\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_\circ}\left(\frac{\vec{p}}{r^3}\right)$
- Torque experienced by a dipole in a uniform electric field: $\vec{\tau} = \vec{p}\times\vec{E}$
- Electrostatic potential at a distance r from the point charge: $V = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r}$
- Electrostatic potential due to an electric dipole: $V = \frac{1}{4\pi\epsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2}$
- The electrostatic potential is the same at all points on an equipotential surface.
- The relation between electric field and electrostatic potential:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

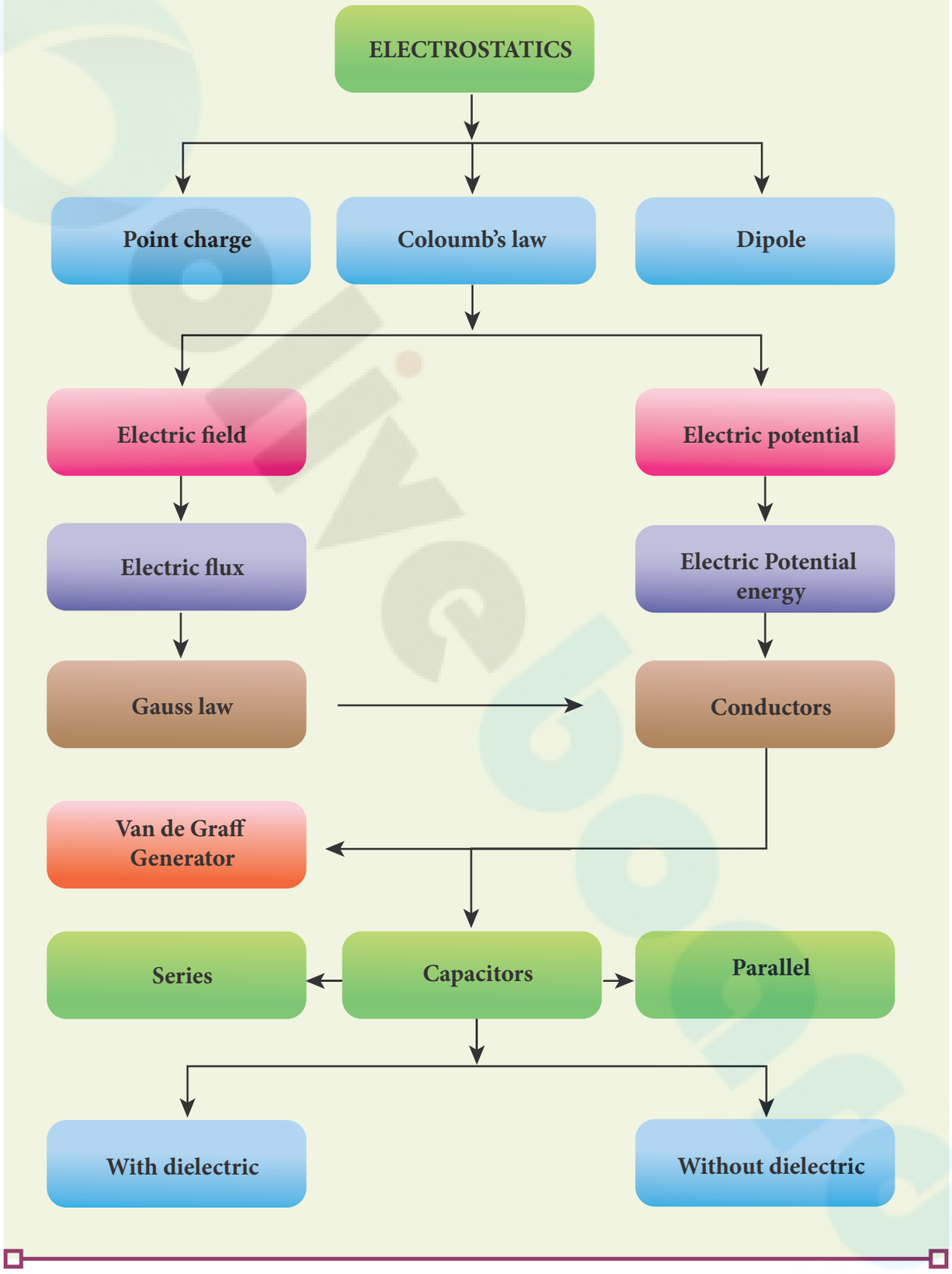
- Electrostatic potential energy for system of charges is equal to the work done to arrange the charges in the given configuration.
- Electrostatic potential energy of a dipole system in a uniform electric field: $U = -\vec{p}\cdot\vec{E}$
- The total electric flux through a closed surface : $\Phi_E = \frac{Q}{\epsilon_\circ}$ where Q is the net charge enclosed by the surface
- Electric field due to a charged infinite wire : $\vec{E} = \frac{1}{2\pi\epsilon_\circ}\frac{\lambda}{r}\hat{r}$
- Electric field due to a charged infinite plane : $\vec{E} = \frac{\sigma}{2\epsilon_\circ}\hat{n}$ ($\hat{n}$ is normal to the plane)
- Electric field inside a charged spherical shell is zero. For points outside: $\vec{E} = \frac{Q}{4\pi\epsilon_\circ r^2}\hat{r}$

UNIT-1(XII-Physics_Vol-1).indd 68 14-02-2020 19:34:00

- Electric field inside a conductor is zero. The electric field at the surface of the conductor is normal to the surface and has magnitude $E = \frac{\sigma}{\epsilon_0}$.
- The surface of the conductor has the same potential, at all points on the surface.
- Conductor can be charged using the process of induction.
- A dielectric or insulator has no free electrons. When an electric field is applied, the dielectric is polarised.
- Capacitance of a conductor is given by $C = \frac{Q}{V}$.
- Capacitance of a parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$
- Electrostatic energy stored in a capacitor: $U = \frac{1}{2}CV^2$
- The equivalent capacitance for parallel combination is equal to the sum of individual capacitance of the capacitors.
- For a series combination: The inverse of equivalent capacitance is equal to sum of inverse of individual capacitance of capacitors.
- The distribution of charges in the conductors depends on the shape of conductor. For sharper edge, the surface charge density is greater. This principle is used in the lightning arrestor
- Van de Graaff generator is used to produce large potential difference (~10^7 V).

UNIT-1(XII-Physics_Vol-1).indd 69 14-02-2020 19:34:01

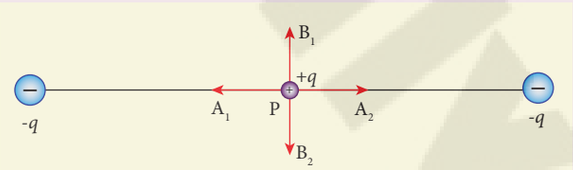
CONCEPT MAP



UNIT-1(XII-Physics_Vol-1).indd 70 14-02-2020 19:34:01

I Multiple choice questions

1. Two identical point charges of magnitude $-q$ are fixed as shown in the figure below. A third charge $+q$ is placed midway between the two charges at the point P. Suppose this charge $+q$ is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will $+q$ be stable with respect to the displacement?



(a) A_1 and A_2 (b) B_1 and B_2

(c) both directions (d) No stable

2. Which charge configuration produces a uniform electric field?

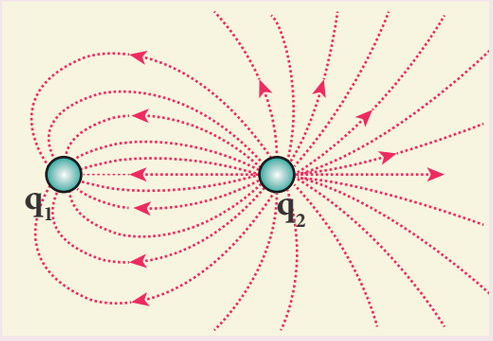
(a) point charge

(b) uniformly charged infinite line

(c) uniformly charged infinite plane

(d) uniformly charged spherical shell

3. What is the ratio of the charges $\left|\frac{q_1}{q_2}\right|$ for the following electric field line pattern?



(a) $\frac{1}{5}$ (b) $\frac{25}{11}$

(c) 5 (d) $\frac{11}{25}$

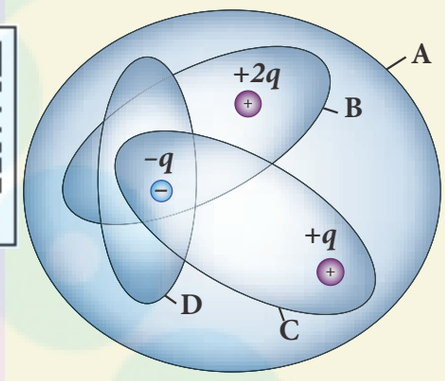
4. An electric dipole is placed at an alignment angle of 30° with an electric field of 2×10^5 N C^{-1}. It experiences a torque equal to 8 N m. The charge on the dipole if the dipole length is 1 cm is

(a) 4 mC (b) 8 mC

(c) 5 mC (d) 7 mC

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.

87W5DE



(a) D < C < B < A

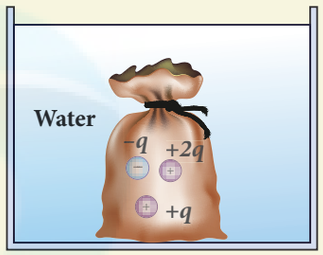
(b) A < B = C < D

(c) C < A = B < D

(d) D > C > B > A

6. The total electric flux for the following closed surface which is kept inside water

UNIT-1(XII-Physics_Vol-1).indd 71 14-02-2020 19:34:02



(a) $\frac{80q}{\epsilon_0}$ (b) $\frac{q}{40\epsilon_0}$

(c) $\frac{q}{80\epsilon_0}$ (d) $\frac{q}{160\epsilon_0}$

7. Two identical conducting balls having positive charges q_1 and q_2 are separated by a centre to centre distance r. If they are made to touch each other and then separated to the same distance, the force between them will be
(NSEP 04-05)

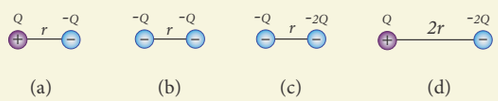
(a) less than before

(b) same as before

(c) more than before

(d) zero

8. Rank the electrostatic potential energies for the given system of charges in increasing order.



(a) 1 = 4 < 2 < 3 (b) 2 = 4 < 3 < 1

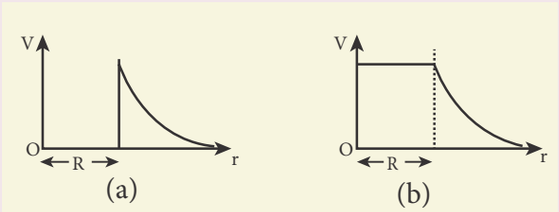
(c) 2 = 3 < 1 < 4 (d) 3 < 1 < 2 < 4

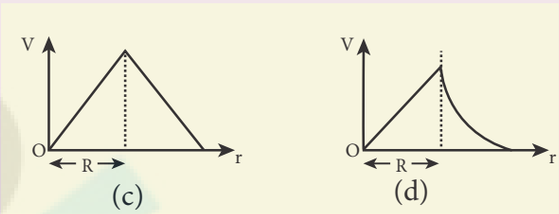
9. An electric field $\vec{E} = 10x\hat{i}$ exists in a certain region of space. Then the potential difference $V = V_o - V_A$, where V_o is the potential at the origin and V_A is the potential at x = 2 m is:

(a) 10 V (b) –20 V

(c) +20 V (d) –10 V

10. A thin conducting spherical shell of radius R has a charge Q which is uniformly distributed on its surface. The correct plot for electrostatic potential due to this spherical shell is





11. Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is

(a) 8.80×10^{-17} J

(b) -8.80×10^{-17} J

(c) 4.40×10^{-17} J

(d) 5.80×10^{-17} J

12. If voltage applied on a capacitor is increased from V to $2V$, choose the correct conclusion.

(a) Q remains the same, C is doubled

(b) Q is doubled, C doubled

(c) C remains same, Q doubled

(d) Both Q and C remain same

UNIT-1(XII-Physics_Vol-1).indd 72 14-02-2020 19:34:04

13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?

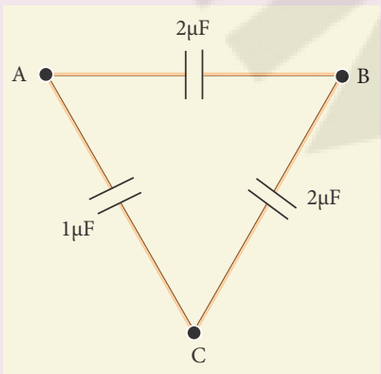
(a) Capacitance

(b) Charge

(c) Voltage

(d) Energy density

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the points A and C is



(a) 1μF (b) 2 μF

(c) 3 μF (d) $\frac{1}{4}$μF

15. Two metallic spheres of radii 1 cm and 3 cm are given charges of -1×10^{-2} C and 5×10^{-2} C respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is (AIIPMT -2012)

(a) 3×10^{-2} C

(b) 4×10^{-2} C

(c) 1×10^{-2} C

(d) 2×10^{-2} C

Answers

1) b	**2)** c	**3)** d	**4)** b	**5)** a
6) b	**7)** c	**8)** a	**9)** c	**10)** b
11) a	**12)** c	**13)** d	**14)** b	**15)** a

II Short Answer Questions

1. What is meant by quantisation of charges?
2. Write down Coulomb's law in vector form and mention what each term represents.
3. What are the differences between Coulomb force and gravitational force?
4. Write a short note on superposition principle.
5. Define 'electric field'.
6. What is mean by 'electric field lines'?
7. The electric field lines never intersect. Justify.
8. Define 'electric dipole'. Give the expression for the magnitiude of its electric dipole moment and the direction.
9. Write the general definition of electric dipole moment for a collection of point charge.
10. Define 'electrostatic potential".
11. What is an equipotential surface?
12. What are the properties of an equipotential surface?
13. Give the relation between electric field and electric potential.
14. Define 'electrostatic potential energy'.
15. Define 'electric flux'.
16. What is meant by electrostatic energy density?

UNIT-1(XII-Physics_Vol-1).indd 73 14-02-2020 19:34:04

17. Write a short note on 'electrostatic shielding'.
18. What is polarisation?
19. What is dielectric strength?
20. Define 'capacitance'. Give its unit.
21. What is corona discharge?

III Long Answer questions

1. Discuss the basic properties of electric charges.
2. Explain in detail Coulomb's law and its various aspects.
3. Define 'electric field' and discuss its various aspects.
4. Calculate the electric field due to a dipole on its axial line and equatorial plane.
5. Derive an expression for the torque experienced by a dipole due to a uniform electric field.
6. Derive an expression for electrostatic potential due to a point charge.
7. Derive an expression for electrostatic potential due to an electric dipole.
8. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.
9. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.
10. Obtain Gauss law from Coulomb's law.
11. Obtain the expression for electric field due to an infinitely long charged wire.
12. Obtain the expression for electric field due to an charged infinite plane sheet.
13. Obtain the expression for electric field due to an uniformly charged spherical shell.
14. Discuss the various properties of conductors in electrostatic equilibrium.
15. Explain the process of electrostatic induction.
16. Explain dielectrics in detail and how an electric field is induced inside a dielectric.
17. Obtain the expression for capacitance for a parallel plate capacitor.
18. Obtain the expression for energy stored in the parallel plate capacitor.
19. Explain in detail the effect of a dielectric placed in a parallel plate capacitor.
20. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.
21. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.
22. Explain in detail the construction and working of a Van de Graaff generator.

Exercises

1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

   Ans: 31.25×10^{10} electrons
2. The total number of electrons in the human body is typically in the order of 10^{28}. Suppose, due to some reason, you

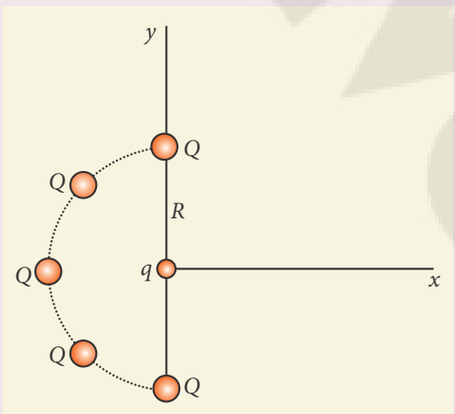
UNIT-1(XII-Physics_Vol-1).indd 74 14-02-2020 19:34:05

and your friend lost 1% of this number of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

Ans: $F_e = 23 \times 10^{23}$ N, W = 588 N,

$$\frac{F_e}{W} = 3.9 \times 10^{21}$$

3. Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the centre of the circle of radius R. Calculate the electrostatic force experienced by the charge q.



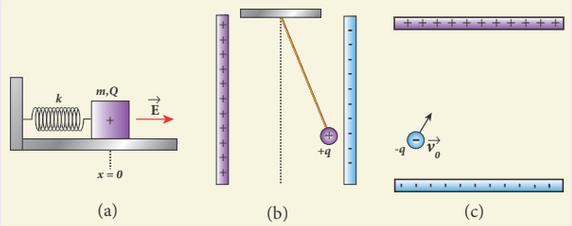
Ans: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{R^2}\left(1+\sqrt{2}\right)\hat{i}$ N

4. Suppose a charge $+q$ on Earth's surface and another $+q$ charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change?

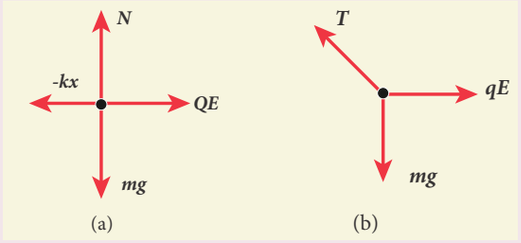
(Take $m_E = 5.9 \times 10^{24}$ kg, $m_M = 7.9 \times 10^{22}$ kg)

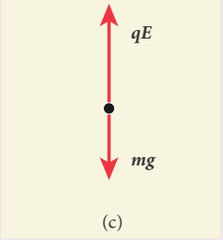
Ans: (a) $q \approx +5.87 \times 10^{13}$ C, (b) no change

5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).

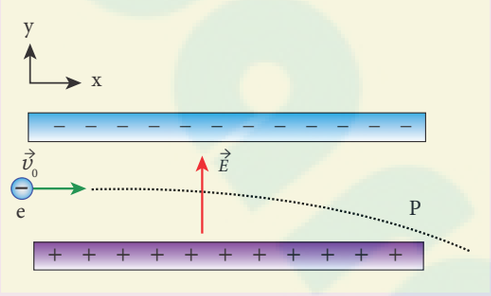


Ans:





6. Consider an electron travelling with a speed v_0 and entering into a uniform electric field $\vec{E}$ which is perpendicular to $\vec{v}_0$ as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.

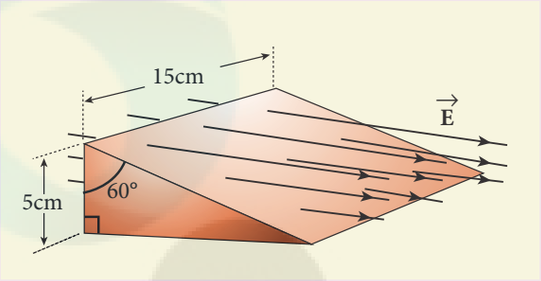


Ans :

$$\vec{a} = -\frac{eE}{m}\hat{j}, \vec{v} = v_0\hat{i} - \frac{eE}{m}t\hat{j}, \vec{r} = v_0 t\hat{i} - \frac{1}{2}\frac{eE}{m}t^2\hat{j}$$

UNIT-1(XII-Physics_Vol-1).indd 75 14-02-2020 19:34:08

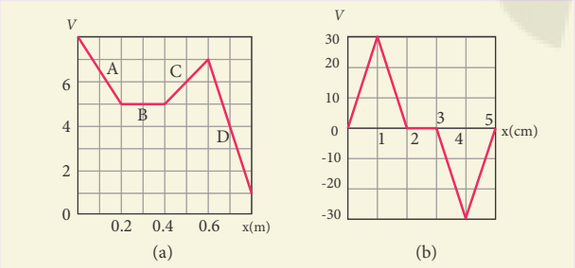
7. A closed triangular box is kept in an electric field of magnitude $E = 2 \times 10^3\ N\,C^{-1}$ as shown in the figure.



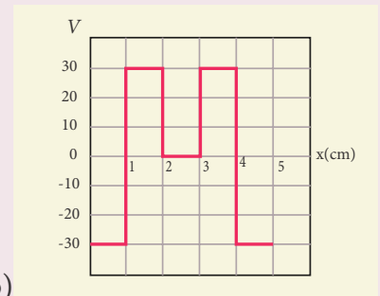
Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface.

Ans: (a) 15 $Nm^2\ C^{-1}$ (b) 15 $Nm^2\ C^{-1}$ (c) zero

8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).



Ans: (a) E_X = 15 Vm^{-1} (region A), E_X = -10 Vm^{-1} (region C)
E_X = 0 (region B), E_X = 30 Vm^{-1} (region D)



(b)

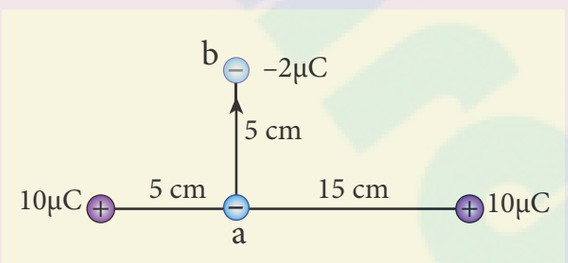
9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure.



To create the spark, an electric field of magnitude 3 × 10^6 Vm^{-1} is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same? (c) find the potential difference if the gap is 1 mm.

Ans: (a) 1800 V, (b) increases (c) 3000 V

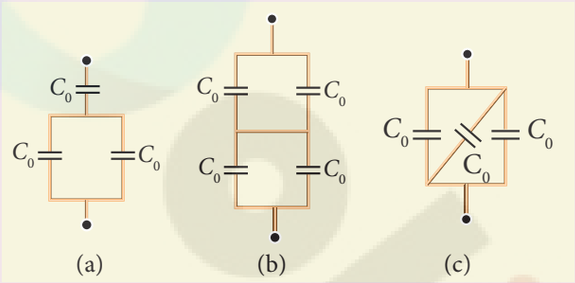
10. A point charge of +10 μC is placed at a distance of 20 cm from another identical point charge of +10 μC. A point charge of -2 μC is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.

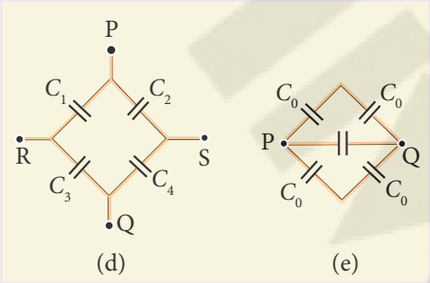


UNIT-1(XII-Physics_Vol-1).indd 76 14-02-2020 19:34:08

Ans: $\Delta U = +1.12$ J, positive sign implies that to move the charge $-2\mu C$ external work is required.

11. Calculate the resultant capacitances for each of the following combinations of capacitors.





Ans: (a) $\frac{2}{3}C_o$ (b) C_o (c) $3C_o$

(d) across PQ:

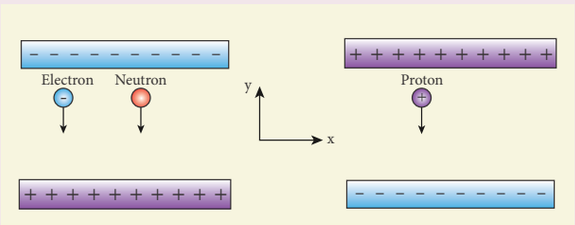
$$\frac{C_1C_2C_3 + C_2C_3C_4 + C_1C_2C_4 + C_1C_3C_4}{(C_1 + C_3)(C_2 + C_4)}$$

across RS:

$$\frac{C_1C_2C_3 + C_2C_3C_4 + C_1C_2C_4 + C_1C_3C_4}{(C_1 + C_2)(C_3 + C_4)}$$

(e) across PQ: $2\,C_o$

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance h = 1 mm as shown in the figure.



(a) Calculate the time of flight for both electron and proton (b) Suppose if a neutron is allowed to fall, what is the time of flight? (c) Among the three, which one will reach the bottom first? (Take $m_p = 1.6 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg and g = 10 m s^{-2})

Ans:

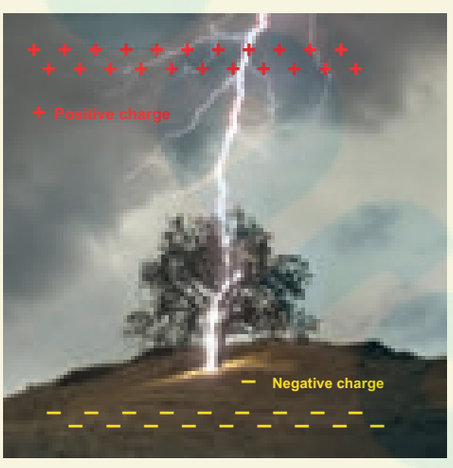
(a) $t_e = \sqrt{\frac{2hm_e}{eE}} \approx 1.5$ ns (ignoring the gravity),

$t_p = \sqrt{\frac{2hm_p}{eE}} \approx 63$ ns (ignoring the gravity)

(b) $t_n = \sqrt{\frac{2h}{g}} \approx 14.1$ ms

(c) electron will reach first

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air (3×10^6 Vm^{-1}), lightning will occur.



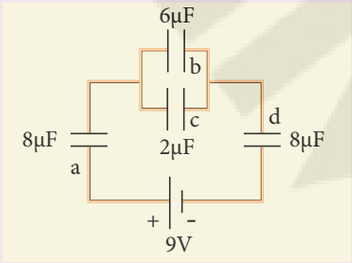
UNIT-1(XII-Physics_Vol-1).indd 77 14-02-2020 19:34:11

(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25 C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Ans: (a) $V = 3 \times 10^9$ V, (b) $U = 75 \times 10^9$ J

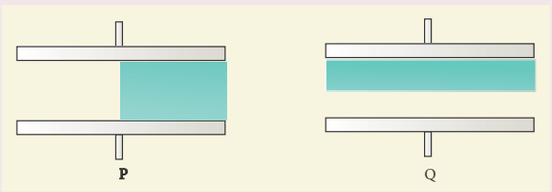
14. For the given capacitor configuration (a) Find the charges on each capacitor (b) potential difference across them (c) energy stored in each capacitor



Ans:

$Q_a = 24\ \mu C$, $Q_b = 18\ \mu C$,
$Q_c = 6\ \mu C$, $Q_d = 24\ \mu C$
$V_a = 3V$, $V_b = 3V$,
$V_c = 3V$, $V_d = 3V$,
$U_a = 36\ \mu J$, $U_b = 27\ \mu J$,
$U_c = 9\ \mu J$, $U_d = 36\ \mu J$

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ϵ_r as shown in the figure. Calculate the capacitance of capacitors P and Q.



Ans : $C_P = \frac{\epsilon_o A}{2d}(1+\epsilon_r)$, $C_Q = \frac{2\epsilon_o A}{d}\left(\frac{\epsilon_r}{1+\epsilon_r}\right)$

BOOKS FOR REFERENCE

1. Douglas C.Giancoli, , "*Physics for Scientist & Engineers with Modern Physics*", Pearson Prentice Hall, Fourth edition.
2. James Walker, "*Physics*", Pearson- Addison Wesley Publishers, Fourth Edition.
3. Purcell, Morin, "*Electricity and Magnetism*", "Cambridge University Press, Third Edition.
4. Serway and Jewett, "*Physics for Scientist and Engineers with Modern Physics*", Brook/Coole Publishers, Eighth Edition.
5. Tipler, Mosca, "*Physics for scientist and Engineers with Modern Physics*", Freeman and Company, Sixth Edition.
6. Tarasov and Tarasova, "*Questions and problems in School Physics*", Mir Publishers.
7. H.C.Verma, "*Concepts of Physics: Vol 2*, Bharthi Bhawan Publishers.
8. Eric Roger, *Physics for the Inquiring Mind,* Princeton University Press.

UNIT-1(XII-Physics_Vol-1).indd 78 14-02-2020 19:34:11

ICT CORNER

Electrostatics

In this activity you will be able to learn about capacitor and the factors affecting capacitance.

Topic: Capacitor lab

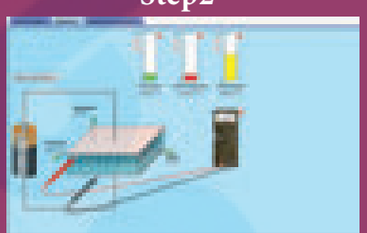
STEPS:

- Open the browser and type "phet.colorado.edu/en/simulation/legacy/capacitor-lab" in the address bar. Go to the tab 'Dielectric'.
- Change the plate area, distance between the plate and dielectric. Identify what you would maximize or minimize to make a capacitor with the greatest capacitance.
- Explore the relationships between charge, voltage, and stored energy for a capacitor. Design a capacitor system to store the greatest energy.
- Charge the capacitor with 1.0 v using the battery. Disconnect the battery. Now insert a dielectric between the plates. Discus how electric field changes in between the plates when dielectric is introduced.
- What is the effect of introducing a dielectric between plates? (Change dielectric materials)

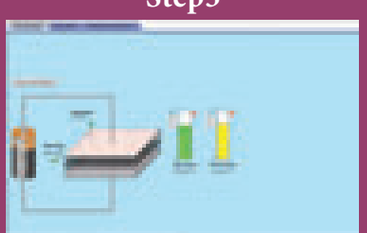
Step1



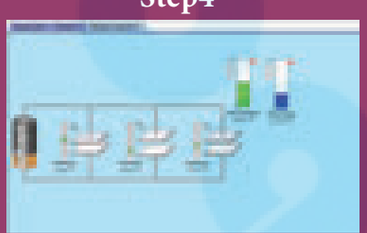
Step2



Step3



Step4



Connect capacitors parallel and series combination and find the effective capacitance.

Note:

Install Java application if it is not in your system. You can download all the phet simulation and works in off line from https://phet.colorado.edu/en/offline-access .

URL:

https://phet.colorado.edu/en/simulation/legacy/capacitor-lab

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT-1(XII-Physics_Vol-1).indd 79 14-02-2020 19:34:12

UNIT 2

CURRENT ELECTRICITY

We will make electricity so cheap that only the rich will burn candles
– Thomas A. Edison

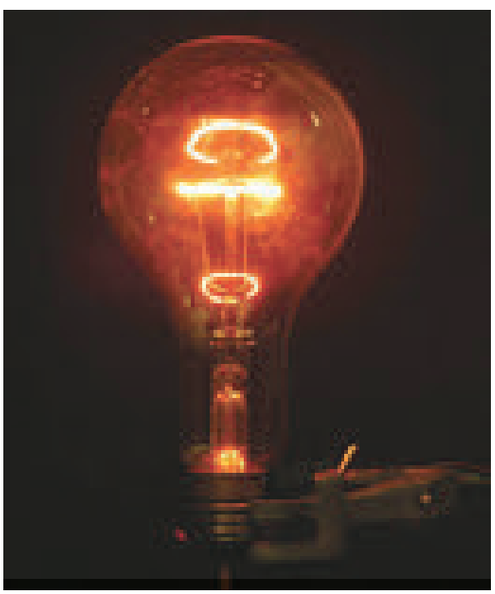
Learning Objectives

In this unit, the student is exposed to

- Flow of charges in a metallic conductor
- Ohm's law, electrical resistance, V-I characteristics
- Carbon resistors and combination of resistors
- Kirchhoff's laws - Wheatstone's bridge and its applications
- Electric power and Electric energy
- Heating effect - Joule's law – applications
- Thermoelectric effects – Seebeck effect – Peltier effect – Thomson effect

K4HZ3A

INTRODUCTION



In unit 1, we studied the properties of charges when they are at rest. In reality, the charges are always moving within the materials. For example, the electrons in a copper wire are never at rest and are continuously in random motion. Therefore it is important to analyse the behaviour of charges when they are in motion. The motion of charges constitutes 'electric current'. Current electricity is the study of flow of electric charges. It owes its origin to Alessandro Volta (1745-1827), who invented the electric battery which produced the first steady flow of electric current. Modern world depends heavily on the use of electricity. It is used to operate machines, communication systems, electronic devices, home appliances etc., In this unit, we will study about the electric current, resistance and related phenomenon in materials.

UNIT-2(XII-Physics_Vol-1).indd 80 14-02-2020 19:36:37

2.1 ELECTRIC CURRENT

Matter is made up of atoms. Each atom consists of a positively charged nucleus with negatively charged electrons moving around the nucleus. Atoms in metals have one or more electrons which are loosely bound to the nucleus. These electrons are called free electrons and can be easily detached from the atoms. The substances which have an abundance of these free electrons are called conductors. These free electrons move randomly throughout the conductor at a given temperature. In general due to this random motion, there is no net transfer of charges from one end of the conductor to other end and hence no current in the conductor. When a potential difference is applied by the battery across the ends of the conductor, the free electrons drift towards the positive terminal of the battery, producing a net electric current. This is easily understandable from the analogy given in the Figure 2.1.

In the XI Volume 2, unit 6, we studied, that the mass move from higher gravitational potential to lower gravitational potential. Likewise, positive charge flows from region of higher electric potential to region of lower electric potential and negative charge flows from region of lower electric potential to region of higher electric potential. So battery or electric cell simply creates potential difference across the conductor.

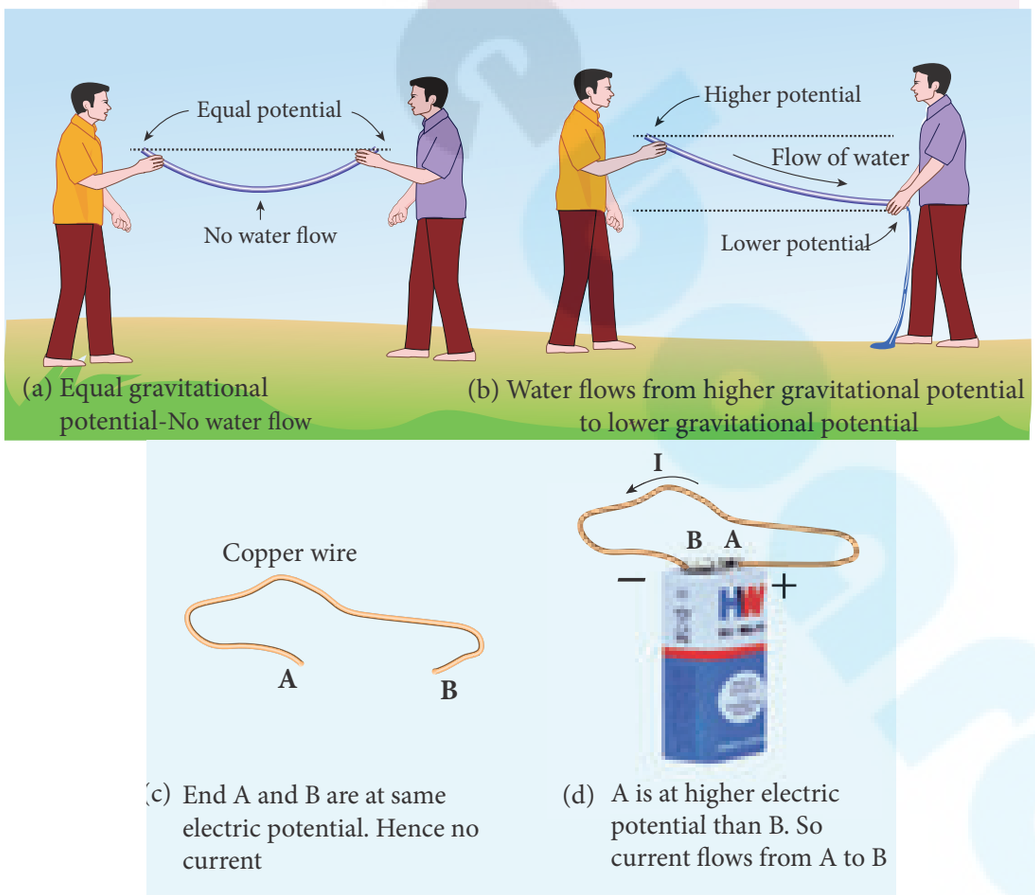


Figure 2.1 Water current and Electric current

UNIT-2(XII-Physics_Vol-1).indd 81 14-02-2020 19:36:38

The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area A. It is shown in the Figure 2.2.

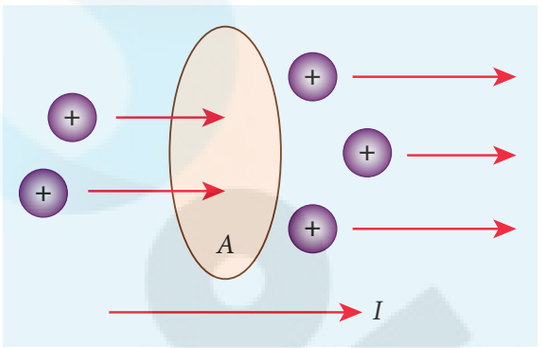


Figure 2.2 Charges flow across the area A

If a net charge Q passes through any cross section of a conductor in time t, then the current is defined as $I = \frac{Q}{t}$. But charge flow is not always constant. Hence current can more generally be defined as

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad (2.1)$$

Where ΔQ is the amount of charge that passes through the conductor at any cross section during the time interval Δt. If the rate at which charge flows changes with time, the current also changes. The instantaneous current I is defined as the limit of the average current, as $\Delta t \to 0$

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad (2.2)$$

The SI unit of current is the **ampere** (A)

$$1\text{A} = \frac{1\text{C}}{1\text{s}}$$

That is, 1A of current is equivalent to 1 coulomb of charge passing through a perpendicular cross section in a conductor in one second. The electric current is a scalar quantity.

EXAMPLE 2.1

Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

Solution

The current (rate of flow of charge) in the wire is

$$I = \frac{Q}{t} = \frac{120}{60} = 2\text{A}$$

2.1.1 Conventional Current

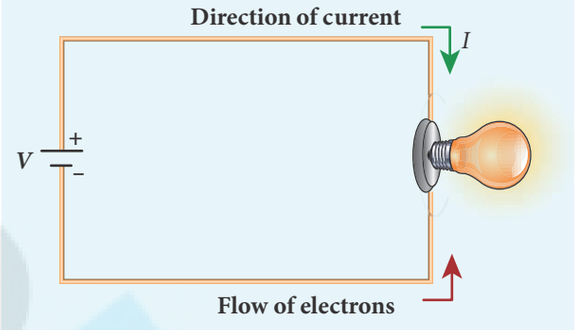


Figure 2.3 Direction of conventional current and electron flow

In an electric circuit, arrow heads are used to indicate the direction of flow of current. By convention, this flow in the circuit should be from the positive terminal of the battery to the negative terminal. This current is called the conventional current or simply current and is in the direction in which a positive test charge would move. In typical circuits the charges that flow are actually electrons, from the negative terminal of the battery to the positive terminal. As a result, the flow of electrons and the direction of conventional current point in opposite direction as shown in Figure 2.3. Mathematically, a transfer of positive charge is the

X14TS6

UNIT-2(XII-Physics_Vol-1).indd 82 14-02-2020 19:36:39

same as a transfer of negative charge in the opposite direction.



Electric current is not only produced by batteries. In nature, lightning bolt produces enormous electric current in a short time. During lightning, very high potential difference is created between the clouds and ground and hence charges flow between the clouds and ground.

2.1.2 Drift velocity

In a conductor the charge carriers are free electrons. These electrons move freely through the conductor and collide repeatedly with the positive ions. If there is no electric field, the electrons move in random directions, and hence their velocities are also randomly oriented. On an average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. As a result, there is no net flow of electrons in any direction and hence there will not be any current.

Suppose a potential difference is set across the conductor by connecting a battery, an electric field $\vec{E}$ is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons, while ions scatter the electrons and change their direction of motion. Thus, we see zigzag motion of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor in a direction opposite to that of $\vec{E}$ as shown in the Figure 2.4.

Ions

Any material is made up of neutral atoms with equal number of electrons and protons. If the outermost electrons leave the atoms, they become free electrons and are responsible for electric current. The atoms after losing their outer most electrons will have more positive charges and hence are called positive ions. These ions will not move freely within the material like the free electrons. Hence the positive ions will not give rise to current.

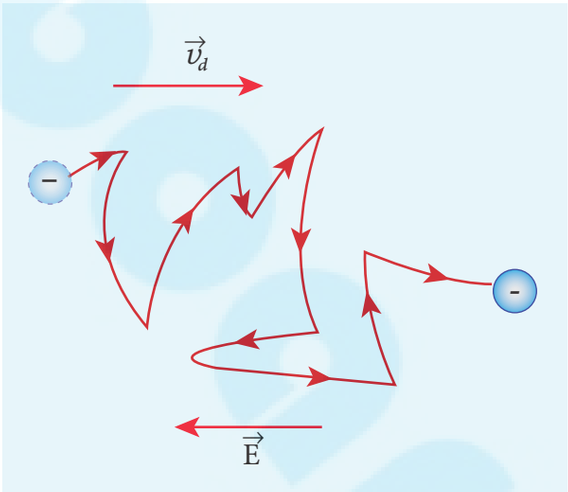


Figure 2.4 Zig-zag motion and drift velocity

This velocity is called drift velocity $\vec{v}_d$. The drift velocity is the average velocity acquired by the electrons inside the conductor when

UNIT-2(XII-Physics_Vol-1).indd 83 14-02-2020 19:36:40

it is subjected to an electric field. The average time between two successive collisions is called the mean free time denoted by τ. The acceleration $\vec{a}$ experienced by the electron in an electric field $\vec{E}$ is given by

$$\vec{a} = \frac{-e\vec{E}}{m} \qquad \left(\text{since } \vec{F} = -e\vec{E}\right) \qquad (2.3)$$

The drift velocity $\vec{v}_d$ is given by

$$\vec{v}_d = \vec{a}\,\tau$$

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E} \qquad (2.4)$$

$$\vec{v}_d = -\mu\vec{E} \qquad (2.5)$$

Here $\mu = \frac{e\tau}{m}$ is the mobility of the electron and it is defined as the magnitude of the drift velocity per unit electric field.

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|} \qquad (2.6)$$

The SI unit of mobility is $m^2V^{-1}s^{-1}$.

Note

The typical drift velocity of electrons in the wire is 10^{-4} m s^{-1}. If an electron drifts with this speed, then the electrons leaving the battery will take hours to reach the light bulb. Then how electric bulbs glow as soon as we switch on the battery? When battery is switched on, the electrons begin to move away from the negative terminal of the battery and this electron exerts force on the nearby electrons. This process creates a propagating influence (electric field) that travels through the wire at the speed of light. In other words, the energy is transported from the battery to bulb at the speed of light through propagating influence (electric field). Due to this reason, the bulb glows as soon as the battery is switched on.

EXAMPLE 2.2

If an electric field of magnitude 570 N C^{-1}, is applied in the copper wire, find the acceleration experienced by the electron.

Solution:

E = 570 N C^{-1}, $e = 1.6 \times 10^{-19}$ C,
$m = 9.11 \times 10^{-31}$ kg and a = ?

$F = ma = eE$

$$a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}$$

$$= \frac{912 \times 10^{-19} \times 10^{31}}{9.11}$$

$$= 1.001 \times 10^{14} \text{ m s}^{-2}$$

Misconception

(i) There is a common misconception that the battery is the source of electrons. It is not true. When a battery is connected across the given wire, the electrons in the closed circuit resulting the current. Battery sets the potential difference (electrical energy) due to which these electrons in the conducting wire flow in a particular direction. The resulting electrical energy is used by electric bulb, electric fan etc. Similarly the electricity board is supplying the electrical energy to our home.

(ii) We often use the phrases like 'charging the battery in my mobile' and 'my mobile phone battery has no charge' etc. These sentences are not correct.



UNIT-2(XII-Physics_Vol-1).indd 84 14-02-2020 19:36:44

When we say 'battery has no charge', it means, that the battery has lost ability to provide energy or provide potential difference to the electrons in the circuit. When we say 'mobile is charging', it implies that the battery is receiving energy from AC power supply and not electrons.

2.1.3 Microscopic model of current

Consider a conductor with area of cross section A and let an electric field $\vec{E}$ be applied to it from right to left. Suppose there are n electrons per unit volume in the conductor and assume that all the electrons move with the same drift velocity $\vec{v}_d$ as shown in Figure 2.5.

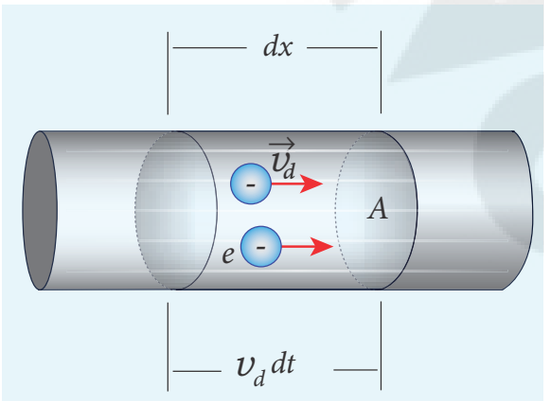


Figure 2.5 Microscopic model of current

The drift velocity of the electrons = v_d

If the electrons move through a distance dx within a small interval of dt, then

$$v_d = \frac{dx}{dt}\,; \quad dx = v_d\,dt \qquad (2.7)$$

Since A is the area of cross section of the conductor, the electrons available in the volume of length dx is

= volume × number of electrons per unit volume

$$= A\,dx \times n \qquad (2.8)$$

Substituting for dx from equation (2.7) in (2.8)

$$= (A\ v_d\,dt\ )\ n$$

Total charge in the volume element dQ = (charge) × (number of electrons in the volume element)

$$dQ = (e)(A v_d dt)n$$

Hence the current $I = \frac{dQ}{dt}$

$$I = ne\,A v_d \qquad (2.9)$$

Current density (J)

The current density (J) is defined as the current per unit area of cross section of the conductor.

$$J = \frac{I}{A}$$

The S.I unit of current density is $\frac{\text{A}}{\text{m}^2}$ (or) A m^{-2}

$J = \frac{neAv_d}{A}$ (from equation 2.9)

$$J = nev_d \qquad (2.10)$$

The above expression is valid only when the direction of the current is perpendicular to the area A. In general, the current density is a vector quantity and it is given by

$$\vec{J} = ne\vec{v}_d$$

Substituting $\vec{v_d}$ from equation (2.4)

$$\vec{J} = -\frac{n \cdot e^2 \tau}{m}\vec{E} \qquad (2.11)$$

$$\vec{J} = -\sigma\vec{E}$$

But conventionally, we take the direction of (conventional) current density as the direction of electric field. So the above equation becomes

$$\vec{J} = \sigma\vec{E} \qquad (2.12)$$

where $\sigma = \frac{ne^2\tau}{m}$ is called conductivity.

The equation (2.12) is called microscopic form of ohm's law.

UNIT-2(XII-Physics_Vol-1).indd 85 14-02-2020 19:36:47

The inverse of conductivity is called resistivity (ρ) [Refer section 2.2.1].

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad (2.13)$$

EXAMPLE 2.3

A copper wire of cross-sectional area 0.5 mm^2 carries a current of 0.2 A. If the free electron density of copper is 8.4×10^{28} m^{-3} then compute the drift velocity of free electrons.

Solution

The relation between drift velocity of electrons and current in a wire of cross-sectional area A is

$$v_d = \frac{I}{neA} = \frac{0.2}{8.4\times10^{28}\times1.6\times10^{-19}\times0.5\times10^{-6}}$$

$$v_d = 0.03 \times 10^{-3} \text{ m s}^{-1}$$

Note

Why current density is a vector but current is a scalar?

In general, the current I is defined as the scalar product of the current density and area vector in which the charges cross.

$$I = \vec{J} \cdot \vec{A}$$

The current I can be positive or negative depending on the choice of the unit vector normal to the surface area A.

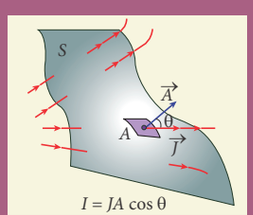


Figure 2.6 Current is a scalar

EXAMPLE 2.4

Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it.

Solution

I = 32 A , t = 1 s

Charge of an electron, e = 1.6×10^{-19} C

The number of electrons flowing per second, n =?

$$I = \frac{q}{t} = \frac{ne}{t}$$

$$n = \frac{It}{e}$$

$$n = \frac{32\times1}{1.6\times10^{-19}\text{C}}$$

$$n = 20 \times 10^{19} = 2 \times 10^{20} \text{ electrons}$$

2.2 OHM'S LAW

The ohm's law can be derived from the equation $J = \sigma E$. Consider a segment of wire of length l and cross sectional area A as shown in Figure 2.7.

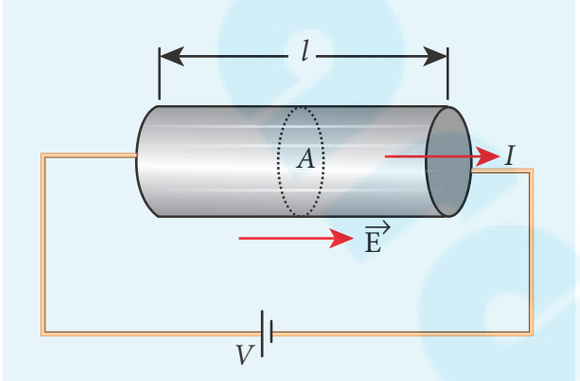


Figure 2.7 Current through the conductor

UNIT-2(XII-Physics_Vol-1).indd 86 14-02-2020 19:36:48

When a potential difference V is applied across the wire, a net electric field is created in the wire which constitutes the current in the wire. For simplicity, we assume that the electric field is uniform in the entire length of the wire, then the potential difference (voltage V) can be written as

$$V = El$$

As we know, the magnitude of current density

$$J = \sigma E = \sigma \frac{V}{l} \qquad (2.14)$$

But $J = \frac{I}{A}$, so we write the equation (2.14) as

$$\frac{I}{A} = \sigma \frac{V}{l}.$$

By rearranging the above equation, we get

$$V = I\left(\frac{l}{\sigma A}\right) \qquad (2.15)$$

The quantity $\frac{l}{\sigma A}$ is called resistance of the conductor and it is denoted as R. Note that the resistance is directly proportional to the length of the conductor and inversely proportional to area of cross section.

Therefore, the macroscopic form of ohm's law can be stated as

$$V = IR \qquad (2.16)$$

From the above equation, **the resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.**

$$R = \frac{V}{I} \qquad (2.17)$$

The SI unit of resistance is ohm (Ω). From the equation (2.16), we infer that the graph between current versus voltage is straight line with a slope equal to the inverse of resistance R of the conductor. It is shown in the Figure 2.8 (a).

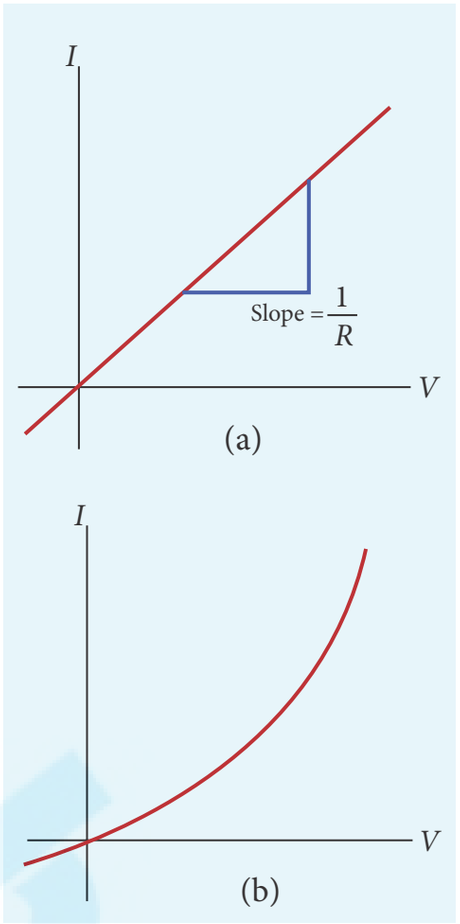


Figure 2.8 Current against voltage for (a) a conductor which obeys Ohm's law and (b) for a non-ohmic device (Diode given in XII physics, unit 9 is an example of a non-ohmic device)

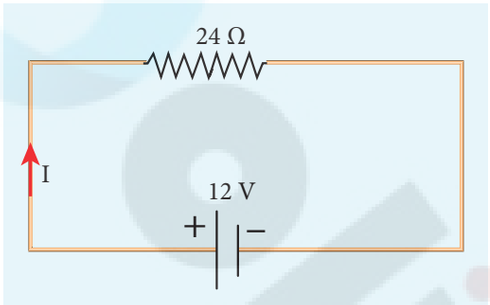
Materials for which the current versus voltage graph is a straight line through the origin, are said to obey Ohm's law and their behaviour is said to be ohmic as shown in Figure 2.8(a). Materials or devices that do not follow Ohm's law are said to be non-ohmic. These materials have more complex relationships between voltage and current. A plot of I versus V for a non-ohmic material is non-linear and they do not have a constant resistance (Figure 2.8(b)).

UNIT-2(XII-Physics_Vol-1).indd 87 14-02-2020 19:36:49

EXAMPLE 2.5

A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?

Solution



V = 12 V and R = 24 Ω

Current, I = ?

From Ohm's law, $I = \frac{V}{R} = \frac{12}{24} = 0.5\,\text{A}$

2.2.1 Resistivity

In the previous section, we have seen that the resistance R of any conductor is given by

$$R = \frac{l}{\sigma A} \qquad (2.18)$$

where σ is called the conductivity of the material and it depends only on the type of the material used and not on its dimension.

The resistivity of a material is equal to the reciprocal of its conductivity.

$$\rho = \frac{1}{\sigma} \qquad (2.19)$$

Now we can rewrite equation (2.18) using equation (2.19)

$$R = \rho\frac{l}{A} \qquad (2.20)$$

The resistance of a material is directly proportional to the length of the conductor and inversely proportional to the area of cross section of the conductor. The proportionality constant ρ is called the resistivity of the material.

If l = 1 m and A = 1 m^2, then the resistance $R = \rho$. In other words, the **electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section.** The SI unit of ρ is ohm-metre (Ω m). Based on the resistivity, materials are classified as conductors, insulators and semiconductors. The conductors have lowest resistivity, insulators have highest resistivity and semiconductors have resistivity greater than conductors but less than insulators. The typical resistivity values of some conductors, insulators and semiconductors are given in the Table 2.1

Table 2.1 Resistivity for various materials

Material	Resistivity, ρ (Ω m) at 20^{0}C
Insulators	
Pure Water	2.5×10^{5}
Glass	$10^{10} - 10^{14}$
Hard Rubber	$10^{13} - 10^{16}$
NaCl	10^{14}
Fused Quartz	10^{16}
Semiconductors	
Germanium	0.46
Silicon	640
Conductors	
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}

UNIT-2(XII-Physics_Vol-1).indd 88 14-02-2020 19:36:50

EXAMPLE 2.6

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

Solution

$R_1 = 20\ \Omega$, $R_2 = ?$

Let the original length of the wire (l_1) be l.

New length, $l_2 = 8l_1$ *(i.,e)* $l_2 = 8l$

Original resistance, $R_1 = \rho\dfrac{l_1}{A_1}$

New resistance $R_2 = \rho\dfrac{l_2}{A_2} = \dfrac{\rho(8l)}{A_2}$

Though the wire is stretched, its volume remains unchanged.

Initial volume = Final volume

$A_1 l_1 = A_2 l_2$, $\quad A_1 l = A_2(8l)$

$$\frac{A_1}{A_2} = \frac{8l}{l} = 8$$

By dividing equation for R_2 by equation for R_1, we get

$$\frac{R_2}{R_1} = \frac{\rho(8l)}{A_2} \times \frac{A_1}{\rho l}$$

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8$$

Substituting the value of $\dfrac{A_1}{A_2}$, we get

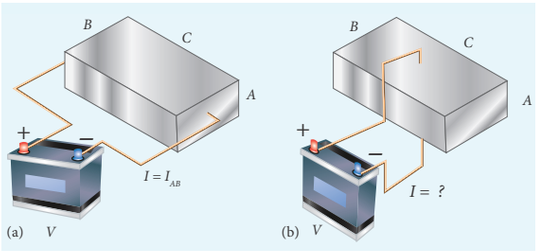
$$\frac{R_2}{R_1} = 8 \times 8 = 64$$

$$R_2 = 64 \times 20 = 1280\ \Omega$$

Hence, stretching the length of the wire has increased its resistance.

EXAMPLE 2.7

Consider a rectangular block of metal of height A, width B and length C as shown in the figure.



If a potential difference of V is applied between the two faces A and B of the block (figure (a)), the current I_{AB} is observed. Find the current that flows if the same potential difference V is applied between the two faces B and C of the block (figure (b)). Give your answers in terms of I_{AB}.

Solution

In the first case, the resistance of the block

$$R_{AB} = \rho\frac{\text{length}}{\text{Area}} = \rho\frac{C}{AB}$$

The current $I_{AB} = \dfrac{V}{R_{AB}} = \dfrac{V}{\rho}\cdot\dfrac{AB}{C}$ (1)

In the second case, the resistance of the block $R_{BC} = \rho\dfrac{A}{BC}$

The current $I_{BC} = \dfrac{V}{R_{BC}} = \dfrac{V}{\rho}\cdot\dfrac{BC}{A}$ (2)

To express I_{BC} interms of I_{AB}, we multiply and divide equation (2) by AC, we get

$$I_{BC} = \frac{V}{\rho}\cdot\frac{BC}{A}\frac{AC}{AC} = \left(\frac{V}{\rho}\cdot\frac{AB}{C}\right)\cdot\frac{C^2}{A^2} = \frac{C^2}{A^2}\cdot I_{AB}$$

Since $C > A$, the current $I_{BC} > I_{AB}$



The human body contains a large amount of water which has low resistance of around 200 Ω and the dry skin has high resistance of around 500 k Ω. But when the skin is wet, the resistance is reduced to around 1000 Ω. This is the reason why repairing the electrical connection with the wet skin is always dangerous.

UNIT-2(XII-Physics_Vol-1).indd 89 14-02-2020 19:36:53

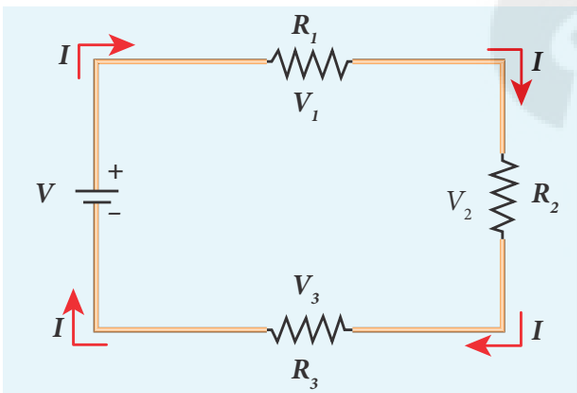
2.2.2 Resistors in series and parallel

An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors.

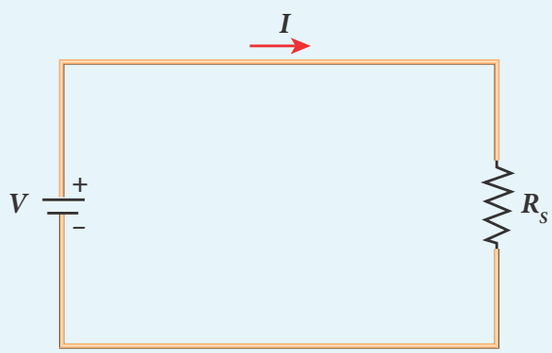
Resistors in series

When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices. Figure 2.9 (a) shows three resistors R_1, R_2 and R_3 connected in series.

The amount of charge passing through resistor R_1 must also pass through resistors R_2 and R_3 since the charges cannot accumulate anywhere in the circuit. Due to this reason, the current I passing through all the three resistors is the same. According to Ohm's law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different. If V_1, V_2 and V_3 be the potential differences (voltage) across each of the resistors R_1, R_2 and R_3 respectively, then we can write $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. But the supply voltage V must be equal to the sum of voltages(potential differences) across each resistor.



(a) Three resistors in series



(b) Equivalent resistance (R_S) has the same current

Figure 2.9 Resistors in series

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad (2.21)$$

$$V = I(R_1 + R_2 + R_3)$$

$$V = IR_S \quad (2.22)$$

where R_S is the equivalent resistance.

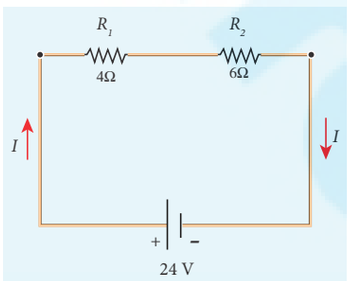
$$R_S = R_1 + R_2 + R_3 \quad (2.23)$$

When several resistors are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the Figure 2.9 (b).

Note: The value of equivalent resistance in series connection will be greater than each individual resistance.

EXAMPLE 2.8

Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across each resistors in the circuit.



UNIT-2(XII-Physics_Vol-1).indd 90 14-02-2020 19:36:54

Solution

Since the resistors are connected in series, the effective resistance in the circuit

$= 4\ \Omega + 6\ \Omega = 10\ \Omega$

current I in the circuit $= \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,\text{A}$

Voltage across 4Ω resistor

$$V_1 = IR_1 = 2.4\,\text{A} \times 4\Omega = 9.6\,\text{V}$$

Voltage across 6 Ω resistor

$$V_2 = IR_2 = 2.4\,\text{A} \times 6\Omega = 14.4\,\text{V}$$

Resistors in parallel

Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a).

In this case, the total current I that leaves the battery is split into three separate components. Let I_1, I_2 and I_3 be the current through the resistors R_1, R_2 and R_3 respectively. Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the three resistors.

$$I = I_1 + I_2 + I_3 \qquad (2.24)$$

Since the voltage across each resistor is the same, applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \qquad (2.25)$$

Substituting these values in equation (2.24),we get

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$I = \frac{V}{R_P}$$

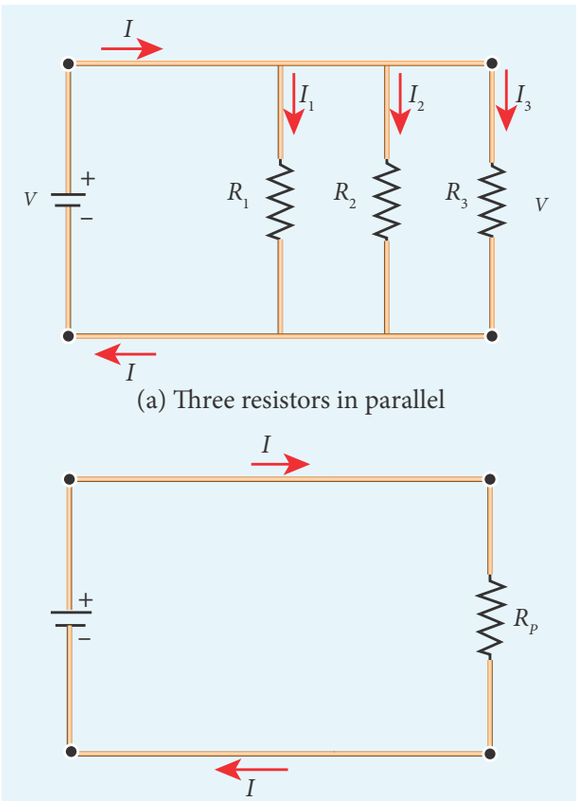


Figure 2.10 Resistors in parallel

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (2.26)$$

Here R_P is the equivalent resistance of the parallel combination of the resistors. Thus, when a number of resistors are connected in parallel, the sum of the reciprocals of resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination as shown in the Figure 2.10 (b).

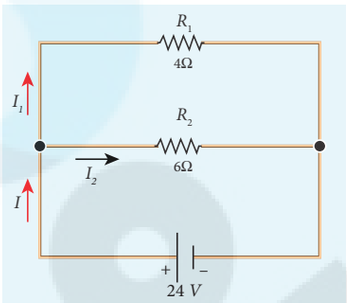
Note: The value of equivalent resistance in parallel connection will be lesser than each individual resistance.

House hold appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.

UNIT-2(XII-Physics_Vol-1).indd 91 14-02-2020 19:36:56

EXAMPLE 2.9

Calculate the equivalent resistance in the following circuit and also find the values of current I, I_1 and I_2 in the given circuit.



Solution

Since the resistances are connected in parallel, the equivalent resistance in the circuit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_P} = \frac{5}{12}\Omega \quad \text{or } R_P = \frac{12}{5}\Omega$$

The resistors are connected in parallel, the potential diffrence (voltage) across them is the same.

$$I_1 = \frac{V}{R_1} = \frac{24\text{V}}{4\Omega} = 6\,\text{A}$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4\,\text{A}$$

The current I is the sum of the currents in the two branches. Then,

$$I = I_1 + I_2 = 6\text{ A} + 4\text{ A} = 10\text{ A}$$

EXAMPLE 2.10

Two resistors when connected in series and parallel, their equivalent resistances are 15 Ω and $\frac{56}{15}\Omega$ respectively. Find the values of the resistances.

Solution

$$R_s = R_1 + R_2 = 15\ \Omega \quad (1)$$

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{56}{15}\Omega \quad (2)$$

From equation (1) substituting for $R_1 + R_2$ in equation (2)

$$\frac{R_1 R_2}{15} = \frac{56}{15}\Omega$$

$$\therefore R_1 R_2 = 56$$

$$R_2 = \frac{56}{R_1}\Omega \quad (3)$$

Substituting for R_2 in equation (1) from equation (3)

$$R_1 + \frac{56}{R_1} = 15$$

Then, $\frac{R_1^2 + 56}{R_1} = 15$

$$R_1^2 + 56 = 15\,R_1$$

$$R_1^2 - 15\,R_1 + 56 = 0$$

The above equation can be solved using factorisation.

$R_1 = 8\ \Omega$ (or) $R_1 = 7\ \Omega$

If $R_1 = 8\ \Omega$

Substituting in equation (1)

$8 + R_2 = 15$

$R_2 = 15 - 8 = 7\ \Omega$,

$R_2 = 7\ \Omega$ i.e , (when $R_1 = 8\ \Omega$; $R_2 = 7\ \Omega$)

If $R_1 = 7\ \Omega$

Substituting in equation (1)

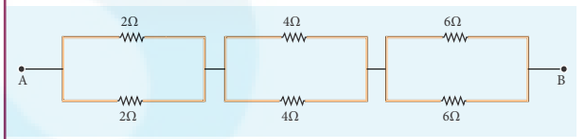
$7 + R_2 = 15$

$R_2 = 8\ \Omega$, i.e , (when $R_1 = 7\ \Omega$; $R_2 = 8\ \Omega$)

UNIT-2(XII-Physics_Vol-1).indd 92 14-02-2020 19:36:58

EXAMPLE 2.11

Calculate the equivalent resistance between A and B in the given circuit.



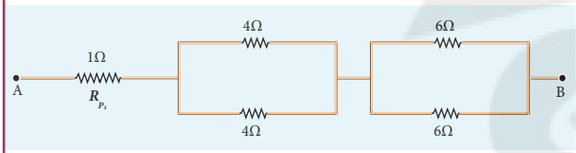
Solution

In all the sections, the resistors are connected in parallel.

Section 1

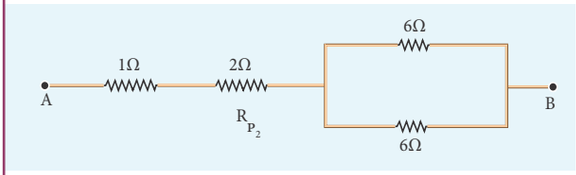
$$\frac{1}{R_{p_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{p_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \qquad R_{p_1} = 1\Omega$$



Section II

$$\frac{1}{R_{p_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}, \quad \frac{1}{R_{p_2}} = \frac{1}{2}, \quad R_{p_2} = 2\Omega$$



Section III

$$\frac{1}{R_{p_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

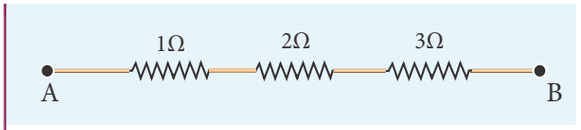
$$\frac{1}{R_{p_3}} = \frac{1}{3}, \quad R_{p_3} = 3\ \Omega$$

Equivalent resistance is given by

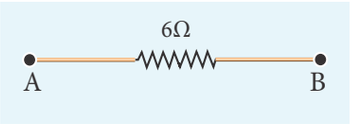
$$R = R_{p_1} + R_{p_2} + R_{p_3}$$

$$R = 1\ \Omega + 2\ \Omega + 3\ \Omega = 6\ \Omega$$

The circuit becomes,

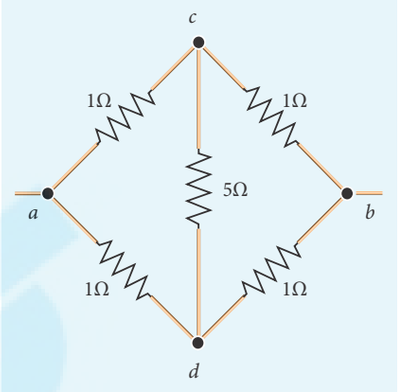


Equivalent resistance between A and B is



EXAMPLE 2.12

Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.

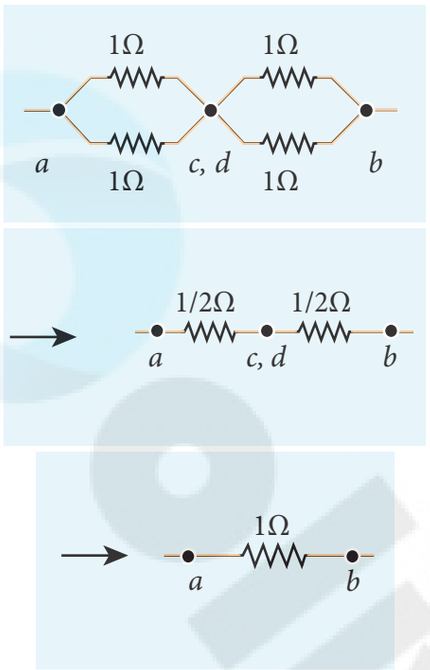


Solution

Case (a)

To find the equivalent resistance between the points a and b, we assume that a current is entering the junction at a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. Hence the points C and D are at the same potential and no current through 5 Ω. It implies that the 5 Ω has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.

UNIT-2(XII-Physics_Vol-1).indd 93 14-02-2020 19:37:01



The equivalent resistance of the circuit between a and b is $R_{eq} = 1\ \Omega$

2.2.3 Colour code for Carbon resistors



Figure 2.11 Resistances used in laboratory

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited as shown in Figure 2.11. These resistors are inexpensive, stable and compact in size. Colour rings are used to indicate the value of the resistance according to the rules given in the Table 2.2.

Three coloured rings are used to indicate the values of a resistor: the first two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth colour, silver or gold, shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12 .If there is no fourth ring, the tolerance is 20%.

Table 2.2 Colour Coding for Resistors

Colour	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Sliver		10^{-2}	10%
Colourless			20%

For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = 10^3 (orange) and tolerance = 5% (gold). The value of resistance = $56 \times 10^3\ \Omega$ or 56 kΩ with the tolerance value 5%.

Note While reading the colour code, hold the resistor with colour bands to your left. Resistors never start with a metallic band on the left.

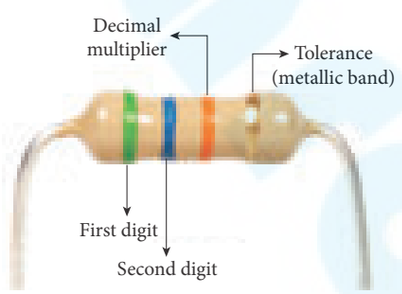


Figure 2.12 Resistor colour coding

UNIT-2(XII-Physics_Vol-1).indd 94 14-02-2020 19:37:02

2.2.4 Temperature dependence of resistivity

The resistivity of a material is dependent on temperature. It is experimentally found that for a wide range of temperatures, the resistivity of a conductor increases with increase in temperature according to the expression,

$$\rho_T = \rho_\circ\left[1+\alpha\left(T-T_\circ\right)\right] \qquad (2.27)$$

where ρ_T is the resistivity of a conductor at $T\,^\circ C$, ρ_o is the resistivity of the conductor at some reference temperature T_o (usually at 20°C) and α is the temperature coefficient of resistivity. **It is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at T_o.**

From the equation (2.27), we can write

$$\rho_T-\rho_\circ = \alpha\rho_\circ\left(T-T_\circ\right)$$

$$\therefore \alpha = \frac{\rho_T-\rho_\circ}{\rho_\circ\left(T-T_\circ\right)} = \frac{\Delta\rho}{\rho_\circ\Delta T}$$

where $\Delta\rho = \rho_T - \rho_o$ is change in resistivity for a change in temperature $\Delta T = T - T_o$. Its unit is per °C.

α of conductors

For conductors α is positive. If the temperature of a conductor increases, the average kinetic energy of electrons in the conductor increases. This results in more frequent collisions and hence the resistivity increases. The graph of the equation (2.27) is shown in Figure 2.13.

Even though, the resistivity of conductors like metals varies linearly for wide range of temperatures, there also exists a non-linear region at very low temperatures. The resistivity approaches some finite value as the temperature approaches absolute zero as shown in Figure 2.13(b).

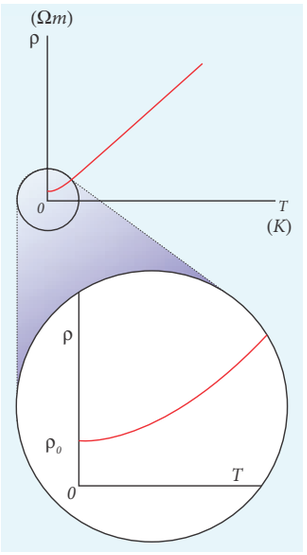


Figure 2.13 (a) Temperature dependence of resistivity for a conductor (b) Non linear region at low temperature

Using the equation $\rho = R\frac{A}{l}$ in equation (2.27), we get the expression for the resistance of a conductor at temperature T °C as

$$R_T = R_\circ\left[1+\alpha\left(T-T_\circ\right)\right] \qquad (2.28)$$

The temperature coefficient of resistivity can also be obtained from the equation (2.28),

$$R_T - R_\circ = \alpha R_\circ\left(T-T_\circ\right)$$

$$\therefore \alpha = \frac{R_T-R_\circ}{R_\circ\left(T-T_\circ\right)} = \frac{1}{R_\circ}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{R_\circ}\frac{\Delta R}{\Delta T} \qquad (2.29)$$

where $\Delta R = R_T - R_\circ$ is change in resistance during the change in temperature $\Delta T = T - T_\circ$

α of semiconductors

For semiconductors, the resistivity decreases with increase in temperature. As the temperature increases, more electrons will be liberated from their atoms (Refer unit 9 for conduction in semi conductors).

UNIT-2(XII-Physics_Vol-1).indd 95 14-02-2020 19:37:04

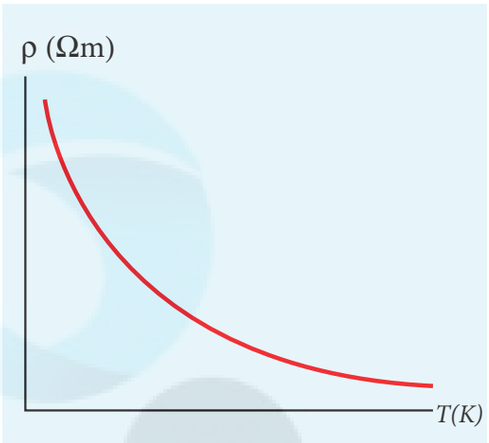


Figure 2.14 Temperature dependence of resistivity for a semiconductor

Hence the current increases and therefore the resistivity decreases as shown in Figure 2.14. A semiconductor with a negative temperature coefficient of resistivity is called a thermistor.

The typical values of temperature coefficients of various materials are given in table 2.3.

Table 2.3

Material	Temperature Coefficient of resistivity α $[(°C)^{-1}]$
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.92×10^{-3}
Lead	3.9×10^{-3}
Nichrome	0.4×10^{-3}
Carbon	-0.5×10^{-3}
Germanium	-48×10^{-3}
Silicon	-75×10^{-3}

We can understand the temperature dependence of resistivity in the following way. In section 2.1.3, we have shown that the electrical conductivity, $\sigma = \frac{ne^2\tau}{m}$. As the resistivity is inverse of σ, it can be written as

$$\rho = \frac{m}{ne^2\tau} \qquad (2.30)$$

The resistivity of materials is

i) inversely proportional to the number density (n) of the electrons

ii) inversely proportional to the average time between the collisions (τ).

In metals, if the temperature increases, the average time between the collision (τ) decreases and n is independent of temperature. In semiconductors when temperature increases, n increases and τ decreases, but increase in n is dominant than decreasing τ, so that overall resistivity decreases.



The resistance of certain materials become zero below certain temperature T_c. This temperature is known as critical temperature or transition temperature. The materials which exhibit this property are known as superconductors. This phenomenon was first observed by Kammerlingh Onnes in 1911. He found that mercury exhibits superconductor behaviour at 4.2 K. Since R = 0, current once induced in a superconductor persists without any potential difference.

EXAMPLE 2.13

If the resistance of coil is 3 Ω at 20^0 C and α = 0.004/^{0}C then determine its resistance at 100 ^{0}C.

UNIT-2(XII-Physics_Vol-1).indd 96 14-02-2020 19:37:04

Solution

$R_0 = 3\ \Omega, \quad T = 100^0C, \quad T_0 = 20^0C$

$\alpha = 0.004/^0C, \quad R_T = ?$

$R_T = R_0(1 + \alpha(T\text{-}T_0))$

$R_{100} = 3(1 + 0.004 \times 80)$

$R_{100} = 3.96\ \Omega$

EXAMPLE 2.14

Resistance of a material at 20^0C and 40^0C are 45 Ω and 85 Ω respectively. Find its temperature coefficient of resistivity.

Solution

$T_0 = 20\ ^0C, T = 40\ ^0C, R_0 = 45\ \Omega, R = 85\ \Omega$

$$\alpha = \frac{1}{R_\circ}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{45}\left(\frac{85-45}{40-20}\right) = \frac{1}{45}(2)$$

$$\alpha = 0.044\ \text{per}^\circ C$$

2.3 ENERGY AND POWER IN ELECTRICAL CIRCUITS

When a battery is connected between the ends of a conductor, a current is established. The battery is supplying energy to the device which is connected in the circuit. Consider a circuit in which a battery of voltage V is connected to the resistor as shown in Figure 2.15.

Assume that a positive charge of dQ moves from point a to b through the battery and moves from point c to d through the resistor and back to point a. When the charge

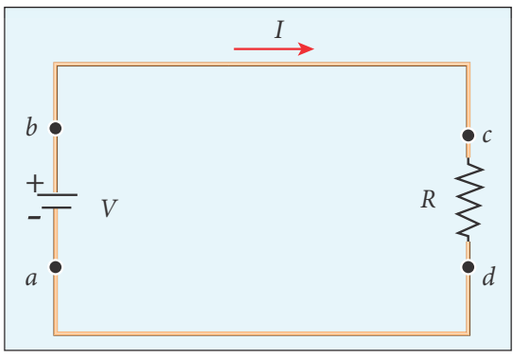


Figure 2.15 Energy given by the battery

moves from point a to b, it gains potential energy $dU = V.dQ$ and the chemical potential energy of the battery decreases by the same amount. When this charge dQ passes through resistor it loses the potential energy $dU = V.dQ$ due to collision with atoms in the resistor and again reaches the point a. This process occurs continuously till the battery is connected in the circuit. The rate at which the charge loses its electrical potential energy in the resistor can be calculated.

The electrical power P is the rate at which the electrical potential energy is delivered,

$$P = \frac{dU}{dt} = \frac{(V.dQ)}{dt} = V\frac{dQ}{dt} \qquad (2.31)$$

Since the electric current $I = \frac{dQ}{dt}$, the equation (2.31) can be rewritten as

$$P = VI \qquad (2.32)$$

This expression gives the power delivered by the battery to any electrical system, where I is the current passing through it and V is the potential difference across it. The SI unit of electrical power is watt ($1W = 1\ J\,s^{-1}$). Commercially, the electrical bulbs used in houses come with the power and voltage rating of 5W-220V, 30W-220V, 60W-220V etc. (Figure 2.16).

UNIT-2(XII-Physics_Vol-1).indd 97 14-02-2020 19:37:05



Figure 2.16 Electrical bulbs with power rating

Usually these voltage rating refers AC RMS voltages. For a given bulb, if the voltage drop across the bulb is greater than voltage rating, the bulb will fuse.

Using Ohm's law, power delivered to the resistance R is expressed in other forms

$$P = IV = I\,(IR) = I^2R \qquad (2.33)$$

$$P = IV = \frac{V}{R}V = \frac{V^2}{R} \qquad (2.34)$$

The electrical power produced (dissipated) by a resistor is I^2R. It depends on the square of the current. Hence, if current is doubled, the power will increase by four times. Similar explanation holds true for voltage also.

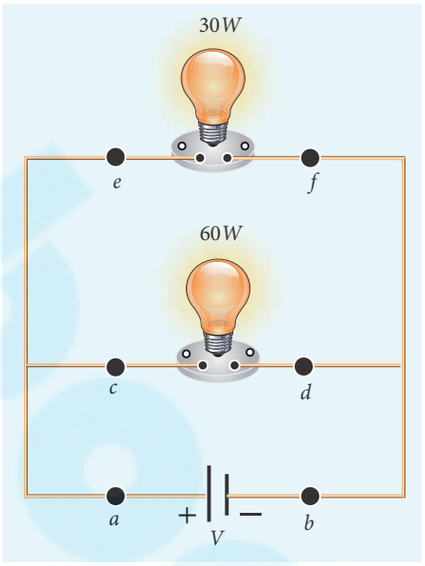
The total electrical energy used by any device is obtained by multiplying the power and duration of the time when it is ON. If the power is in watts and the time is in seconds, the energy will be in joules. In practice, electrical energy is measured in kilowatt hour (kWh). 1 kWh is known as 1 unit of electrical energy.

(1 kWh = 1000 Wh = (1000 W) (3600 s) = 3.6×10^6 J)

The Tamilnadu Electricity Board is charging for the amount of energy you use and not for the power. A current of 1A flowing through a potential difference of 1V produces a power of 1W.

EXAMPLE 2.15

A battery of voltage V is connected to 30 W bulb and 60 W bulb as shown in the figure. (a) Identify brightest bulb (b) which bulb has greater resistance? (c) Suppose the two bulbs are connected in series, which bulb will glow brighter?



Solution

(a) The power delivered by the battery $P = VI$. Since the bulbs are connected in parallel, the voltage drop across each bulb is the same. If the voltage is kept fixed, then the power is directly proportional to current ($P \propto I$). So 60 W bulb draws twice as much as current as 30 W and it will glow brighter than 30 W bulb.

UNIT-2(XII-Physics_Vol-1).indd 98 14-02-2020 19:37:06

(b) To calculate the resistance of the bulbs, we use the relation $P = \frac{V^2}{R}$. In both the bulbs, the voltage drop is the same. So the power is inversely proportional to the resistance or resistance is inversely proportional to the power $\left(R \propto \frac{1}{P}\right)$. It implies that, the 30W has twice as much as resistance as 60 W bulb.

(c) When the bulbs are connected in series, the current passing through each bulb is the same. It is equivalent to two resistors connected in series. The bulb which has higher resistance has higher voltage drop. So 30W bulb will glow brighter than 60W bulb. So the higher power rating does not always imply more brightness and it depends whether bulbs are connected in series or parallel.

EXAMPLE 2.16

Two electric bulbs marked 20 W – 220 V and 100 W – 220 V are connected in series to 440 V supply. Which bulb will get fused?

Solution

To check which bulb will get fused, the voltage drop across each bulb has to be calculated.

The resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{(\text{Rated voltage})^2}{\text{Rated power}}$$

For 20W-220V bulb,

$$R_1 = \frac{(220)^2}{20}\Omega = 2420\,\Omega$$

DQNPP1

For 100W-220V bulb,

$$R_2 = \frac{(220)^2}{100}\Omega = 484\,\Omega$$

Both the bulbs are connected in series. So same current will pass through both the bulbs. The current that passes through the circuit, $I = \frac{V}{R_{tot}}$.

$$R_{tot} = (R_1 + R_2)$$

$$R_{tot} = (484 + 2420)\Omega = 2904\Omega$$

$$I = \frac{440V}{2904\Omega} \approx 0.151\text{A}$$

The voltage drop across the 20W bulb is

$$V_1 = IR_1 = \frac{440}{2904} \times 2420 \approx 366.6\text{ V}$$

The voltage drop across the 100W bulb is

$$V_2 = IR_2 = \frac{440}{2904} \times 484 \approx 73.3\text{ V}$$

The 20 W bulb will get fused because the voltage across it is more than the voltage rating.

2.4 ELECTRIC CELLS AND BATTERIES

An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrodes (carbon and zinc) immersed in an electrolyte (sulphuric acid) as shown in Figure 2.17.

Several electric cells connected together form a battery. When a cell or battery is connected to a circuit, electrons flow from the negative terminal to the positive terminal through the circuit. By using chemical reactions, a battery

UNIT-2(XII-Physics_Vol-1).indd 99 14-02-2020 19:37:09

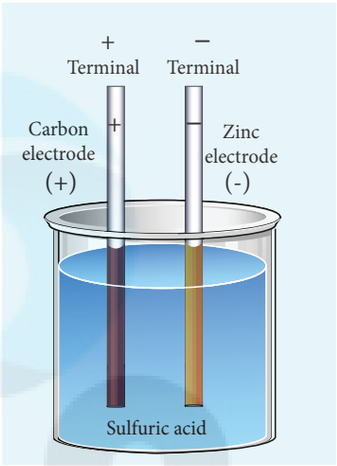


Figure 2.17 Simple electric cell

produces potential difference across its terminals. This potential difference provides the energy to move the electrons through the circuit. Commercially available electric cells and batteries are shown in Figure 2.18.

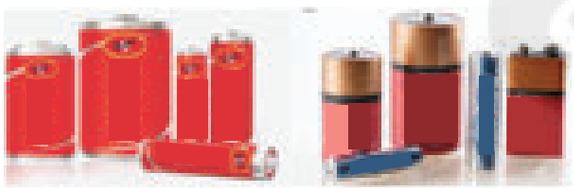


Figure 2.18 Electric cells and Batteries

2.4.1 Electromotive force and internal resistance

A battery or cell is called a source of electromotive force (emf). The term 'electromotive force' is a misnomer since it does not really refer to a force but describes a potential difference in volts. The emf of a battery or cell is the voltage provided by the battery when no current flows in the external circuit. It is shown in Figure 2.19.

Electromotive force determines the amount of work a battery or cell has to do



Figure 2.19 Measuring the emf of a cell

move a certain amount of charge around the circuit. It is denoted by the symbol ε. An ideal battery has zero internal resistance and the potential difference (terminal voltage) across the battery equals to its emf. In reality, the battery is made of electrodes and electrolyte, there is resistance to the flow of charges within the battery. This resistance is called internal resistance *r*. For a real battery, the terminal voltage is not equal to the emf of the battery. A freshly prepared cell has low internal resistance and it increases with ageing.

2.4.2 Determination of internal resistance

The circuit connections are made as shown in Figure 2.20.

The emf of cell ε is measured by connecting a high resistance voltmeter across it without connecting the external resistance *R* as shown in Figure 2.20(a). Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell. Then, external resistance *R* is included in the circuit and current *I* is established in the circuit. The potential difference across

UNIT-2(XII-Physics_Vol-1).indd 100 14-02-2020 19:37:09

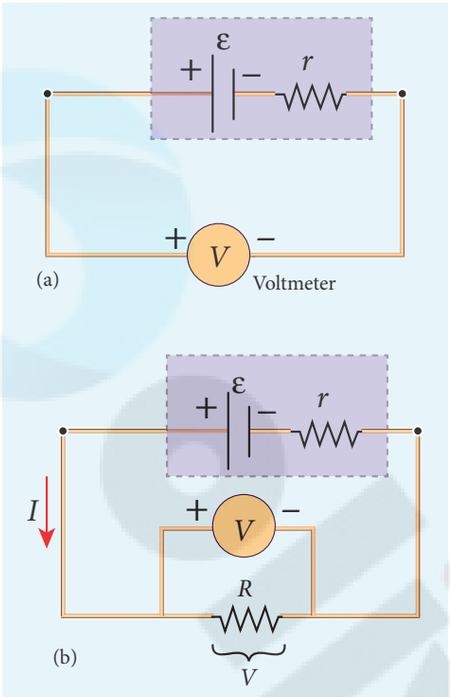


Figure 2.20 Internal resistance of the cell

R is equal to the potential difference across the cell (V) as shown in Figure 2.20(b).

The potential drop across the resistor R is

$$V = IR \qquad (2.35)$$

Due to internal resistance r of the cell, the voltmeter reads a value V, which is less than the emf of cell ε. It is because, certain amount of voltage (Ir) has dropped across the internal resistance r.

Then $V = \varepsilon - Ir$

$$Ir = \varepsilon - V \qquad (2.36)$$

Dividing equation (2.36) by equation (2.35), we get

$$\frac{Ir}{IR} = \frac{\varepsilon - V}{V}$$

$$r = \left[\frac{\varepsilon - V}{V}\right] R \qquad (2.37)$$

Since ε, V and R are known, internal resistance r can be determined. We can also find the total current that flows in the circuit.

Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery. For a battery of emf ε, with an internal resistance r, the power delivered to the circuit of resistance R is given by

$P = I\varepsilon = I\,(V + Ir)$ (from equation 2.36)

Here V is the voltage drop across the resistance R and it is equal to IR.

Therefore, $P = I\,(IR + Ir)$

$$P = I^2 R + I^2 r \qquad (2.38)$$

Here I^2r is the power delivered to the internal resistance and I^2R is the power delivered to the electrical device (here it is the resistance R). For a good battery, the internal resistance r is very small, then $I^2r << I^2R$ and almost entire power is delivered to the external resistance.

EXAMPLE 2.17

A battery has an emf of 12 V and connected to a resistor of 3 Ω. The current in the circuit is 3.93 A. Calculate (a) terminal voltage and the internal resistance of the battery (b) power delivered by the battery and power delivered to the resistor

Solution

The given values $I = 3.93$ A, $\varepsilon = 12$ V, $R = 3\ \Omega$

(a) The terminal voltage of the battery is equal to voltage drop across the resistor

$V = IR = 3.93 \times 3 = 11.79$ V

UNIT-2(XII-Physics_Vol-1).indd 101 14-02-2020 19:37:10

The internal resistance of the battery,

$$r = \left[\frac{\varepsilon - V}{V}\right] R = \left[\frac{12 - 11.79}{11.79}\right] \times 3 = 0.05\ \Omega$$

(b) The power delivered by the battery $P = I\varepsilon = 3.93 \times 12 = 47.1$ W

The power delivered to the resistor = I^2R = 46.3 W

The remaining power P = (47.1 – 46.3) = 0.8 W is delivered to the internal resistance and cannot be used to do useful work. (It is equal to I^2r).

2.4.3 Cells in series

Several cells can be connected to form a battery. In series connection, the negative terminal of one cell is connected to the positive terminal of the second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery.

Suppose n cells, each of emf ε volts and internal resistance r ohms are connected in series with an external resistance R as shown in Figure 2.21

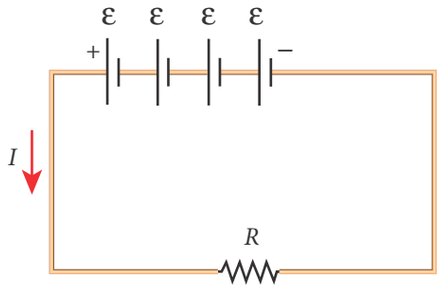


Figure 2.21 cells in series

The total emf of the battery = $n\varepsilon$

The total resistance in the circuit = $nr + R$

By Ohm's law, the current in the circuit is

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n\varepsilon}{nr + R} \qquad (2.39)$$

Case (a) If $r << R$, then,

$$I = \frac{n\varepsilon}{R} \approx nI_1 \qquad (2.40)$$

where, I_1 is the current due to a single cell $\left(I_1 = \frac{\varepsilon}{R}\right)$

Thus, if r is negligible when compared to R the current supplied by the battery is n times that supplied by a single cell.

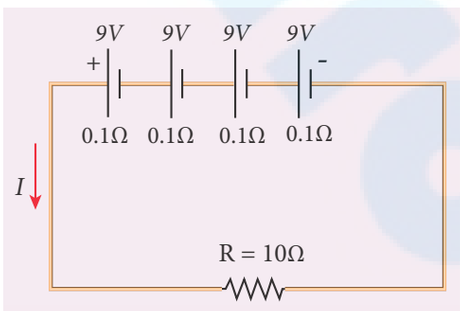
Case (b) If $r>>R$, $I = \frac{n\varepsilon}{nr} \approx \frac{\varepsilon}{r}$ (2.41)

It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells.

Thus series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with R.

EXAMPLE 2.18

From the given circuit,



UNIT-2(XII-Physics_Vol-1).indd 102 14-02-2020 19:37:11

Find

i) Equivalent emf of the combination

ii) Equivalent internal resistance

iii) Total current

iv) Potential difference across external resistance

v) Potential difference across each cell

Solution

i) Equivalent emf of the combination $\varepsilon_{eq} = n\varepsilon = 4 \times 9 = 36$ V

ii) Equivalent internal resistance $r_{eq} = nr$ $= 4 \times 0.1 = 0.4\ \Omega$

iii) Total current $I = \dfrac{n\varepsilon}{R + nr}$

$$= \frac{4 \times 9}{10 + (4 \times 0.1)}$$

$$= \frac{4 \times 9}{10 + 0.4} = \frac{36}{10.4}$$

$$I = 3.46 \text{ A}$$

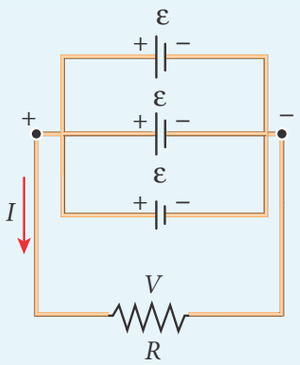
iv) Potential difference across external resistance $V = IR = 3.46 \times 10 = 34.6$ V. The remaining 1.4 V is dropped across the internal resistances of cells.

v) Potential difference across each cell $\dfrac{V}{n} = \dfrac{34.6}{4} = 8.65 \text{V}$

2.4.4 Cells in parallel

In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and negative terminals of the battery.

Let n cells be connected in parallel between the points A and B and a resistance R is connected between the points A and B as shown in Figure 2.22. Let ε be the emf and r the internal resistance of each cell.



Cells in parallel (circuit diagram)

Figure 2.22 Cells in parallel

The equivalent internal resistance of the battery is $\dfrac{1}{r_{eq}} = \dfrac{1}{r} + \dfrac{1}{r} + ... \dfrac{1}{r} (n \text{ terms}) = \dfrac{n}{r}$. So $r_{eq} = \dfrac{r}{n}$ and the total resistance in the circuit $= R + \dfrac{r}{n}$. The total emf is the potential difference between the points A and B, which is equal to ε. The current in the circuit is given by

$$I = \frac{\varepsilon}{\frac{r}{n} + R}$$

$$I = \frac{n\varepsilon}{r + nR} \qquad (2.42)$$

Case (a) If $r >> R$, $I = \dfrac{n\varepsilon}{r} = nI_1$ (2.43)

where I_1 is the current due to a single cell $\left(\dfrac{\varepsilon}{r}\right)$ when R is negligible. Thus, the current through the external resistance due to the whole battery is n times the current due to a single cell.

Case (b) If $r << R$, $I = \dfrac{\varepsilon}{R}$ (2.44)

UNIT-2(XII-Physics_Vol-1).indd 103 14-02-2020 19:37:14

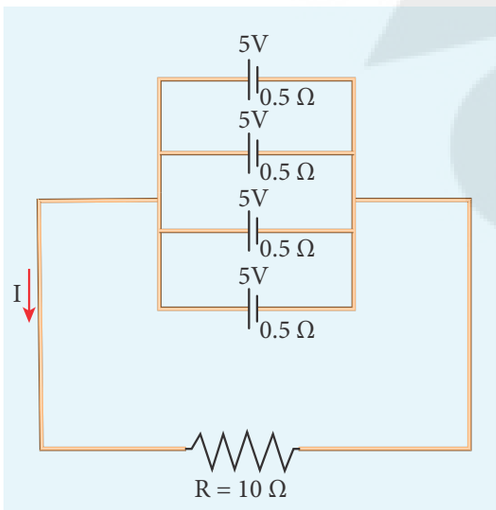
The above equation implies that current due to the whole battery is the same as that due to a single cell. Hence it is advantageous to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.



When the car engine is started with headlights turned on, they sometimes become dim. This is due to the internal resistance of the car battery.

EXAMPLE 2.19

For the given circuit



Find

i) Equivalent emf
ii) Equivalent internal resistance
iii) Total current (I)
iv) Potential difference across each cell
v) Current from each cell

Solution

i) Equivalent emf $\varepsilon_{eq} = 5$ V

ii) Equivalent internal resistance,

$$R_{eq} = \frac{r}{n} = \frac{0.5}{4} = 0.125\Omega$$

iii) total current, $I = \dfrac{\varepsilon}{R + r/n}$

$$I = \frac{5}{10 + 0.125} = \frac{5}{10.125}$$

$$I \approx 0.5 \text{ A}$$

iv) Potential difference across each cell

$$V = IR = 0.5 \times 10 = 5 \text{ V}$$

v) Current from each cell, $I' = \dfrac{I}{n}$

$$I' = \frac{0.5}{4} = 0.125 \text{ A}$$

2.5 KIRCHHOFF'S RULES

Ohm's law is useful only for simple circuits. For more complex circuits, Kirchhoff's rules can be used to find current and voltage. There are two generalized rules: i) Kirchhoff's current rule ii) Kirchhoff's voltage rule.

2.5.1 Kirchhoff's first rule (Current rule or Junction rule)

It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of law of conservation of electric charge. The charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction. By convention, current entering the junction is taken as positive and current leaving the junction is taken as negative.

UNIT-2(XII-Physics_Vol-1).indd 104 14-02-2020 19:37:15

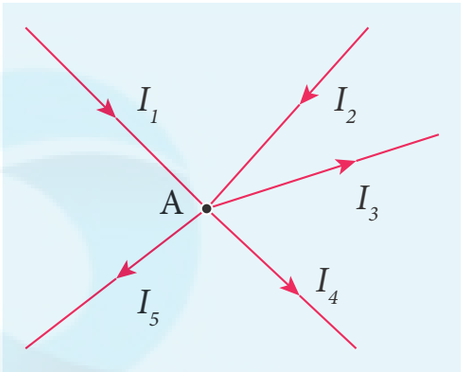


Figure 2.23 Kirchhoff's current rule

Applying this law to the junction A in Figure 2.23

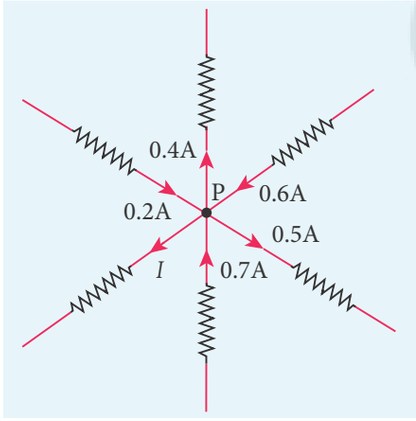
$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

(or)

$$I_1 + I_2 = I_3 + I_4 + I_5$$

EXAMPLE 2.20

For the given circuit find the value of *I*.



Solution

Applying Kirchhoff's rule to the point P in the circuit,

The arrows pointing towards P are positive and away from P are negative.

Therefore, 0.2A – 0.4A + 0.6A – 0.5A + 0.7A – I = 0

1.5A – 0.9A – I = 0

0.6A – I = 0

I = 0.6 A

2.5.2 Kirchhoff's Second rule (Voltage rule or Loop rule)

It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors). The product of current and resistance is taken as positive when the direction of the current is followed. Suppose if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistor is negative. It is shown in Figure 2.24 (a) and (b). The emf is considered positive when proceeding from the negative to the positive terminal of the cell. It is shown in Figure 2.24 (c) and (d).

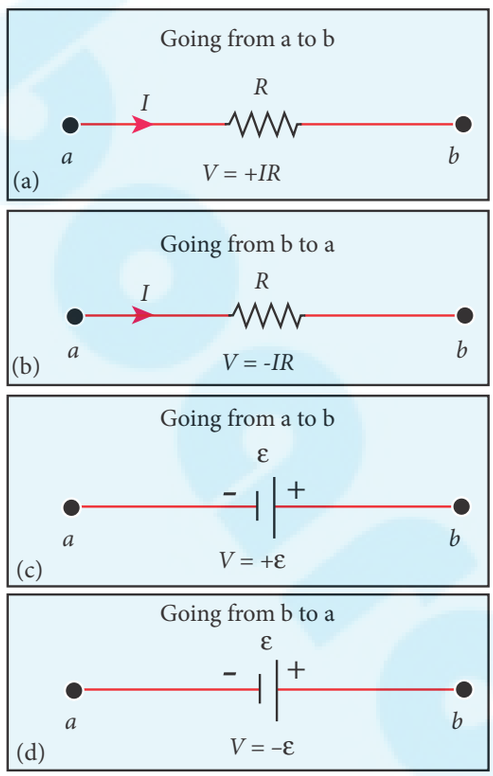


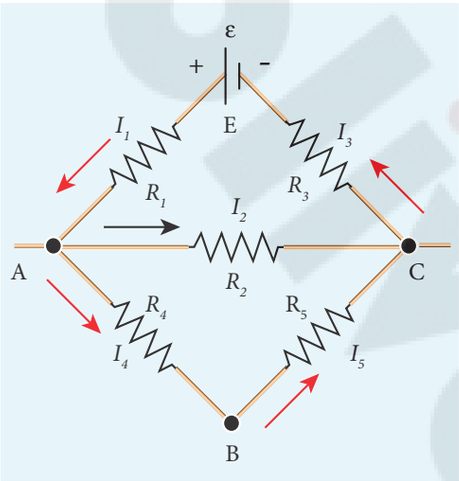
Figure 2.24 Kirchhoff voltage rule

UNIT-2(XII-Physics_Vol-1).indd 105 14-02-2020 19:37:15

Kirchhoff voltage rule has to be applied only when all currents in the circuit reach a steady state condition (the current in various branches are constant).

EXAMPLE 2.21

The following figure shows a complex network of conductors which can be divided into two closed loops like EACE and ABCA. Apply Kirchhoff's voltage rule (KVR),



Solution

Thus applying Kirchhoff's second law to the closed loop EACE

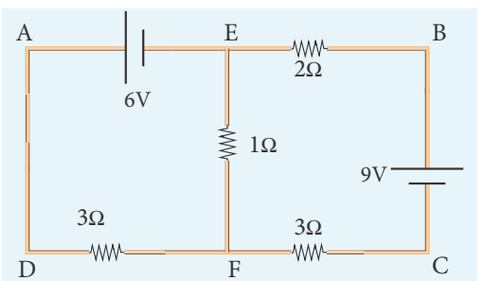
$$I_1R_1 + I_2R_2 + I_3R_3 = \varepsilon$$

and for the closed loop ABCA

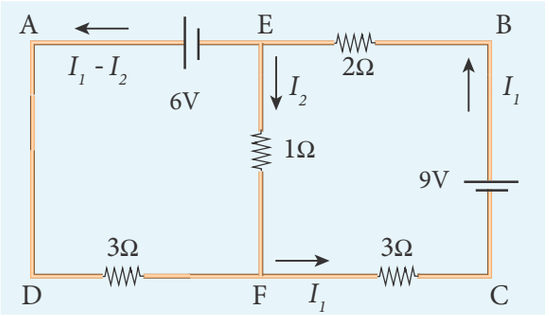
$$I_4R_4 + I_5R_5 - I_2R_2 = 0$$

EXAMPLE 2.22

Calculate the current that flows in the 1 Ω resistor in the following circuit.



Solution



We can denote the current that flows from 9V battery as I_1 and it splits up into I_2 and $(I_1 - I_2)$ at the junction E according Kirchhoff's current rule (KCR).

Now consider the loop EFCBE and apply KVR, we get

$$1I_2 + 3I_1 + 2I_1 = 9$$

$$5I_1 + I_2 = 9 \qquad (1)$$

Applying KVR to the loop EADFE, we get

$$3(I_1 - I_2) - 1I_2 = 6$$

$$3I_1 - 4I_2 = 6 \qquad (2)$$

Solving equation (1) and (2), we get

$$I_1 = 1.83 \text{ A and } I_2 = -0.13 \text{ A}$$

It implies that the current in the 1 ohm resistor flows from F to E.

2.5.3 Wheatstone's bridge

An important application of Kirchhoff's rules is the Wheatstone's bridge. It is used to compare resistances and in determining the unknown resistance in electrical network. The bridge consists of four resistances *P*, *Q*, *R* and *S* connected as shown in Figure 2.25. *A* galvanometer *G* is connected between the points *B* and *D*. The battery is connected between the points *A* and *C*. The current

UNIT-2(XII-Physics_Vol-1).indd 106 14-02-2020 19:37:16

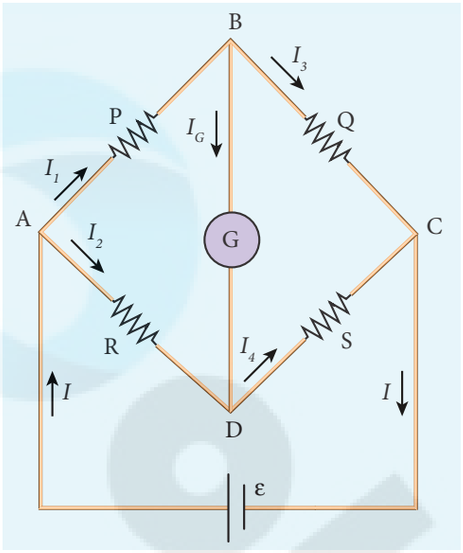


Figure 2.25 Wheatstone's bridge

through the galvanometer is I_G and its resistance is G.

Applying Kirchhoff's current rule to junction B and D respectively.

$$I_1 - I_G - I_3 = 0 \quad (2.45)$$

$$I_2 + I_G - I_4 = 0 \quad (2.46)$$

Applying Kirchhoff's voltage rule to loop $ABDA$,

$$I_1P + I_GG - I_2R = 0 \quad (2.47)$$

Applying Kirchhoff's voltage rule to loop $ABCDA$,

$$I_1P + I_3Q - I_4S - I_2R = 0 \quad (2.48)$$

When the points B and D are at the same potential, the bridge is said to be balanced. As there is no potential difference between B and D, no current flows through galvanometer ($I_G = 0$). Substituting $I_G = 0$ in equation (2.45), (2.46) and (2.47), we get

$$I_1 = I_3 \quad (2.49)$$

$$I_2 = I_4 \quad (2.50)$$

$$I_1P = I_2R \quad (2.51)$$

Using equation (2.51) in equation (2.48)

$$I_3Q = I_4S \quad (2.52)$$

Dividing equation (2.52) by equation (2.51), we get

$$\frac{P}{Q} = \frac{R}{S} \quad (2.53)$$

This is the condition for bridge balance. Only under this condition, galvanometer shows null deflection. Suppose we know the values of two adjacent resistances, the other two resistances can be compared. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.



A galvanometer is an instrument used for detecting and measuring even very small electric currents. It is extensively useful to compare the potential difference between various parts of the circuit.

EXAMPLE 2.23

In a Wheatstone's bridge $P = 100\ \Omega$, $Q = 1000\ \Omega$ and $R = 40\ \Omega$. If the galvanometer shows zero deflection, determine the value of S.

UNIT-2(XII-Physics_Vol-1).indd 107 14-02-2020 19:37:16

Solution

$$\frac{P}{Q} = \frac{R}{S}$$

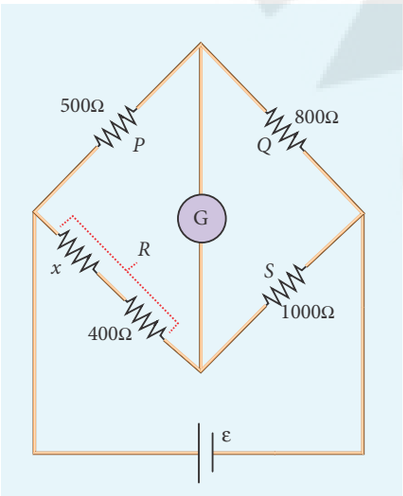
$$S = \frac{Q}{P} \times R$$

$$S = \frac{1000}{100} \times 40 \quad S = 400\ \Omega$$

EXAMPLE 2.24

What is the value of x when the Wheatstone's network is balanced?

$P = 500\ \Omega$, $Q = 800\ \Omega$, $R = x + 400$, $S = 1000\ \Omega$



Solution

$\frac{P}{Q} = \frac{R}{S}$, when the network is balanced

$$\frac{500}{800} = \frac{x + 400}{1000}$$

$$x + 400 = \frac{5}{8} \times 1000$$

$$x + 400 = 625$$

$$x = 625 - 400$$

$$x = 225\ \Omega$$

2.5.4 Meter bridge

The meter bridge is another form of Wheatstone's bridge. It consists of a uniform wire of manganin AB of one meter length. This wire is stretched along a metre scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps G_1 and G_2 as shown in Figure 2.26.

An unknown resistance P is connected in G_1 and a standard resistance Q is connected in G_2. A jockey (conducting wire-contact maker) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR). The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected between the ends of the bridge wire.

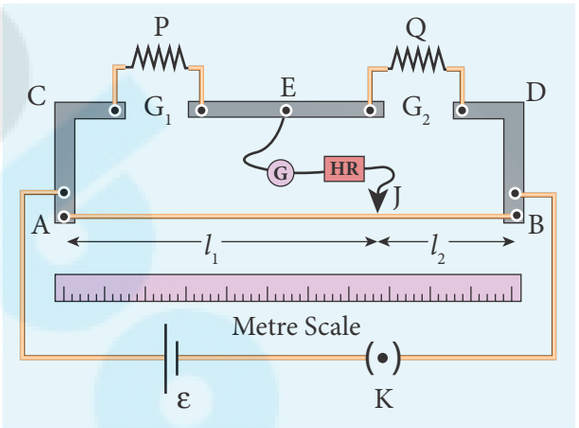


Figure 2.26 Meter bridge

The position of the jockey on the wire is adjusted so that the galvanometer shows zero deflection. Let the position of jockey at the wire be at J. The resistances corresponding to AJ and JB of the bridge wire form the resistances R and S of the Wheatstone's bridge. Then for the bridge balance

$$\frac{P}{Q} = \frac{R}{S} = \frac{r.\mathrm{AJ}}{r.\mathrm{JB}} \qquad (2.54)$$

UNIT-2(XII-Physics_Vol-1).indd 108 14-02-2020 19:37:18

where r is the resistance per unit length of wire.

$$\frac{P}{Q} = \frac{AJ}{JB} = \frac{l_1}{l_2} \quad (2.55)$$

$$P = Q\frac{l_1}{l_2} \quad (2.56)$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistances. This error can be eliminated, if another set of readings is taken with P and Q interchanged and the average value of P is found.

To find the specific resistance of the material of the wire in the coil P, the radius a and length l of the wire are measured. The specific resistance or resistivity ρ can be calculated using the relation.

$$\text{Resistance} = \rho\frac{l}{A}$$

By rearranging the above equation, we get

$$\rho = \text{Resistance} \times \frac{A}{l} \quad (2.57)$$

If P is the unknown resistance equation (2.57) becomes,

$$\rho = P\frac{\pi a^2}{l}$$

EXAMPLE 2.25

In a meter bridge experiment with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance.

Solution

$Q = 15\ \Omega, \quad l_1 : l_2 = 3:2$

$$\frac{l_1}{l_2} = \frac{3}{2}$$

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

$$P = Q\,\frac{l_1}{l_2}$$

$$P = 15 \times \frac{3}{2} = 22.5\ \Omega$$

EXAMPLE 2.26

In a meter bridge experiment, the value of resistance in the resistance box connected in the right gap is 10 Ω. The balancing length is l_1 = 55 cm. Find the value of unknown resistance.

Solution

$Q = 10\ \Omega$

$$\frac{P}{Q} = \frac{l_1}{100 - l_1} = \frac{l_1}{l_2}$$

$$P = Q \times \frac{l_1}{100 - l_1}$$

$$P = \frac{10 \times 55}{100 - 55}$$

$$P = \frac{550}{45} = 12.2\ \Omega$$

2.5.5 Potentiometer

Potentiometer is used for the accurate measurement of potential differences, current and resistances. It consists of ten meter long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board. The two free ends A and B are brought to the same side and fixed to copper strips with binding screws. A meter scale is fixed parallel to the wire. A jockey is provided for making contact.

The principle of the potentiometer is illustrated in Figure 2.27. A steady current is maintained across the wire CD by a battery Bt.

UNIT-2(XII-Physics_Vol-1).indd 109 14-02-2020 19:37:21

The battery, key and the potentiometer wire connected in series form the primary circuit. The positive terminal of a primary cell of emf ε is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.

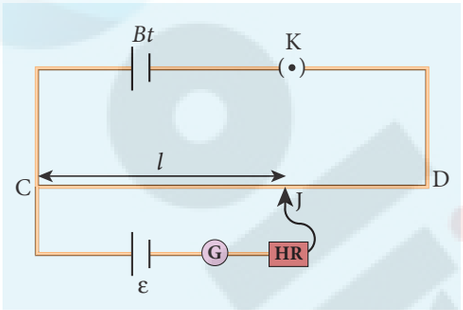


Figure 2.27 Potentiometer

Let the contact be made at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell ε, then no current will flow through the galvanometer and it will show zero deflection. CJ is the balancing length *l*. The potential difference across CJ is equal to *Irl* where *I* is the current flowing through the wire and *r* is the resistance per unit length of the wire.

$$\text{Hence } \varepsilon = Irl \qquad (2.58)$$

Since *I* and *r* are constants, $\varepsilon \propto l$. The emf of the cell is directly proportional to the balancing length.

2.5.6 Comparison of emf of two cells with a potentiometer

To compare the emf of two cells, the circuit connections are made as shown in Figure 2.28. Potentiometer wire CD is connected to a battery *Bt* and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR. The cells whose emf ε_1 and ε_2 to be compared are connected to the terminals M_1,N_1 and M_2,N_2 of the DPDT switch. The positive terminals of *Bt*, ε_1 and ε_2 should be connected to the same end C.

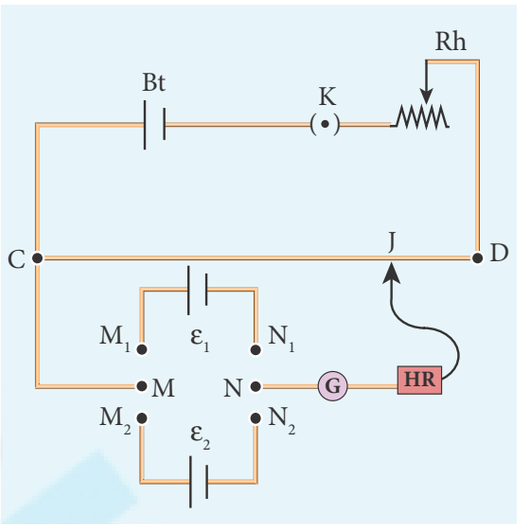


Figure 2.28 Comparison of emf of two cells

The DPDT switch is pressed towards M_1, N_1 so that cell ε_1 is included in the secondary circuit and the balancing length l_1 is found by adjusting the jockey for zero deflection. Then the second cell ε_2 is included in the circuit and the balancing length l_2 is determined. Let *r* be the resistance per unit length of the potentiometer wire and *I* be the current flowing through the wire.

$$\text{we have} \quad \varepsilon_1 = Irl_1 \qquad (2.59)$$

$$\varepsilon_2 = Irl_2 \qquad (2.60)$$

By dividing equation (2.59) by (2.60)

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \qquad (2.61)$$

UNIT-2(XII-Physics_Vol-1).indd 110 14-02-2020 19:37:21

By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it.

2.5.7 Measurement of internal resistance of a cell by potentiometer

To measure the internal resistance of a cell, the circuit connections are made as shown in Figure 2.29. The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K_1. This forms the primary circuit.

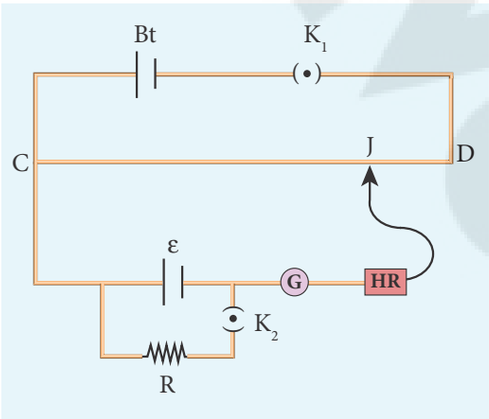


Figure 2.29 measurement of internal resistance

The positive terminal of the cell of emf ε whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell ε is connected to a jockey through a galvanometer and a high resistance. A resistance box R and key K_2 are connected across the cell ε. With K_2 open, the balancing point J is obtained and the balancing length CJ = l_1 is measured. Since the cell is in open circuit, its emf is

$$\varepsilon \propto l_1 \qquad (2.62)$$

A suitable resistance (say, 10 Ω) is included in the resistance box and key K_2 is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

$$I = \frac{\varepsilon}{R+r}$$

The potential difference across R is

$$V = \frac{\varepsilon R}{R+r}$$

When this potential difference is balanced on the potentiometer wire, let l_2 be the balancing length.

$$\text{Then } \frac{\varepsilon R}{R+r} \propto l_2 \qquad (2.63)$$

From equations (2.62) and (2.63)

$$\frac{R+r}{R} = \frac{l_1}{l_2} \qquad (2.64)$$

$$1+\frac{r}{R} = \frac{l_1}{l_2};$$

$$r = R\left[\frac{l_1}{l_2} - 1\right]$$

$$\therefore r = R\left(\frac{l_1 - l_2}{l_2}\right) \qquad (2.65)$$

Substituting the values of the R, l_1 and l_2, the internal resistance of the cell is determined. The experiment can be repeated for different values of R. It is found that the internal resistance of the cell is not constant but increases with increase of external resistance connected across its terminals.

2.6 HEATING EFFECT OF ELECTRIC CURRENT

When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of

UNIT-2(XII-Physics_Vol-1).indd 111 14-02-2020 19:37:23

current is known as Joule's heating effect. Just as current produces thermal energy, thermal energy may also be suitably used to produce an electromotive force. This is known as thermoelectric effect.

2.6.1 Joule's law

If a current I flows through a conductor kept across a potential difference V for a time t, the work done or the electric potential energy spent is

$$W = VIt \qquad (2.66)$$

In the absence of any other external effect, this energy is spent in heating the conductor. The amount of heat(H) produced is

$$H = VIt \qquad (2.67)$$

For a resistance R,

$$H = I^2Rt \qquad (2.68)$$

This relation was experimentally verified by Joule and is known as Joule's law of heating. It states that **the heat developed in an electrical circuit due to the flow of current varies directly as**

(i) the square of the current
(ii) the resistance of the circuit and
(iii) the time of flow.

EXAMPLE 2.27

Find the heat energy produced in a resistance of 10 Ω when 5 A current flows through it for 5 minutes.

Solution

$R = 10\ \Omega$, $I = 5$ A, $t = 5$ minutes $= 5 \times 60$ s

$H = I^2Rt$

$= 5^2 \times 10 \times 5 \times 60$

$= 25 \times 10 \times 300$

$= 25 \times 3000$

$= 75000$ J (or) 75 kJ

2.6.2 Application of Joule's heating effect

1. Electric heaters

Electric iron, electric heater, electric toaster shown in Figure 2.30 are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.



Figure 2.30 (a) Electric Iron box, (b) electric heater (c) electric Toaster

EXAMPLE 2.28

An electric heater of resistance 10 Ω connected to 220 V power supply is immersed in the water of 1 kg. How long the electrical heater has to be switched on to increase its temperature from 30°C to 60°C. (Specific heat capacity of water is $s = 4200$ J kg^{-1} K^{-1})

UNIT-2(XII-Physics_Vol-1).indd 112 14-02-2020 19:37:24

Solution

According to Joule's heating law $H = I^2Rt$

The current passed through the electrical heater $= \frac{220\text{V}}{10\Omega} = 22\,\text{A}$

Heat produced in one second by the electrical heater $H = I^2R$

Heat produced in one second
$H = (22)^2 \times 10 = 4840$ J $= 4.84$ k J. In fact the power rating of this electrical heater is 4.84 k W.

The amount of heat energy to increase the temperature of 1kg water from 30°C to 60°C is

$Q = ms\,\Delta T$ (Refer XI physics vol 2, unit 8)

Here $m = 1$ kg,
$s = 4200$ J kg^{-1} K^{-1},
$\Delta T = 30$ K,

so $Q = 1 \times 4200 \times 30 = 126$ kJ

The time required to produce this heat energy $t = \frac{Q}{I^2R} = \frac{126 \times 10^3}{4840} \approx 26.03$ s

2. Electric fuses

Fuses as shown in Figure 2.31, are connected in series in a circuit to protect the electric devices from the heat developed by the passage of excessive current. It is a short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain value. An alloy of lead - tin is used for fuses when current rating is below 15 A and when current rating is above 15 A, copper fuse wires are used.

The only disadvantage with the above fuses is that once fuse wire is burnt due to excessive current, they need to be replaced. Nowadays in houses, circuit breakers

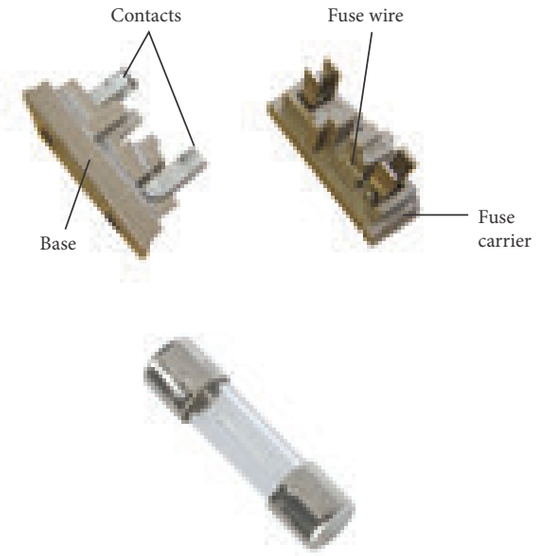


Figure 2.31 Electric Fuse

(trippers) are also used instead of fuses. Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch. It is shown in the Figure 2.32.



Figure 2.32 circuit breakers

3. Electric furnace

Furnaces as shown in Figure 2.33 are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc. To produce temperatures up to 1500°C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000°C.

UNIT-2(XII-Physics_Vol-1).indd 113 14-02-2020 19:37:25



Figure 2.33 Electric furnace

4. Electrical lamp

It consists of a tungsten filament (melting point 3380^{0}C) kept inside a glass bulb and heated to incandescence by current. In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current as shown in Figure 2.34.

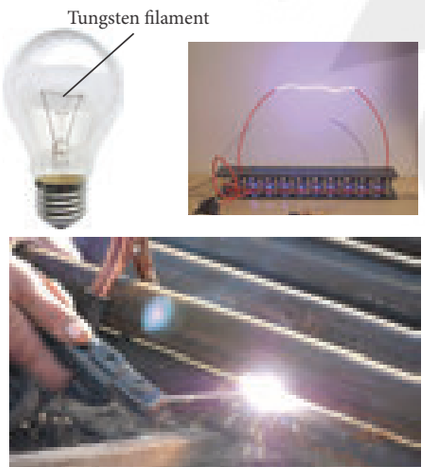


Figure 2.34 Electric bulb, electric arc and electric welding

2.7 THERMOELECTRIC EFFECT

Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect. A thermoelectric device generates voltage when there is a temperature difference on each side. If a voltage is applied, it generates a temperature difference.

2.7.1 Seebeck effect

Seebeck discovered that in a closed circuit consisting of two dissimilar metals, when the junctions are maintained at different temperatures an emf (potential difference) is developed. The current that flows due to the emf developed is called thermoelectric current. The two dissimilar metals connected to form two junctions is known as thermocouple (Figure 2.35).

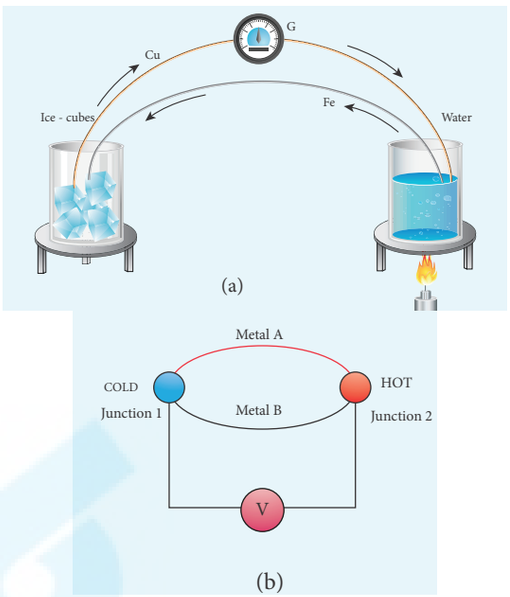


Figure 2.35 Seebeck effect (Thermocouple)

If the hot and cold junctions are interchanged, the direction of current also reverses. Hence the effect is reversible.

The magnitude of the emf developed in a thermocouple depends on (i) the nature of the metals forming the couple and (ii) the temperature difference between the junctions.

Applications of Seebeck effect

1. Seebeck effect is used in thermoelectric generators (Seebeck generators).These thermoelectric generators are used in power plants to convert waste heat into electricity.

UNIT-2(XII-Physics_Vol-1).indd 114 14-02-2020 19:37:26

2. This effect is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency.
3. Seebeck effect is used in thermocouples and thermopiles to measure the temperature difference between the two objects.

2.7.2 Peltier effect

In 1834, Peltier discovered that **when an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect.**

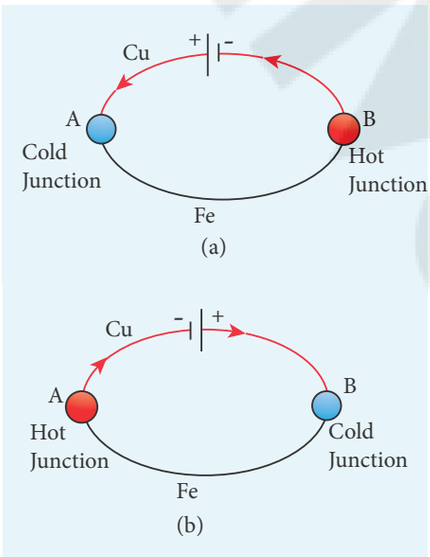


Figure 2.36 Peltier effect: Cu – Fe thermocouple

In the Cu-Fe thermocouple the junctions A and B are maintained at the same temperature. Let a current from a battery flow through the thermocouple (Figure 2.36 (a)). At the junction A, where the current flows from Cu to Fe, heat is absorbed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu heat is liberated and it becomes hot. When the direction of current is reversed, junction A gets heated and junction B gets cooled as shown in the Figure 2.36(b). Hence Peltier effect is reversible.

2.7.3 Thomson effect

Thomson showed that **if two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is also reversible.**

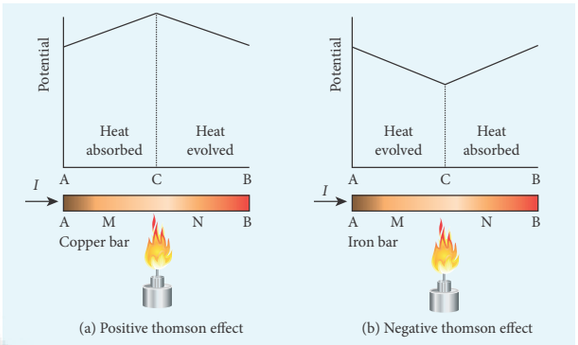


Figure 2.37 (a) Positive Thomson effect (b) Negative Thomson effect

If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is absorbed along AC and evolved along CB of the conductor as shown in Figure 2.37(a). Thus heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Similar effect is observed in metals like silver, zinc, and cadmium.

When the copper bar is replaced by an iron bar, heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect as shown in the Figure 2.37(b). Similar effect is observed in metals like platinum, nickel, cobalt, and mercury.

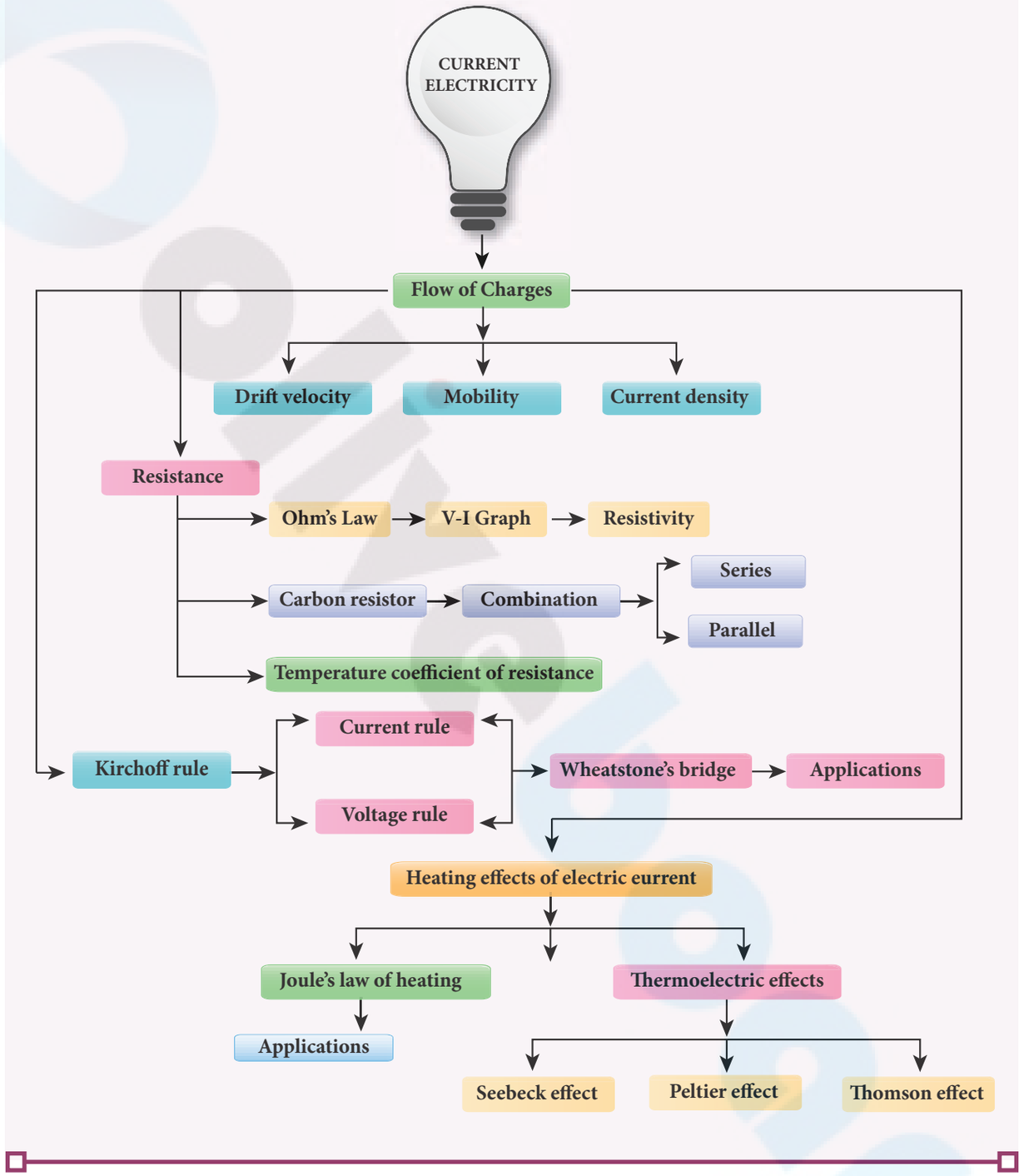
UNIT-2(XII-Physics_Vol-1).indd 115 14-02-2020 19:37:26

SUMMARY

- The current, I flowing in a conductor $I = \frac{dQ}{dt}$, where dQ is the charge that flows through a cross-section in a time interval dt. SI unit of current is ampere (A). 1A = 1 C s^{-1}.
- The current density J in a conductor is the current flowing per unit area. $\left(J = \frac{I}{A}\right)$
- Current is a scalar but current density is a vector.
- The general form of Ohm's law $\vec{J} = \sigma\vec{E}$
- Practical form of Ohm's law states that $V \propto I$, or $V = IR$ where I is the current and R is the resistance, V potential difference between ends of the conductor.
- The resistance R of a conductor is $R = \frac{V}{I}$. SI unit of resistance is ohm (Ω) and $1\ \Omega = \frac{1V}{1A}$
- The resistance of a conductor $R = \rho\frac{l}{A}$ where l is length of the conductor and A is its area of cross section.
- The resistivity of a material determines how much resistance it offers to the flow of current.
- The equivalent resistance (R_S) of several resistances (R_1, R_2, R_3……..) connected in series is $R_S = (R_1 + R_2 + R_3$……..)
- The equivalent resistance (R_P) of several resistances (R_1, R_2, R_3……..) connected in parallel is $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- Kirchhoff's first rule (Current rule or junction rule): The algebraic sum of the currents at any junction is zero.
- Kirchhoff's second rule (Voltage rule or loop rule): In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- Electric power is the rate at which electric energy is delivered.
- If a current I flows across a potential difference V, the power delivered to the circuit is $P = IV$.
- In a resistor R, the electrical power converted to heat is $P = I^2R = \frac{V^2}{R}$
- The energy equivalent of one kilowatt-hour (kWh) is 1kWh = 3.6 X 10^6 J.
- Metre bridge is one form of Wheatstone's network.
- Potentiometer is used to compare emf of cells.
- Joule's law of heating is $H = VIt$ (or) $H = I^2Rt$.

UNIT-2(XII-Physics_Vol-1).indd 116 14-02-2020 19:37:28

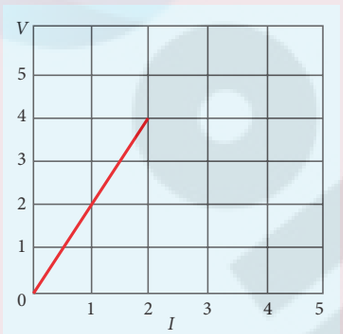
CONCEPT MAP



UNIT-2(XII-Physics_Vol-1).indd 117 14-02-2020 19:37:28

I Multiple Choice Questions

1. The following graph shows current versus voltage values of some unknown conductor. What is the resistance of this conductor?

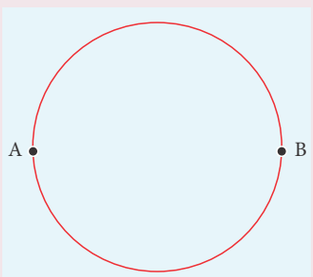


QMAJDW

(a) 2 ohm (b) 4 ohm

(c) 8 ohm (d)1 ohm

2. A wire of resistance 2 ohms per meter is bent to form a circle of radius 1m. The equivalent resistance between its two diametrically opposite points, A and B as shown in the figure is



(a) $\pi\ \Omega$ (c) $\frac{\pi}{2}\ \Omega$

(c) $2\pi\ \Omega$ (d) $\frac{\pi}{4}\ \Omega$

3. A toaster operating at 240 V has a resistance of 120 Ω. Its power is

a) 400 W b) 2 W

c) 480 W d) 240 W

4. A carbon resistor of $(47 \pm 4.7)\ k\Omega$ to be marked with rings of different colours for its identification. The colour code sequence will be

a) Yellow – Green – Violet – Gold

b) Yellow – Violet – Orange – Silver

c) Violet – Yellow – Orange – Silver

d) Green – Orange – Violet - Gold

5. What is the value of resistance of the following resistor?



(a)100 k Ω (b)10 k Ω

(c) 1k Ω (d)1000 k Ω

6. Two wires of A and B with circular cross section are made up of the same material with equal lengths. Suppose $R_A = 3\ R_B$, then what is the ratio of radius of wire A to that of B?

(a) 3 (b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

7. A wire connected to a power supply of 230 V has power dissipation P_1. Suppose the wire is cut into two equal pieces and connected parallel to the same power supply. In this case power dissipation is P_2. The ratio $\frac{P_2}{P_1}$ is

(a)1 (b) 2

(c) 3 (d) 4

UNIT-2(XII-Physics_Vol-1).indd 118 14-02-2020 19:37:29

8. In India electricity is supplied for domestic use at 220 V. It is supplied at 110 V in USA. If the resistance of a 60W bulb for use in India is R, the resistance of a 60W bulb for use in USA will be

(a) R (b) $2R$

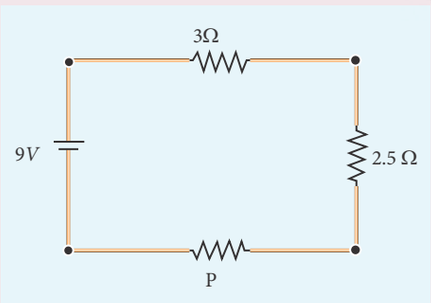
(c) $\frac{R}{4}$ (d) $\frac{R}{2}$

9. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1kW are connected. The voltage of electric mains is 220 V. The maximum capacity of the main fuse of the building will be (IIT-JEE 2014)

(a) 14 A (b) 8 A

(c) 10 A (d) 12 A

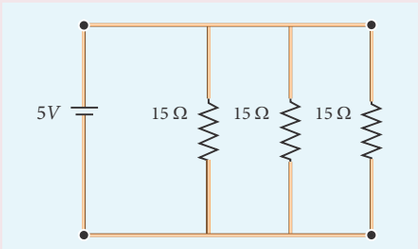
10. There is a current of 1.0 A in the circuit shown below. What is the resistance of P ?



a) 1.5 Ω b) 2.5 Ω

c) 3.5 Ω d) 4.5 Ω

11. What is the current drawn out from the battery?



a) 1A b) 2A

c) 3A d) 4A

12. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 20°C, its resistance is 1 Ω. The resistance of the wire will be 2 Ω at

a) 800 °C b) 700 °C

c) 850 °C d) 820 °C

13. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is

a) 0.2 Ω b) 0.5 Ω

c) 0.8 Ω d) 1.0 Ω

14. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

a) each of them increases

b) each of them decreases

c) copper increases and germanium decreases

d) copper decreases and germanium increases

15. In Joule's heating law, when R and t are constant, if the H is taken along the y axis and I^2 along the x axis, the graph is

a) straight line b) parabola

c) circle d) ellipse

Answers

1) a	**2)** a	**3)** c	**4)** b	**5)** a
6) c	**7)** d	**8)** c	**9)** d	**10)** c
11) a	**12)** d	**13)** b	**14)** d	**15)** a

II Short Answer Questions

1. Why current is a scalar?
2. Define current density.
3. Distinguish between drift velocity and mobility.

UNIT-2(XII-Physics_Vol-1).indd 119 14-02-2020 19:37:30

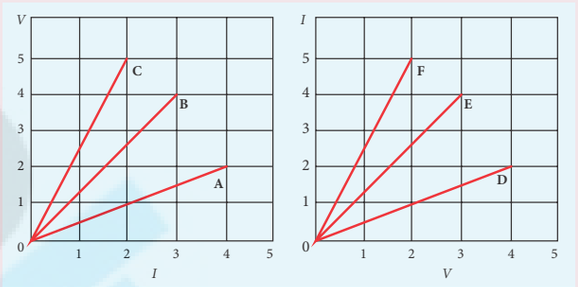
4. State microscopic form of Ohm's law.
5. State macroscopic form of Ohm's law.
6. What are ohmic and non ohmic devices?
7. Define electrical resistivity.
8. Define temperature coefficient of resistance.
9. Write a short note on superconductors?
10. What is electric power and electric energy?
11. Derive the expression for power P=VI in electrical circuit.
12. Write down the various forms of expression for power in electrical circuit.
13. State Kirchhoff's current rule.
14. State Kirchhoff's voltage rule.
15. State the principle of potentiometer.
16. What do you mean by internal resistance of a cell?
17. State Joule's law of heating.
18. What is Seebeck effect?
19. What is Thomson effect?
20. What is Peltier effect?
21. State the applications of Seebeck effect.

III Long Answer Questions

1. Describe the microscopic model of current and obtain general form of Ohm's law.
2. Obtain the macroscopic form of Ohm's law from its microscopic form and discuss its limitation.
3. Explain the equivalent resistance of a series and parallel resistor network.
4. Explain the determination of the internal resistance of a cell using voltmeter.
5. State and explain Kirchhoff's rules.
6. Obtain the condition for bridge balance in Wheatstone's bridge.
7. Explain the determination of unknown resistance using meter bridge.
8. How the emf of two cells are compared using potentiometer?

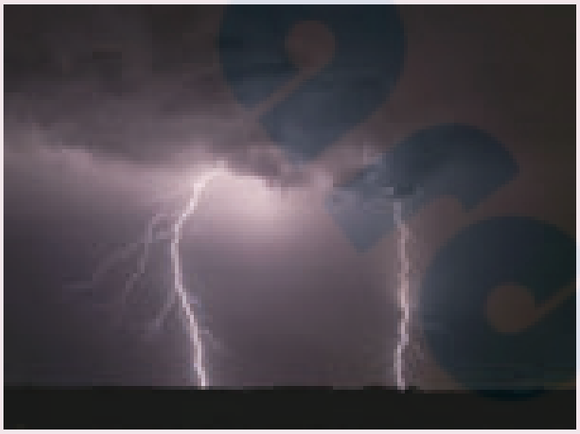
IV Numerical problems

1. The following graphs represent the current versus voltage and voltage versus current for the six conductors A,B,C,D,E and F. Which conductor has least resistance and which has maximum resistance?



Ans: Least: $R_F = 0.4\ \Omega$, maximum $R_C = 2.5\ \Omega$

2. Lightning is very good example of natural current. In typical lightning, there is 10^9 J energy transfer across the potential difference of 5×10^7 V during a time interval of 0.2 s.



UNIT-2(XII-Physics_Vol-1).indd 120 14-02-2020 19:37:30

Using this information, estimate the following quantities (a) total amount of charge transferred between cloud and ground (b) the current in the lightning bolt (c) the power delivered in 0.2 s.

Ans: charge = 20 C, I = 100 A, P = 5 GW

3. A copper wire of 10^{-6} m^2 area of cross section, carries a current of 2 A. If the number of free electrons per cubic meter in the wire is 8×10^{28}, calculate the current density and average drift velocity of electrons.

Ans: J = 2×10^6 Am^{-2}
$v_d = 15.6 \times 10^{-5}$ ms^{-1}

4. The resistance of a nichrome wire at 20^{0}C is 10 Ω. If its temperature coefficient of resistivity of nichrom is 0.004/^{0}C, find the resistance of the wire at boiling point of water. Comment on the result.

Ans: R_T = 13.2 Ω.
As the temperature increases the resistance of the wire also increases.

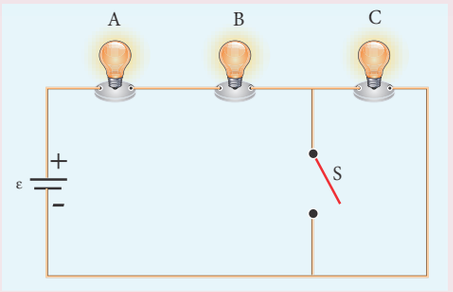
5. The rod given in the figure is made up of two different materials.



Both have square cross sections of 3 mm side. The resistivity of the first material is 4×10^{-3} Ωm and that of second material has resistivity of 5×10^{-3} Ωm. What is the resistance of rod between its ends?

Ans: 500 Ω

6. Three identical lamps each having a resistance R are connected to the battery of emf ε as shown in the figure.



Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A,B and C. (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decrease, increase or remain same?

Ans:

Electrical quantities	Switch S is open	Switch S is closed
a) Current	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{2R}$
b) Voltage	$V_A = \frac{\varepsilon}{3}$ $V_B = \frac{\varepsilon}{3}$ $V_C = \frac{\varepsilon}{3}$	$V_A = \frac{\varepsilon}{2}$ $V_B = \frac{\varepsilon}{2}$ $V_C = 0$
c) Power	$P_A = \frac{\varepsilon^2}{9R}$ $P_B = \frac{\varepsilon^2}{9R}$ $P_C = \frac{\varepsilon^2}{9R}$	$P_A = \frac{\varepsilon^2}{4R}$ $P_B = \frac{\varepsilon^2}{4R}$ $P_C = 0$ Total power increases
d) Intensity	All the bulbs glow with equal intensity	The intensities of the bulbs A and B equally increase. Bulb C will not glow since no current pass through it.
e) Increases		

UNIT-2(XII-Physics_Vol-1).indd 121 14-02-2020 19:37:31

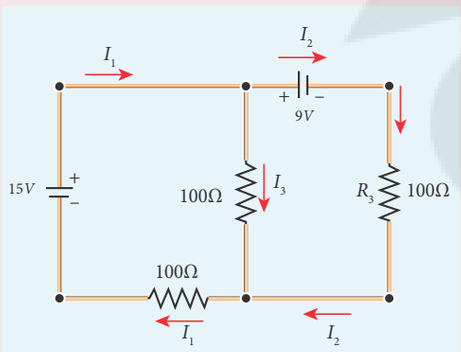
7. An electronics hobbyist is building a radio which requires 150 Ω in her circuit. But she has only 220 Ω, 79 Ω and 92 Ω resistors available. How can she connect the available resistors to get the desired value of resistance?

Ans: Parallel combination of 220 Ω and 79 Ω in series with 92 Ω

8. A cell supplies a current of 0.9 A through a 2 Ω resistor and a current of 0.3 A through a 7 Ω resistor. Calculate the internal resistance of the cell.

Ans: 0.5 Ω

9. Calculate the currents in the following circuit.

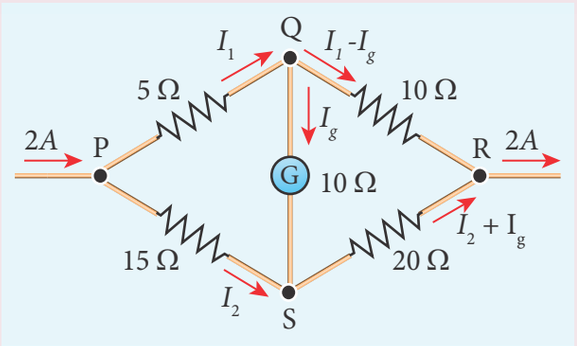


Ans : I_1 = 0.070 A, I_2 = -0.010 A and I_3 = 0.080 A

10. A potentiometer wire has a length of 4 m and resistance of 20 Ω. It is connected in series with resistance of 2980 Ω and a cell of emf 4 V. Calculate the potential gradient along the wire.

Ans: Potential = 0.65×10^{-2} V m^{-1}.

11. Determine the current flowing through the galvanometer (G) as shown in the figure.



Ans: $I_g = \frac{1}{11}$ A

12. Two cells each of 5V are connected in series with a 8 Ω resistor and three parallel resistors of 4 Ω, 6 Ω and 12 Ω. Draw a circuit diagram for the above arrangement. Calculate i) the current drawn from the cells (ii) current through each resistor

Ans:

i) Current through 8 Ω = 1A

ii) The current through 4 Ω, $I = \frac{2}{4} = 0.5\text{A}$,

the current through 6 Ω, $I = \frac{2}{6} = 0.33\text{A}$,

the current through 12 Ω, $I = \frac{2}{12} = 0.17\text{A}$

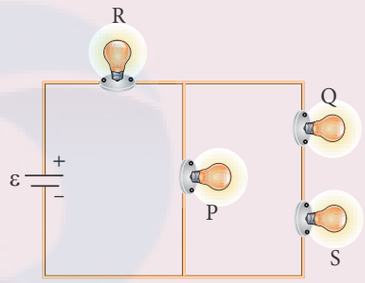
13. Four bulbs P, Q, R, S are connected in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed.

	P	Q	R	S
P removed	*	on	on	on
Q removed	on	*	on	off
R removed	off	off	*	off
S removed	on	off	on	*

Draw the circuit diagram for these bulbs.

UNIT-2(XII-Physics_Vol-1).indd 122 14-02-2020 19:37:32

Ans:



14. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?

    Ans: emf of the second cell is 2.25 V

BOOKS FOR REFERENCE:

1. Douglas C.Giancoli, , "*Physics for Scientist &Engineers with Modern Physics*", Pearson Prentice Hall, Fourth edition.
2. James Walker, *Physics*, Pearson- Addison Wesley publishers, Fourth edition.
3. Tipler, Mosca, "*Physics for scientist and Engineers with Modern Physics*", Freeman and Company, sixth edition.
4. Purcell, Morin, *Electricity and magnetism*, Cambridge university press, third edition
5. Serway and Jewett, "*Physics for Scientist and Engineers with Modern Physics*", Brook/Coole publishers, eighth edition.
6. Tarasov and Tarasova, "*Questions and problems in School Physics*", Mir Publishers.
7. H.C.Verma, "*Concepts of Physics Vol 2*, Bharthi Bhawan publishers.
8. Eric Roger, *Physics for the Inquiring Mind*, Princeton University press.

UNIT-2(XII-Physics_Vol-1).indd 123 14-02-2020 19:37:33

ICT CORNER

Electric current

In this activity you will be able to
(a) measure the potential difference of cells
(b) measure the internal resistance of a given primary cell

Topic: Potentiometer

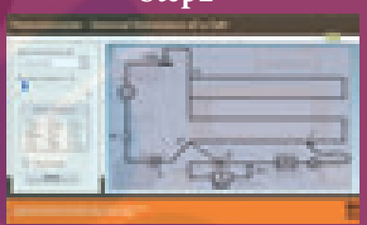
STEPS:

- Open the browser and type "olabs.edu.in" in the address bar. Click physics tab and then click "Potentiometer-Internal Resistance of a Cell" in class 12 section. Go to "simulator" tab to do the experiment.
- Construct the electric circuit as per the connection diagram by clicking "show circuit diagram" tab. You can connect wires between electric component by dragging the mouse between the component.
- To check whether the connections are correct or not, drag the jockey and place it at the two end points of the wire. If the galvanometer shows opposite deflections, the connections are correct. (keep both keys on)

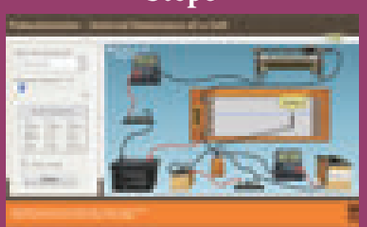
Step1



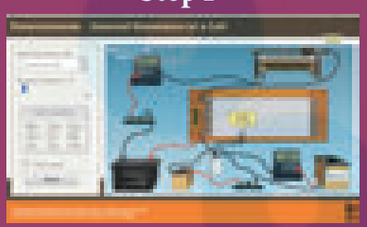
Step2



Step3



Step4



Find the balancing length. Calculate the internal resistance for the observed balancing lengths. Repeat the experiment for five times and take the average.

Note:

1. One time sign up is needed to do simulation. Then login using that username and password.
2. Read theory, procedure and animation to get the theory by clicking the corresponding tab.

URL:

http://amrita.olabs.edu.in/?sub=1&brch=6&sim=147&cnt=4

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT-2(XII-Physics_Vol-1).indd 124 14-02-2020 19:37:33

UNIT 3

MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

"The magnetic force is animate, or imitates a soul; in many respects it surpasses the human soul while it is united to an organic body" – William Gilbert

Learning Objectives

In this unit, the student is exposed to

3IWE3S

- Earth's magnetic field and magnetic elements
- Basic properties of magnets
- Statement of Coulomb inverse square law of magnetism
- Magnetic dipole
- Magnetic field due to a dipole along axial line and equatorial line
- Torque acting on a bar magnet in a uniform magnetic field
- Potential energy of a bar magnet placed in a uniform magnetic field
- Magnetic properties – permeability, susceptibility etc
- Classification of magnetic materials – dia, para and ferro magnetic materials
- Concept of Hysteresis
- Magnetic effects of electric current – long straight conductor and circular coil
- Right hand thumb rule and Maxwell's right hand cork screw rule
- Biot-Savart's law – applications
- Tangent law and Tangent Galvanometer
- Current loop as a magnetic dipole
- Magnetic dipole moment of revolving electron
- Ampère's circuital law – applications
- Solenoid and toroid
- Lorentz force – charged particle moving in an electromagnetic field
- Cyclotron
- Force on a current carrying conductor in a magnetic field
- Force between two long parallel current carrying conductor
- Torque on a current loop in a magnetic field
- Moving Coil Galvanometer

UNIT-3(XII-Physics_Vol-1).indd 125 14-02-2020 20:02:42

3.1 INTRODUCTION TO MAGNETISM

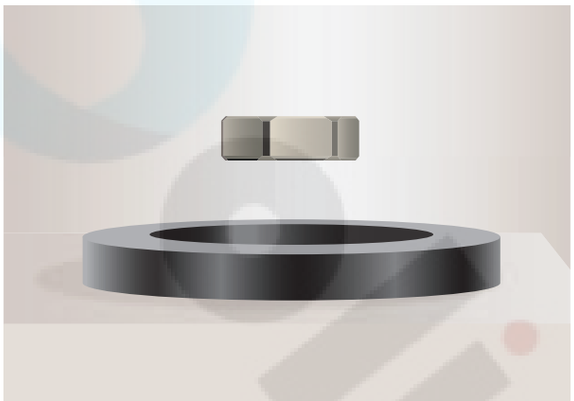


Figure 3.1: Magnetic levitation

Magnets! No doubt, their behaviour will attract everyone. The world enjoys their benefits, to lead a modern luxurious life. The study of magnets fascinated scientists around our globe for many centuries and even now, door for research on magnets is still open (Figure 3.1).



Many birds and animals have magnetic sense in their eyes using Earth's magnetic field for navigation.



Magnetic sensing in eyes - for Zebra finch bird, due to protein cryptochromes Cry4 present in retina; the bird uses Earth's magnetic field for navigation

Magnetism exists everywhere from tiny particles like electrons to the entire universe. Historically the word 'magnetism' was derived from iron ore magnetite (Fe_3O_4). In olden days, magnets were used as magnetic compass for navigation, magnetic therapy for treatment and also used in magic shows.

In modern days, many things we use in our daily life contain magnets (Figure 3.2). Motors, cycle dynamo, loudspeakers, magnetic tapes used in audio and video recording, mobile phones, head phones, CD, pen-drive, hard disc of laptop, refrigerator door, generator are a few examples.

Earlier, both electricity and magnetism were thought to be two independent branches in physics. In 1820, H.C. Oersted observed the deflection of magnetic compass needle kept near a current carrying wire. This unified the two different branches, electricity and magnetism as a single subject 'electromagnetism' in physics.

In this unit, basics of magnets and their properties are given. Later, how a current carrying conductor (here only steady current, not time-varying current is considered) behaves like a magnet is presented.

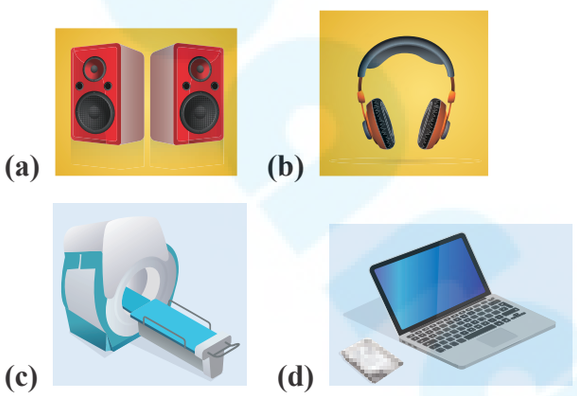


Figure 3.2 Uses of magnets in modern world – (a) speakers (b) head phones (c) MRI scan (d) Hard disc of laptop

UNIT-3(XII-Physics_Vol-1).indd 126 14-02-2020 20:02:43

3.1.1 Earth's magnetic field and magnetic elements

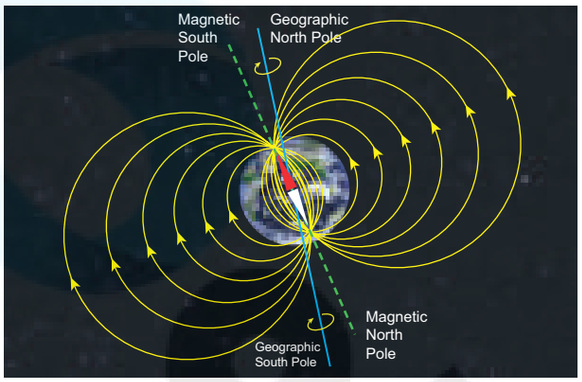


Figure 3.3 Earth's magnetic field

From the activities performed in lower classes, you might have noticed that the needle in a magnetic compass or freely suspended magnet comes to rest in a position which is approximately along the geographical north-south direction of the Earth.



William Gilbert in 1600 proposed that Earth itself behaves like a gigantic powerful bar magnet. But this theory is not successful because the temperature inside the Earth is very high and so it will not be possible for a magnet to retain its magnetism.

Gover suggested that the Earth's magnetic field is due to hot rays coming out from the Sun. These rays will heat up the air near equatorial region. Once air becomes hotter, it rises above and will move towards northern and southern hemispheres and get electrified. This may be responsible to magnetize the ferromagnetic materials near the Earth's surface. Till date, so many theories have been proposed. But none of the theorium completely explains the cause for the Earth's magnetism.

The north pole of magnetic compass needle is attracted towards the magnetic south pole of the Earth which is near the geographic north pole (Figure 3.3). Similarly, the south pole of magnetic compass needle is attracted towards the magnetic north-pole of the Earth which is near the geographic south pole. **The branch of physics which deals with the Earth's magnetic field is called Geomagnetism or Terrestrial magnetism.**

There are three quantities required to specify the magnetic field of the Earth on its surface, which are often called as the elements of the Earth's magnetic field. They are

(a) magnetic declination (D)

(b) magnetic dip or inclination (I)

(c) the horizontal component of the Earth's magnetic field (B_H)

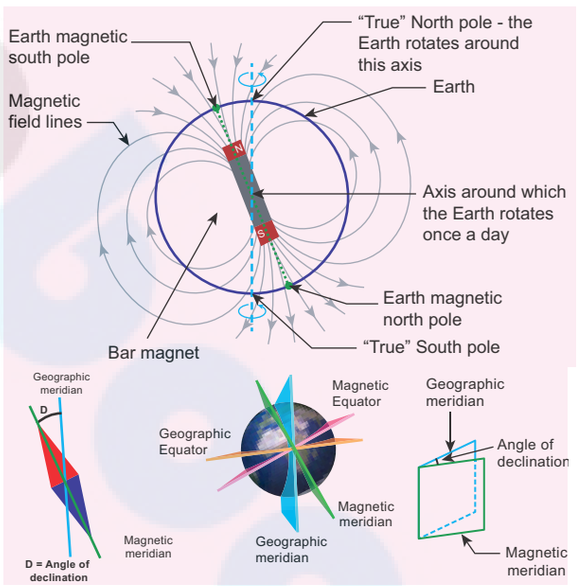


Figure 3.4 Declination angle

Day and night occur because Earth spins about an axis called geographic axis. A vertical plane passing through the geographic axis is called geographic meridian and a great circle perpendicular to Earth's geographic axis is called geographic equator.

The straight line which connects magnetic poles of Earth is known as magnetic axis.

UNIT-3(XII-Physics_Vol-1).indd 127 14-02-2020 20:02:44

A vertical plane passing through magnetic axis is called magnetic meridian and a great circle perpendicular to Earth's magnetic axis is called magnetic equator.

When a magnetic needle is freely suspended, the alignment of the magnet does not exactly lie along the geographic meridian as shown in Figure 3.4. **The angle between magnetic meridian at a point and geographical meridian is called the *declination or magnetic declination*** (D). At higher latitudes, the declination is greater whereas near the equator, the declination is smaller. In India, declination angle is very small and for Chennai, magnetic declination angle is $-1^{\circ}16'$ (which is negative (west)).

The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination (I) **at that point** (Figure 3.5). For Chennai, inclination angle is $14^{\circ}28'$. **The component of Earth's magnetic field along the horizontal direction in the magnetic meridian is called horizontal component of Earth's magnetic field, denoted by B_H.**

Let B_E be the net Earth's magnetic field at any point on the surface of the Earth. B_E can be resolved into two perpendicular components.

$$\text{Horizontal component} \quad B_H = B_E \cos I \qquad (3.1)$$

$$\text{Vertical component} \quad B_V = B_E \sin I \qquad (3.2)$$

Dividing equation (3.2) and (3.1), we get

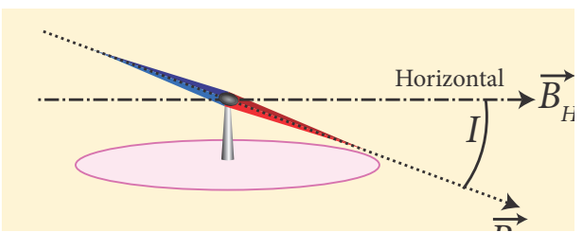


Figure 3.5 Inclination angle

$$\tan I = \frac{B_V}{B_H} \qquad (3.3)$$

(i) At magnetic equator

The Earth's magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip, $I = 0^{\circ}$.

$$B_H = B_E$$
$$B_V = 0$$

This implies that the horizontal component is maximum and vertical component is zero at the equator.

(ii) At magnetic poles

The Earth's magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle of magnetic compass rests vertically at an angle of dip, $I = 90^{\circ}$. Hence,

$$B_H = 0$$
$$B_V = B_E$$

This implies that the vertical component is maximum at poles and horizontal component is zero at poles.

EXAMPLE 3.1

The horizontal component and vertical component of Earth's magnetic field at a place are 0.15 G and 0.26 G respectively. Calculate the angle of dip and resultant magnetic field. (G-gauss, cgs unit for magnetic field $1\text{G} = 10^{-4}$ T)

Solution:

$B_H = 0.15$ G and $B_V = 0.26$ G

$$\tan I = \frac{0.26}{0.15} \Rightarrow I = \tan^{-1}(1.732) = 60^{\circ}$$

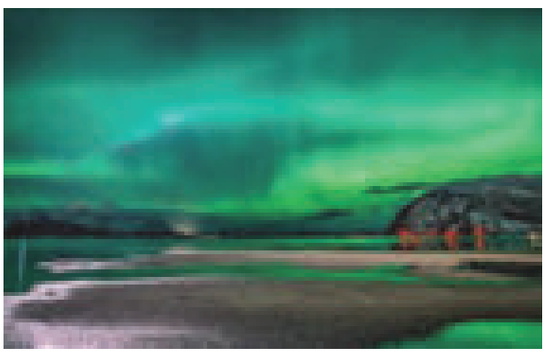
The resultant magnetic field of the Earth is

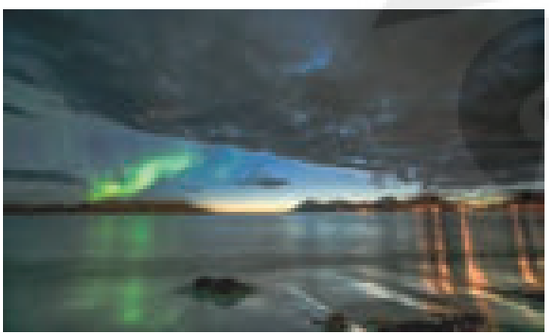
$$B = \sqrt{B_H^2 + B_V^2} = 0.3 \text{ G}$$

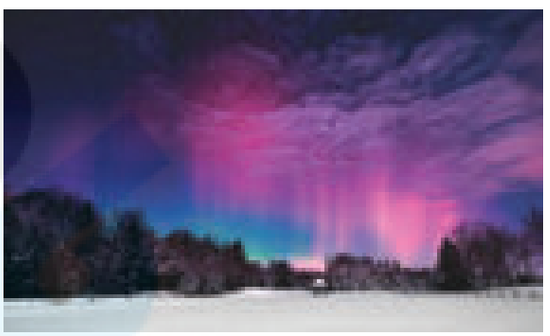
UNIT-3(XII-Physics_Vol-1).indd 128 14-02-2020 20:02:45

Aurora Borealis and Aurora Australis

People living at high latitude regions (near Arctic or Antarctic) might experience dazzling coloured natural lights across the night sky. This ethereal display on the sky is known as aurora borealis (northern lights) or aurora australis (southern lights). These lights are often called as polar lights. The lights are seen above the magnetic poles of the northern and southern hemispheres. They are called as "Aurora borealis" in the north and "Aurora australis" in the south. This occurs as a result of interaction between the gaseous particles in the Earth's atmosphere with highly charged particles released from the Sun's atmosphere through solar wind. These particles emit light due to collision and variations in colour are due to the type of the gas particles that take part in the collisions. A pale yellowish – green colour is produced when the ionized oxygen takes part in the collision and a blue or purplish – red aurora is produced due to ionized nitrogen molecules.









UNIT-3(XII-Physics_Vol-1).indd 129 14-02-2020 20:02:45

3.1.2 Basic properties of magnets

Some basic terminologies and properties used in describing bar magnet.

(a) Magnetic dipole moment

Consider a bar magnet as shown in Figure 3.6. Let q_m be the pole strength of the magnetic pole and let l be the distance between the geometrical centre of bar magnet O and one end of the pole. **The magnetic dipole moment is defined as the product of its pole strength and magnetic length**. It is a vector quantity, denoted by $\vec{p}_m$.

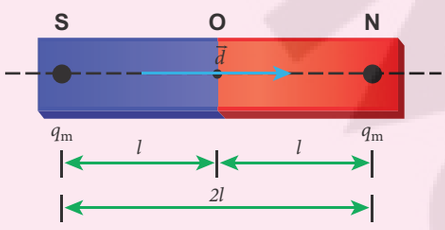


Figure 3.6 A bar magnet

$$\vec{p}_m = q_m \vec{d} \qquad (3.4)$$

where $\vec{d}$ is the vector drawn from south pole to north pole and its magnitude $|\vec{d}| = 2l$.

The magnitude of magnetic dipole moment is $p_m = 2q_m l$

The SI unit of magnetic moment is A m^2. The direction of magnetic moment is from south pole to north pole.

(b) Magnetic field

Magnetic field is the region or space around every magnet within which its influence will be felt by keeping another magnet in that region. **The magnetic field $\vec{B}$ at a point is defined as a force experienced by the bar magnet of unit pole strength.**

$$\vec{B} = \frac{1}{q_m}\vec{F} \qquad (3.5)$$

Its unit is N A^{-1} m^{-1}.

(c) Types of magnets

Magnets are classified into natural magnets and artificial magnets. For example, iron, cobalt, nickel, etc. are natural magnets. Strengths of natural magnets are very weak and the shapes of the magnet are irregular. Artificial magnets are made in order to have desired shape and strength. If the magnet is in the form of rectangular shape or cylindrical shape, then it is known as bar magnet.

Properties of magnet

The following are the properties of bar magnet (Figure 3.7),

1. A freely suspended bar magnet will always point along the north-south direction.
2. A magnet attracts or repels another magnet or magnetic substances towards itself. The attractive or repulsive force is maximum near the end of the bar magnet. When a bar magnet is dipped into iron filling, they cling to the ends of the magnet.
3. When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.
4. Two poles of a magnet have pole strength equal to one another.
5. The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called magnetic length. Magnetic length is always slightly smaller than geometrical length. The ratio of magnetic length and geometrical length is $\frac{5}{6}$.

$$\frac{Magnetic\ length}{Geometrical\ length} = \frac{5}{6} = 0.833$$

UNIT-3(XII-Physics_Vol-1).indd 130 14-02-2020 20:02:46

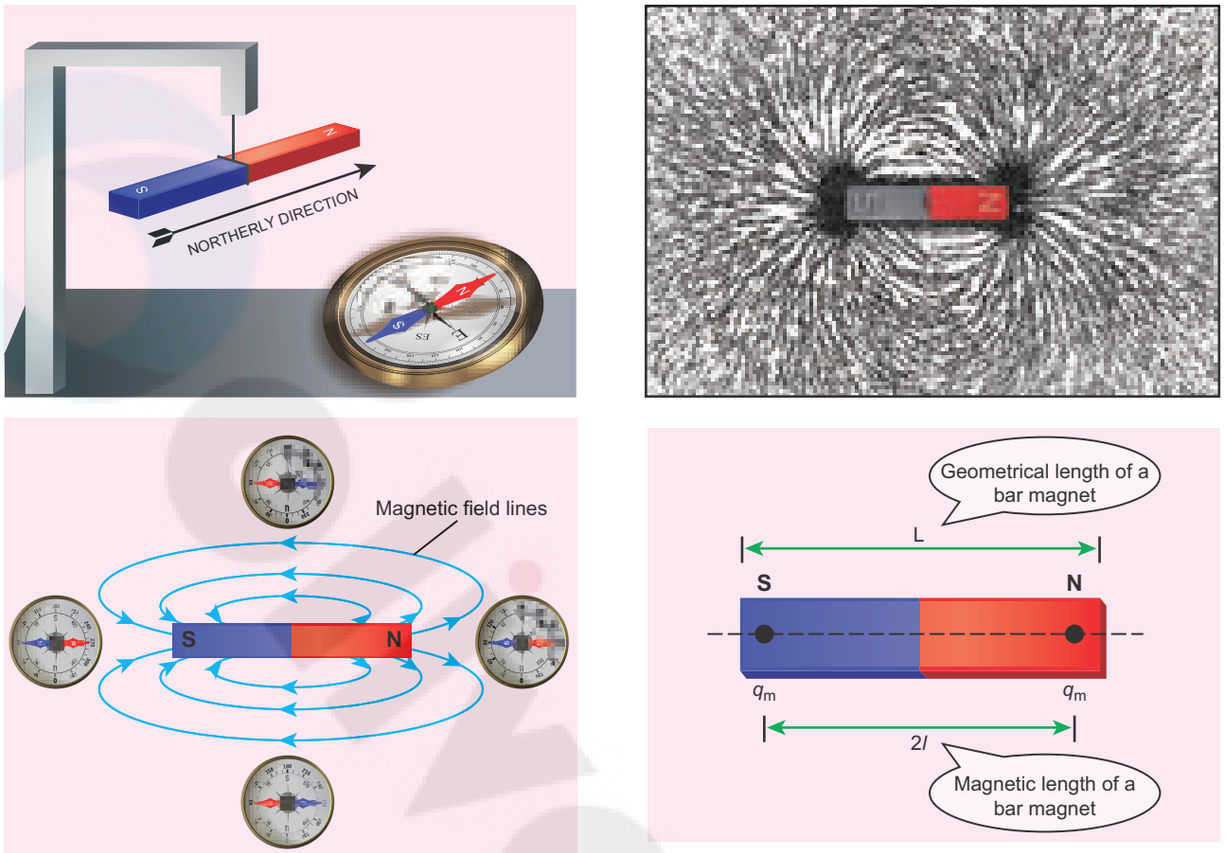


Figure 3.7 Properties of bar magnet

EXAMPLE 3.2

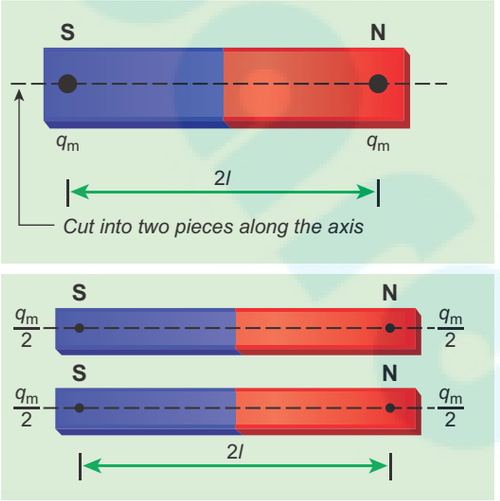
Let the magnetic moment of a bar magnet be $\vec{p}_m$ whose magnetic length is $d = 2l$ and pole strength is q_m. Compute the magnetic moment of the bar magnet when it is cut into two pieces

(a) along its length

(b) perpendicular to its length.

Solution

(a) a bar magnet cut into two pieces along its length:



UNIT-3(XII-Physics_Vol-1).indd 131 14-02-2020 20:02:48

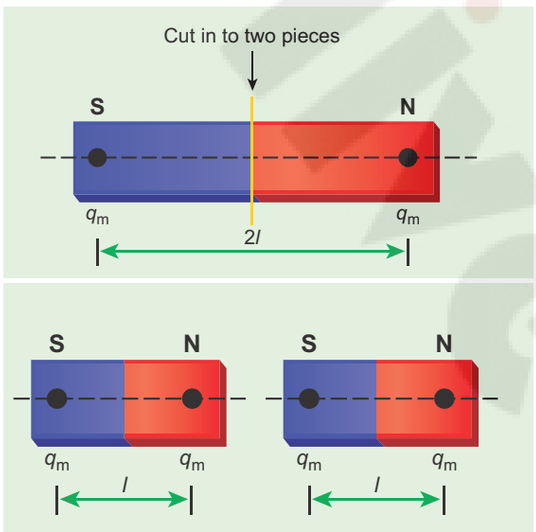
When the bar magnet is cut along the axis into two pieces, new magnetic pole strength is $q'_m = \frac{q_m}{2}$ but magnetic length does not change. So, the magnetic moment is

$$p'_m = q'_m 2l$$

$$p'_m = \frac{q_m}{2} 2l = \frac{1}{2}(q_m 2l) = \frac{1}{2} p_m$$

In vector notation, $\vec{p}'_m = \frac{1}{2}\vec{p}_m$

(b) a bar magnet cut into two pieces perpendicular to the axis:



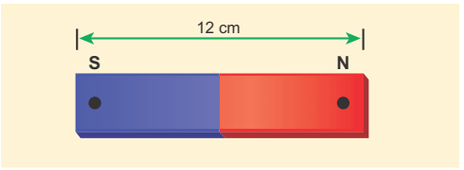
When the bar magnet is cut perpendicular to the axis into two pieces, magnetic pole strength will not change but magnetic length will be halved. So the magnetic moment is

$$p'_m = q_m \times \frac{1}{2}(2l) = \frac{1}{2}(q_m \cdot 2l) = \frac{1}{2} p_m$$

In vector notation, $\vec{p}'_m = \frac{1}{2}\vec{p}_m$

EXAMPLE 3.3

Compute the magnetic length of a uniform bar magnet if the geometrical length of the magnet is 12 cm. Mark the positions of magnetic pole points.



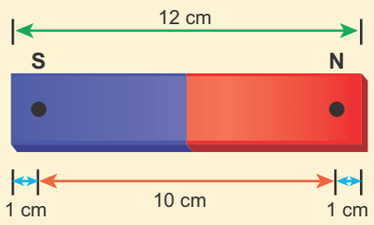
Solution

Geometrical length of the bar magnet is 12 cm

$$\text{Magnetic length} = \frac{5}{6} \times (\text{geometrical length})$$

$$= \frac{5}{6} \times 12 = 10\,\text{cm}$$

In this figure, the dot implies the pole points.



Note

(i) Pole strength is a scalar quantity with dimension $[M^0LT^0A]$. Its SI unit is NT^{-1} (newton per tesla) or A m (ampere-metre).

(ii) Like positive and negative charges in electrostatics, north pole of a magnet experiences a force in the direction of magnetic field while south pole of a magnet experiences force opposite to the magnetic field.

(iii) Pole strength depends on the nature of materials of the magnet, area of cross-section and the state of magnetization.

(iv) If a magnet is cut into two equal halves along the length then pole strength is reduced to half.

(v) If a magnet is cut into two equal halves perpendicular to the length, then pole strength remains same.

(vi) If a magnet is cut into two pieces, we will not get separate north and south poles. Instead, we get two magnets. In other words, isolated monopole does not exist in nature.

UNIT-3(XII-Physics_Vol-1).indd 132 14-02-2020 20:02:50

Magnetic field lines

1. Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from North pole to South pole outside the magnet and from South pole to North pole inside the magnet.
2. The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic field lines at that point.
3. Magnetic field lines never intersect each other. Otherwise, the magnetic compass needle would point towards two different directions, which is not possible.
4. The degree of closeness of the field lines determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak where magnetic field lines are well separated.

(d) Magnetic flux

The number of magnetic field lines crossing any area normally is defined as magnetic flux Φ_B through the area. Mathematically, the magnetic flux through a surface of area $\vec{A}$ in a uniform magnetic field $\vec{B}$ is defined as

$$\Phi_B = \vec{B}.\vec{A} = BA\cos\theta = B_\perp A \qquad (3.6)$$

where θ is the angle between $\vec{B}$ and $\vec{A}$ as shown in Figure 3.8.

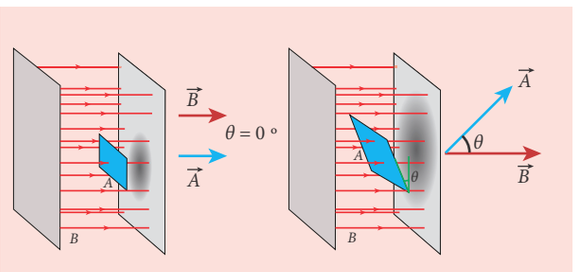


Figure 3.8 Magnetic flux

Special cases

(a) When $\vec{B}$ is normal to the surface i.e., $\theta = 0°$, the magnetic flux is $\Phi_B = BA$ (maximum).

(b) When $\vec{B}$ is parallel to the surface i.e., $\theta = 90°$, the magnetic flux is $\Phi_B = 0$.

Suppose the magnetic field is not uniform over the surface, the equation (3.6) can be written as

$$\Phi_B = \int \vec{B}.\mathrm{d}\vec{A}$$

Magnetic flux is a scalar quantity. The SI unit for magnetic flux is weber, which is denoted by symbol Wb. Dimensional formula for magnetic flux is $\left[ML^2T^{-2}A^{-1}\right]$. The CGS unit of magnetic flux is maxwell.

1 weber = 10^8 maxwell

The magnetic flux density is defined as the number of magnetic field lines crossing per unit area kept normal to the direction of lines of force. Its unit is Wb m^{-2} or tesla (T).

(e) Uniform magnetic field and Non-uniform magnetic field

Uniform magnetic field

Magnetic field is said to be uniform if it has same magnitude and direction at all the

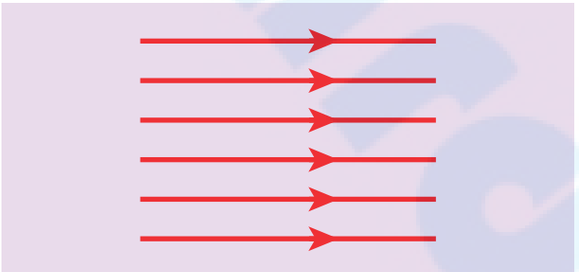


Figure 3.9 Uniform magnetic field

UNIT-3(XII-Physics_Vol-1).indd 133 14-02-2020 20:02:51

points in a given region. Example, locally Earth's magnetic field is uniform.

The magnetic field of Earth has same value over the entire area of your school!

Non-uniform magnetic field

Magnetic field is said to be non-uniform if the magnitude or direction or both vary at different points in a region. Example: magnetic field of a bar magnet

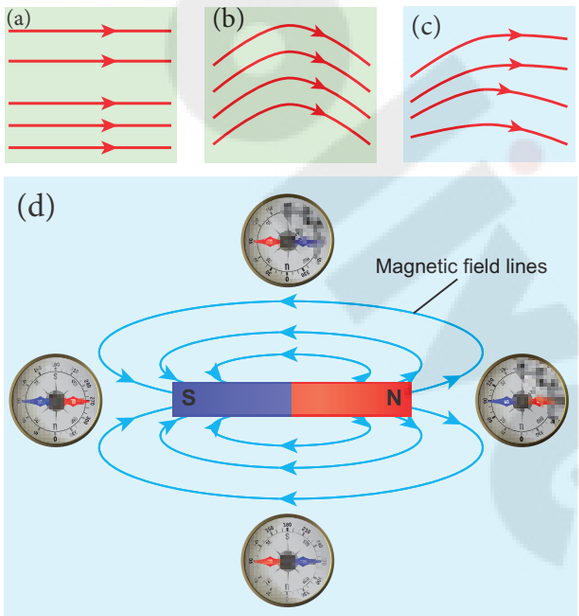
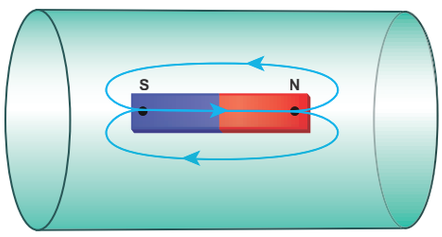


Figure 3.10 Non-uniform magnetic field (a) direction is constant (b) direction is not a constant (c) both magnitude and direction are not constant (d) magnetic field of a bar magnet

EXAMPLE 3.4

Calculate the magnetic flux coming out from closed surface containing magnetic dipole (say, a bar magnet) as shown in figure.



Solution

The total flux emanating from the closed surface S enclosing the dipole is zero. So,

$$\Phi_B = \oint \vec{B}.\mathrm{d}\vec{A} = 0$$

Here the integral is taken over closed surface. Since no isolated magnetic pole (called magnetic monopole) exists, this integral is always zero,

$$\oint \vec{B}.\mathrm{d}\vec{A} = 0$$

This is similar to Gauss's law in electrostatics.

3.2 COULOMB'S INVERSE SQUARE LAW OF MAGNETISM

Consider two bar magnets A and B as shown in Figure 3.11. When the north pole of magnet A and the north pole of magnet B or the south pole of magnet A and the south pole of magnet B are brought closer, they repel each other.

On the other hand, when the north pole of magnet A and the south pole of magnet B or the south pole of magnet A and the north pole of magnet B are brought closer, their poles attract each other.

This looks similar to Coulomb's law for static charges studied in Unit I (opposite charges attract and like charges repel each other). So analogous to Coulomb's law in electrostatics, we can state Coulomb's law for magnetism (Figure 3.12) as follows:

UNIT-3(XII-Physics_Vol-1).indd 134 14-02-2020 20:02:52

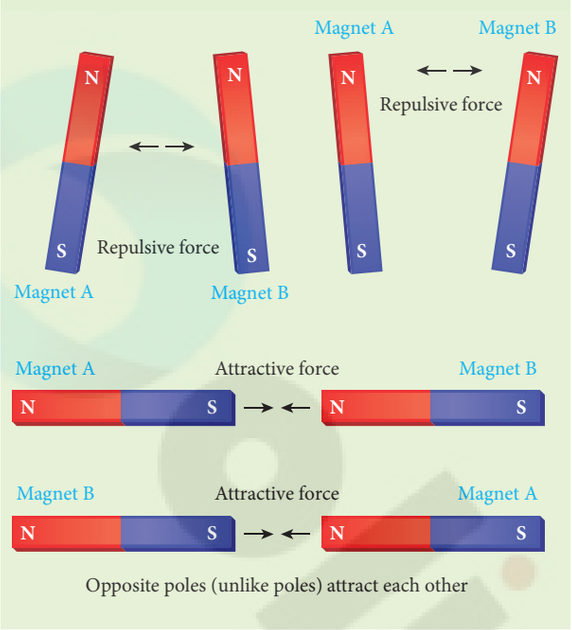


Figure 3.11: Magnetic poles behave like electric charges – like poles repel and unlike poles attract

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

Mathematically, we can write

$$\vec{F} \propto \frac{q_{m_A} q_{m_B}}{r^2} \hat{r}$$

where q_{m_A} and q_{m_B} are pole strengths of two poles and r is the distance between two magnetic poles.

$$\vec{F} = k \frac{q_{m_A} q_{m_B}}{r^2} \hat{r} \qquad (3.7)$$

In magnitude, $F = k \dfrac{q_{m_A} q_{m_B}}{r^2}$ (3.8)

where k is a proportionality constant whose value depends on the surrounding medium. In SI unit, the value of k for free space is $k = \dfrac{\mu_\circ}{4\pi} \approx 10^{-7}\,\text{H m}^{-1}$, where μ_o is the absolute permeability of free space (air or vacuum) and H stands for henry.

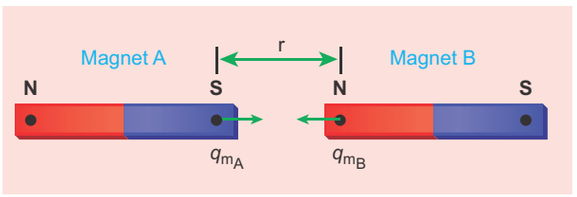


Figure 3.12 Coulomb's law – force between two magnetic poles

EXAMPLE 3.5

The repulsive force between two magnetic poles in air is 9×10^{-3} N. If the two poles are equal in strength and are separated by a distance of 10 cm, calculate the pole strength of each pole.

***Solution*:**

The magnitude of the force between two poles is given by

$$F = k \frac{q_{m_A} q_{m_B}}{r^2}$$

Given: $F = 9 \times 10^{-3}\text{N}, r = 10\,\text{cm} = 10 \times 10^{-2}\,\text{m}$

Since $q_{m_A} = q_{m_B} = q_m$, we have

$$9 \times 10^{-3} = 10^{-7} \times \frac{q_m^2}{\left(10 \times 10^{-2}\right)^2} \Rightarrow q_m = 30\,\text{N T}^{-1}$$

3.2.1 Magnetic field at a point along the axial line of the magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.13. Let N be the north pole and S be the south pole of the bar magnet, each of pole strength q_m and are separated by a distance of $2l$. The magnetic field at a point C (lies along the axis of the magnet)

UNIT-3(XII-Physics_Vol-1).indd 135 14-02-2020 20:02:54

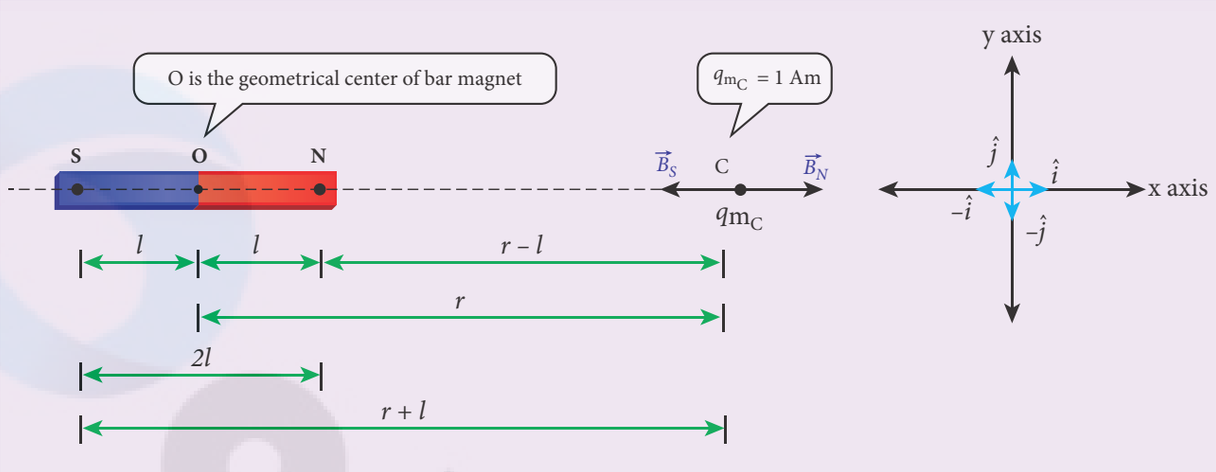


Figure 3.13 Magnetic field at a point along the axial line due to magnetic dipole

at a distance r from the geometrical centre O of the bar magnet can be computed by keeping unit north pole (q_{m_C} = 1 Am) at C.

The magnetic field at C due to the north pole is

$$\vec{B}_N = \frac{\mu_\circ}{4\pi}\frac{q_m}{(r-l)^2}\hat{i} \qquad (3.9)$$

where $(r - l)$ is the distance between north pole of the bar magnet and unit north pole at C.

The magnetic field at C due to the south pole is

$$\vec{B}_S = -\frac{\mu_\circ}{4\pi}\frac{q_m}{(r+l)^2}\hat{i} \qquad (3.10)$$

where $(r + l)$ is the distance between south pole of the bar magnet and unit north pole at C.

The net magnetic field due to the magnetic dipole at point C

$$\vec{B} = \vec{B}_N + \vec{B}_S$$

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{q_m}{(r-l)^2}\hat{i} + \left(-\frac{\mu_\circ}{4\pi}\frac{q_m}{(r+l)^2}\hat{i}\right)$$

$$\vec{B} = \frac{\mu_\circ q_m}{4\pi}\left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right)\hat{i}$$

$$\vec{B} = \frac{\mu_\circ 2r}{4\pi}\left(\frac{q_m\cdot(2l)}{\left(r^2-l^2\right)^2}\right)\hat{i} \qquad (3.11)$$

Since the magnitude of magnetic dipole moment is $|\vec{p}_m| = p_m = q_m \cdot 2l$, the magnetic field at a point C can be written as

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2rp_m}{\left(r^2-l^2\right)^2}\right)\hat{i} \qquad (3.12)$$

If the distance between two poles in a bar magnet is small (looks like short magnet) when compared to the distance between geometrical centre O of bar magnet and the location of point C ($r >> l$),

$$(r^2-l^2)^2 \approx r^4 \qquad (3.13)$$

Therefore, using equation (3.13) in equation (3.12), we get

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i} = \frac{\mu_\circ}{4\pi}\frac{2}{r^3}\vec{p}_m \qquad (3.14)$$

where $\vec{p}_m = p_m\hat{i}$.

UNIT-3(XII-Physics_Vol-1).indd 136 14-02-2020 20:02:57

3.2.2. Magnetic field at a point along the equatorial line due to a magnetic dipole (bar magnet)

Consider a bar magnet NS as shown in Figure 3.14. Let N be the north pole and S be the south pole of the bar magnet, each with pole strength q_m and separated by a distance of 2*l*. The magnetic field at a point C (lies along the equatorial line) at a distance *r* from the geometrical centre O of the bar magnet can be computed by keeping unit north pole (q_{mC} = 1 A m) at C.

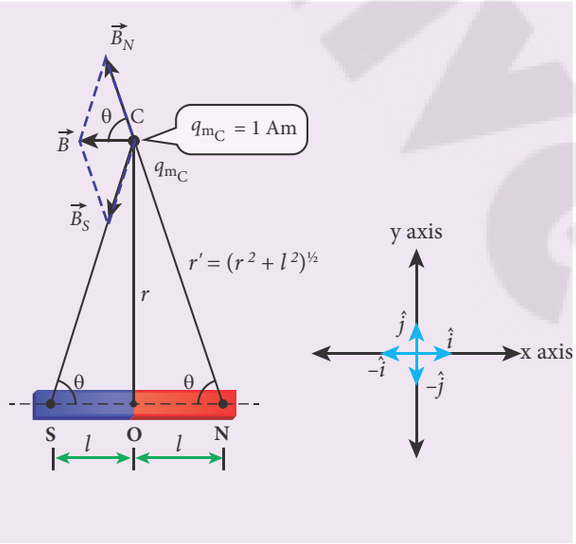


Figure 3.14 Magnetic field at a point along the equatorial line due to a magnetic dipole

The magnetic field at C due to the north pole is

$$\vec{B}_N = -B_N \cos\theta \hat{i} + B_N \sin\theta \hat{j} \qquad (3.15)$$

where $B_N = \frac{\mu_\circ}{4\pi} \frac{q_m}{r'^2}$

Here $r' = (r^2 + l^2)^{\frac{1}{2}}$

The magnetic field at C due to the south pole is

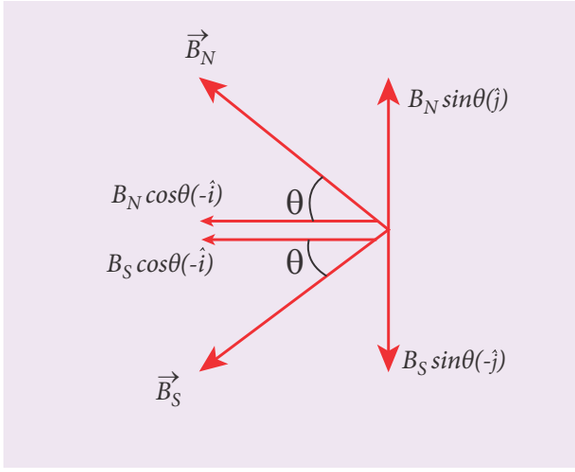


Figure 3.15 Components of magnetic field

$$\vec{B}_S = -B_S \cos\theta \hat{i} - B_S \sin\theta \hat{j} \qquad (3.16)$$

where, $B_S = \frac{\mu_\circ}{4\pi} \frac{q_m}{r'^2}$

From equations (3.15) and (3.16), the net magnetic field at point C due to the dipole is $\vec{B} = \vec{B}_N + \vec{B}_S$.

$$\vec{B} = -(B_N + B_S)\cos\theta \hat{i} \quad \text{Since, } B_N = B_S$$

$$\vec{B} = -\frac{2\mu_\circ}{4\pi} \frac{q_m}{r'^2} \cos\theta \hat{i} = -\frac{2\mu_\circ}{4\pi} \frac{q_m}{(r^2 + l^2)} \cos\theta \hat{i} \qquad (3.17)$$

In a right angle triangle NOC as shown in Figure 3.14

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{l}{r'} = \frac{l}{(r^2 + l^2)^{\frac{1}{2}}} \qquad (3.18)$$

Substituting equation (3.18) in equation (3.17), we get

$$\vec{B} = -\frac{\mu_\circ}{4\pi} \frac{q_m \times (2l)}{(r^2 + l^2)^{\frac{3}{2}}} \hat{i} \qquad (3.19)$$

UNIT-3(XII-Physics_Vol-1).indd 137 14-02-2020 20:03:00

Since, magnitude of magnetic dipole moment is $|\vec{p}_m| = p_m = q_m \cdot 2l$ and substituting in equation (3.19), the magnetic field at a point C is

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{(r^2+l^2)^{\frac{3}{2}}}\hat{i} \qquad (3.20)$$

If the distance between two poles in a bar magnet is small (looks like short magnet) when compared to the distance between geometrical centre O of bar magnet and the location of point C ($r >> l$),

$$(r^2+l^2)^{\frac{3}{2}} \approx r^3 \qquad (3.21)$$

Therefore, using equation (3.21) in equation (3.20), we get

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

Since $p_m\hat{i} = \vec{p}_m$, the magnetic field at equatorial point is given by

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{\vec{p}_m}{r^3} \qquad (3.22)$$

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

EXAMPLE 3.6

A short bar magnet has a magnetic moment of 0.5 J T^{-1}. Calculate magnitude and direction of the magnetic field produced by the bar magnet which is kept at a distance of 0.1 m from the centre of the bar magnet along (a) axial line of the bar magnet and (b) normal bisector of the bar magnet.

Solution

Given magnetic moment = 0.5 J T^{-1} and distance r = 0.1 m

(a) When the point lies on the axial line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{axial} = \frac{\mu_\circ}{4\pi}\left(\frac{2p_m}{r^3}\right)\hat{i}$$

$$\vec{B}_{axial} = 10^{-7} \times \left(\frac{2\times 0.5}{(0.1)^3}\right)\hat{i} = 1\times 10^{-4}\ \hat{i}\ \text{T}$$

Hence, the magnitude of the magnetic field along axial is $B_{axial} = 1 \times 10^{-4}$ T and direction is towards South to North.

(b) When the point lies on the normal bisector (equatorial) line of the bar magnet, the magnetic field for short magnet is given by

$$\vec{B}_{equatorial} = -\frac{\mu_\circ}{4\pi}\frac{p_m}{r^3}\hat{i}$$

$$\vec{B}_{equatorial} = -10^{-7}\left(\frac{0.5}{(0.1)^3}\right)\hat{i} = -0.5\times 10^{-4}\ \hat{i}\ \text{T}$$

Hence, the magnitude of the magnetic field along axial is $B_{equatorial}$ = 0.5 × 10^{-4} T and direction is towards North to South.

Note that magnitude of B_{axial} is twice that of magnitude of $B_{equatorial}$ and the direction of B_{axial} and $B_{equatorial}$ are opposite.

3.3 TORQUE ACTING ON A BAR MAGNET IN UNIFORM MAGNETIC FIELD

Consider a magnet of length $2l$ and pole strength q_m kept in a uniform magnetic field $\vec{B}$ as shown in Figure 3.16. Each pole experiences a force of magnitude q_mB but acting in opposite directions. Therefore, the net force exerted on the magnet is zero and hence, there is no translatory motion. These two equal and opposite forces constitute a couple (about midpoint of bar magnet) tend to align the magnet in the direction of the magnetic field $\vec{B}$.

UNIT-3(XII-Physics_Vol-1).indd 138 14-02-2020 20:03:03

The force experienced by north pole,

$$\vec{F}_N = q_m \vec{B} \qquad (3.23)$$

The force experienced by south pole,

$$\vec{F}_S = -q_m \vec{B} \qquad (3.24)$$

Adding equations (3.23) and (3.24), we get the net force acting on the dipole as

$$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}$$

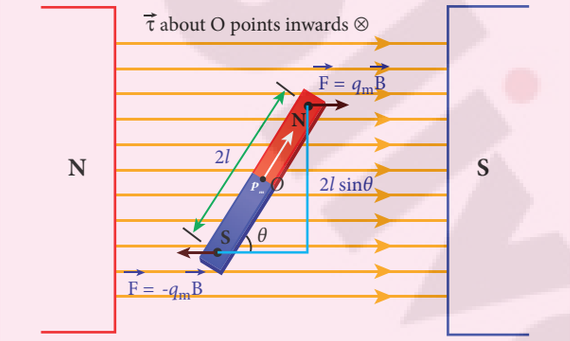


Figure 3.16 Magnetic dipole kept in a uniform magnetic field

The moment of force or torque experienced by north and south pole about point O is

$$\vec{\tau} = \overrightarrow{ON} \times \vec{F}_N + \overrightarrow{OS} \times \vec{F}_S$$
$$\vec{\tau} = \overrightarrow{ON} \times q_m \vec{B} + \overrightarrow{OS} \times \left(-q_m \vec{B}\right)$$

By using right hand cork screw rule, we conclude that the total torque is pointing into the paper. Since the magnitudes $\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OS}\right| = l$ and $\left|q_m \vec{B}\right| = \left|-q_m \vec{B}\right|$, the magnitude of total torque about point O

$$\tau = l \times q_m B \sin\theta + l \times q_m B \sin\theta$$
$$= 2l \times q_m B \sin\theta$$
$$\tau = p_m B \sin\theta \qquad \left(\because q_m \times 2l = p_m\right)$$

In vector notation, $\vec{\tau} = \vec{p}_m \times \vec{B}$ (3.25)



(a) Why a freely suspended bar magnet in your laboratory experiences only torque (rotational motion) but not any translatory motion even though Earth has non-uniform magnetic field?

It is because Earth's magnetic field is locally(physics laboratory)uniform.

(b) Suppose we keep a freely suspended bar magnet in a non-uniform magnetic field. What will happen?

It will undergo translatory motion (net force) and rotational motion (torque).

3.3.1. Potential energy of a bar magnet in a uniform magnetic field

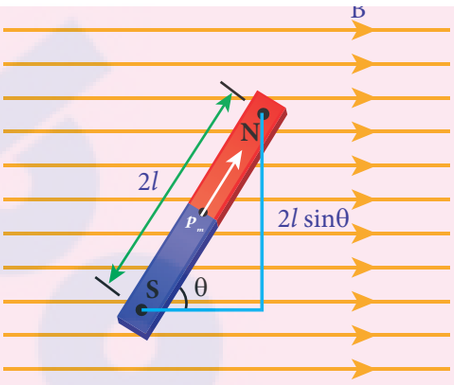


Figure 3.17: A bar magnet (magnetic dipole) in a uniform magnetic field

When a bar magnet (magnetic dipole) of dipole moment $\vec{p}_m$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$, as shown in Figure 3.17 the magnitude of the torque acting on the dipole is

$$\left|\vec{\tau}_B\right| = \left|\vec{p}_m\right|\left|\vec{B}\right|\sin\theta$$

If the dipole is rotated through a very small angular displacement $d\theta$ against the

UNIT-3(XII-Physics_Vol-1).indd 139 14-02-2020 20:03:06

torque τ_B at constant angular velocity, then the work done by external torque $(\vec{\tau}_{ext})$ for this small angular displacement is given by

$$dW = |\vec{\tau}_{ext}| d\theta$$

The bar magnet has to be moved at constant angular velocity, which implies that $|\vec{\tau}_B| = |\vec{\tau}_{ext}|$

$$dW = p_m B \sin\theta \, d\theta$$

Total work done in rotating the dipole from θ' to θ is

$$W = \int_{\theta'}^{\theta} \tau d\theta = \int_{\theta'}^{\theta} p_m B \sin\theta d\theta = p_m B \left[-\cos\theta\right]_{\theta'}^{\theta}$$

$$W = -p_m B(\cos\theta - \cos\theta')$$

This work done is stored as potential energy in bar magnet at an angle θ (when it is rotated from θ' to θ) and it can be written as

$$U = -p_m B(\cos\theta - \cos\theta') \qquad (3.26)$$

In fact, the equation (3.26) gives the difference in potential energy between the angular positions θ' and θ. If we choose the reference point as $\theta' = 90^o$, so that second term in the equation becomes zero, the equation (3.26) can be written as

$$U = -p_m B (\cos\theta) \qquad (3.27)$$

The potential energy stored in a bar magnet in a uniform magnetic field is given by

$$U = -\vec{p}_m \cdot \vec{B} \qquad (3.28)$$

Case 1

(i) If $\theta = 0^o$, then

$$U = -p_m B (\cos 0^\circ) = -p_m B$$

(ii) If $\theta = 180^o$, then

$$U = -p_m B (\cos 180^\circ) = p_m B$$

From the above two results, We infer that the potential energy of the bar magnet is minimum when it is aligned along the external magnetic field and maximum when the bar magnet is aligned anti-parallel to external magnetic field.

EXAMPLE 3.7

Consider a magnetic dipole which on switching ON external magnetic field orient only in two possible ways i.e., one along the direction of the magnetic field (parallel to the field) and another anti-parallel to magnetic field. Compute the energy for the possible orientation.

Solution

Let $\vec{p}_m$ be the dipole and before switching ON the external magnetic field, there is no orientation. Therefore, the energy U = 0.

As soon as external magnetic field is switched ON, the magnetic dipole orient parallel ($\theta = 0^o$) to the magnetic field with energy,

$$U_{parallel} = U_{minimum} = -p_m B \cos 0$$

$$U_{parallel} = -p_m B$$

since $\cos 0^o = 1$

Otherwise, the magnetic dipole orients anti-parallel ($\theta = 180^o$) to the magnetic field with energy,

$$U_{anti-parallel} = U_{maximum} = -p_m B \cos 180$$

$$\Rightarrow U_{anti-parallel} = p_m B$$

since $\cos 180^o = -1$

UNIT-3(XII-Physics_Vol-1).indd 140 14-02-2020 20:03:10

3.4 MAGNETIC PROPERTIES

All materials are not magnetic in nature. Further, all the magnetic materials will not behave identically. So, in order to differentiate one magnetic material from another, some basic parameters are used. They are:

(a) Magnetising field

The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. Magnetising field is a vector quantity and is denoted by $\vec{H}$ and its unit is A m^{-1}.

(b) Magnetic permeability

The magnetic permeability is the measure of ability of the material to allow the passage of magnetic field lines through it or measure of the capacity of the substance to take magnetisation or the degree of penetration of magnetic field through the substance.

In free space, the permeability (or absolute permeability) is denoted by μ_0 and for any other medium it is denoted by μ. **The relative permeability μ_r is defined as the ratio between absolute permeability of the medium to the permeability of free space.**

$$\mu_r = \frac{\mu}{\mu_\circ} \qquad (3.29)$$

Relative permeability is a dimensionless number and has no units. For free space (air or vacuum), the relative permeability is unity i.e., $\mu_r = 1$.

(c) Intensity of magnetisation

Any bulk material (any object of finite size) contains a large number of atoms. Each atom consists of electrons which undergo orbital motion. Due to orbital motion, electron has magnetic moment which is a vector quantity. In general, these magnetic moments orient randomly, therefore, the net magnetic moment is zero per unit volume of the material.

When such a material is kept in an external magnetic field, atomic dipoles are induced and hence, they will try to align partially or fully along the direction of external field. **The net magnetic moment per unit volume of the material** is known **as intensity of magnetisation.** It is a vector quantity. Mathematically,

$$\vec{M} = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{\vec{p}_m}{V} \qquad (3.30)$$

The SI unit of intensity of magnetisation is ampere metre^{-1}. For a bar magnet of pole strength q_m, length $2l$ and area of cross-section A, the magnetic moment of the bar magnet is $\vec{p}_m = q_m \vec{2l}$ and volume of the bar magnet is $V = A\left|\vec{2l}\right| = 2lA$. The intensity of magnetisation for a bar magnet is

$$\vec{M} = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{q_m \vec{2l}}{2lA} \qquad (3.31)$$

In magnitude, equation (3.31) is

$$\left|\vec{M}\right| = M = \frac{q_m \times 2l}{2l \times A} \Rightarrow M = \frac{q_m}{A}$$

UNIT-3(XII-Physics_Vol-1).indd 141 14-02-2020 20:03:11

This means, **for a bar magnet the intensity of magnetisation can be defined as the pole strength per unit area (face area).**

(d) Magnetic induction or total magnetic field

When a substance like soft iron bar is placed in a uniform magnetising field $\vec{H}$, the substance gets magnetised. **The magnetic induction (total magnetic field) inside the specimen $\vec{B}$ is equal to the sum of the magnetic field $\vec{B}_o$ produced in vacuum due to the magnetising field and the magnetic field $\vec{B}_m$ due to the induced magnetism of the substance**.

$$\vec{B} = \vec{B}_o + \vec{B}_m = \mu_\circ \vec{H} + \mu_\circ \vec{M}$$

$$\Rightarrow \vec{B} = \vec{B}_o + \vec{B}_m = \mu_\circ (\vec{H} + \vec{M}) \qquad (3.32)$$

(e) Magnetic susceptibility

When a substance is kept in a magnetising field $\vec{H}$, magnetic susceptibility gives information about how a material responds to the external (applied) magnetic field. In other words, the magnetic susceptibility measures how easily and how strongly a material can be magnetised. It is defined **as the ratio of the intensity of magnetisation $(\vec{M})$ induced in the material to the magnetising field $(\vec{H})$**

$$\chi_m = \frac{|\vec{M}|}{|\vec{H}|} \qquad (3.33)$$

It is a dimensionless quantity. Magnetic susceptibility for some of the isotropic substances is given in Table 3.1.

Table 3.1 Magnetic susceptibility for various materials

Material	Magnetic susceptibility (χ_m)
Aluminium	2.3×10^{-5}
Copper	-0.98×10^{-5}
Diamond	-2.2×10^{-5}
Gold	-3.6×10^{-5}
Mercury	-3.2×10^{-5}
Silver	-2.6×10^{-5}
Titanium	7.06×10^{-5}
Tungsten	6.8×10^{-5}
Carbon dioxide (1 atm)	-2.3×10^{-9}
Oxygen (1 atm)	2090×10^{-9}

EXAMPLE 3.8

Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 200 g, 2 A m^2 and 8 g cm^{-3}, respectively.

Solution

Density of the magnet is

$$Density = \frac{Mass}{Volume} \Rightarrow Volume = \frac{Mass}{Density}$$

$$Volume = \frac{200 \times 10^{-3}\,\text{kg}}{\left(8 \times 10^{-3}\,\text{kg}\right) \times 10^{6}\,\text{m}^{-3}}$$

$$= 25 \times 10^{-6}\,\text{m}^3$$

Magnitude of magnetic moment $p_m = 2\,\text{A m}^2$

Intensity of magnetization,

$$M = \frac{Magnetic\,moment}{Volume} = \frac{2}{25 \times 10^{-6}}$$

$$M = 0.8 \times 10^{5}\,\text{A m}^{-1}$$

UNIT-3(XII-Physics_Vol-1).indd 142 14-02-2020 20:03:18

EXAMPLE 3.9

Using the relation $\vec{B} = \mu_\circ(\vec{H} + \vec{M})$, show that $\chi_m = \mu_r - 1$.

Solution

$$\vec{B} = \mu_\circ(\vec{H} + \vec{M})$$

But from equation (3.36), in vector form,

$$\vec{M} = \chi_m \vec{H}$$

Hence, $\vec{B} = \mu_\circ(\chi_m + 1)\vec{H} \Rightarrow \vec{B} = \mu\vec{H}$

where, $\mu = \mu_\circ(\chi_m + 1) \Rightarrow \chi_m + 1 = \dfrac{\mu}{\mu_\circ} = \mu_r$

$$\Rightarrow \chi_m = \mu_r - 1$$

EXAMPLE 3.10

Two materials X and Y are magnetised whose values of intensity of magnetisation are 500 A m^{-1} and 2000 A m^{-1} respectively. If the magnetising field is 1000 A m^{-1}, then which one among these materials can be easily magnetized?.

Solution

The susceptibility of material X is

$$\chi_{m,\mathrm{X}} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{500}{1000} = 0.5$$

The susceptibility of material Y is

$$\chi_{m,\mathrm{Y}} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{2000}{1000} = 2$$

Since, susceptibility of material Y is greater than that of material X, which implies that material Y can be easily magnetized.

3.5 CLASSIFICATION OF MAGNETIC MATERIALS

The magnetic materials are generally classified into three types based on their behaviour in a magnetising field. They are diamagnetic, paramagnetic and ferromagnetic materials.

(a) Diamagnetic materials

The orbital motion of electrons around the nucleus produces a magnetic field perpendicular to the plane of the orbit. Thus each electron orbit has finite orbital magnetic dipole moment. Since the orbital planes of the other electrons are oriented in random manner, the vector sum of magnetic moments is zero and there is no resultant magnetic moment for each atom.

In the presence of a uniform external magnetic field, some electrons are speeded up and some are slowed down. The electrons whose moments were anti-parallel are speeded up according to Lenz's law and this produces an induced magnetic moment in a direction opposite to the field. The induced moment disappears as soon as the external field is removed.

When placed in a non-uniform magnetic field, the interaction between induced magnetic moment and the external field creates a force which tends to move the material from stronger part to weaker part of the external field. It means that diamagnetic material is repelled by the field.

This action is called diamagnetic action and such materials are known as diamagnetic materials. Examples: Bismuth, Copper and Water etc.

UNIT-3(XII-Physics_Vol-1).indd 143 14-02-2020 20:03:20

The properties of diamagnetic materials are

i) Magnetic susceptibility is negative.

ii) Relative permeability is slightly less than unity.

iii) The magnetic field lines are repelled or expelled by diamagnetic materials when placed in a magnetic field.

iv) Susceptibility is nearly temperature independent.

Note

Superconductors are perfect diamagnetic materials. The expulsion of magnetic flux from a superconductor during its transition to the superconducting state is known as Meissner effect. (see figure 3.18)

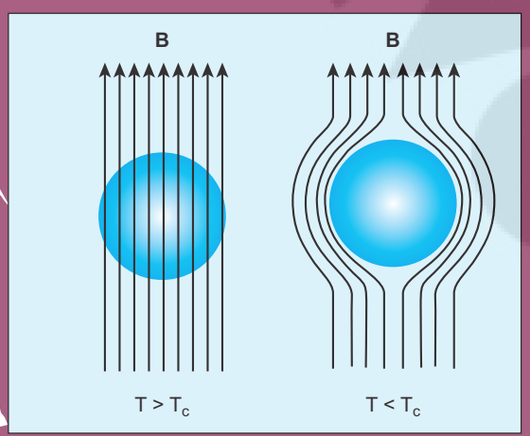


Figure 3.18 Meissner effect – superconductors behave like perfect diamagnetic materials below transition temperature T_C.

(b) Paramagnetic materials

In some magnetic materials, each atom or molecule has net magnetic dipole moment which is the vector sum of orbital and spin magnetic moments of electrons. Due to the random orientation of these magnetic moments, the net magnetic moment of the materials is zero.

Magnetic levitated train

Magnetic levitated train is also called Maglev train. This train floats few centimetres above the guideway because of electromagnet used. Maglev train does not need wheels and also achieve greater speed. The basic mechanism of working of Maglev train involves two sets of magnets. One set is used to repel which makes train to float above the track and another set is used to move the floating train ahead at very great speed. These trains are quieter, smoother and environmental friendly compared conventional trains and have potential for moving with much higher speeds with technology in future.



In the presence of an external magnetic field, the torque acting on the atomic dipoles will align them in the field direction. As a result, there is net magnetic dipole moment induced in the direction of the applied field. The induced dipole moment is present as long as the external field exists.

When placed in a non-uniform magnetic field, the paramagnetic materials will have a tendency to move from weaker to stronger part of the field. Materials which exhibit weak magnetism in the direction of the applied field are known as paramagnetic materials.

UNIT-3(XII-Physics_Vol-1).indd 144 14-02-2020 20:03:21

Examples: Aluminium, Platinum, Chromium and Oxygen etc.

The properties of paramagnetic materials are:

i) Magnetic susceptibility is positive and small.
ii) Relative permeability is greater than unity.
iii) The magnetic field lines are attracted into the paramagnetic materials when placed in a magnetic field.
iv) Susceptibility is inversely proportional to temperature.

Curie's law

When temperature is increased, thermal vibration will upset the alignment of magnetic dipole moments. Therefore, the magnetic susceptibility decreases with increase in temperature. In many cases, the susceptibility of the materials is

$$\chi_m \propto \frac{1}{T} \text{ or } \chi_m = \frac{C}{T}$$

This relation is called Curie's law. Here C is called Curie constant and temperature T is in kelvin. The graph drawn between magnetic susceptibility and temperature is shown in Figure 3.19, which is a rectangular hyperbola.

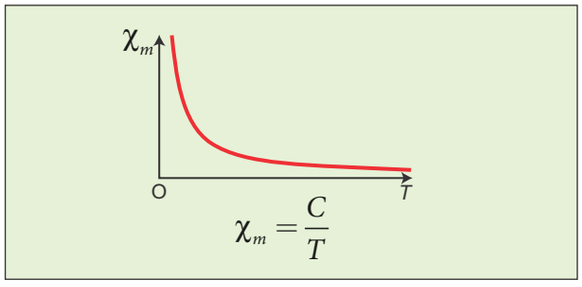


Figure 3.19 Curie's law – susceptibility vs temperature

(c) Ferromagnetic materials

An atom or a molecule in a ferromagnetic material possesses net magnetic dipole moment as in a paramagnetic material. A ferromagnetic material is made up of smaller regions, called ferromagnetic domains (Figure 3.20). Within each domain, the magnetic moments are spontaneously aligned in a direction. This alignment is caused by strong interaction arising from electron spin which depends on the inter-atomic distance. Each domain has net magnetisation in a direction. However the direction of magnetisation varies from domain to domain and thus net magnetisation of the specimen is zero.

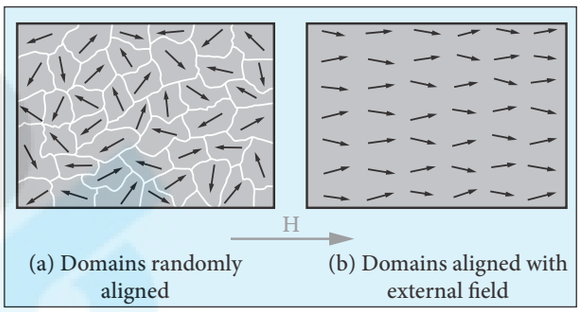


Figure 3.20 Magnetic domains – ferromagnetic materials

In the presence of external magnetic field, two processes take place

(1) The domains having magnetic moments parallel to the field grow bigger in size

(2) The other domains (not parallel to field) are rotated so that they are aligned with the field.

As a result of these mechanisms, there is a strong net magnetisation of the material in the direction of the applied field (Figure 3.21).

UNIT-3(XII-Physics_Vol-1).indd 145 14-02-2020 20:03:21

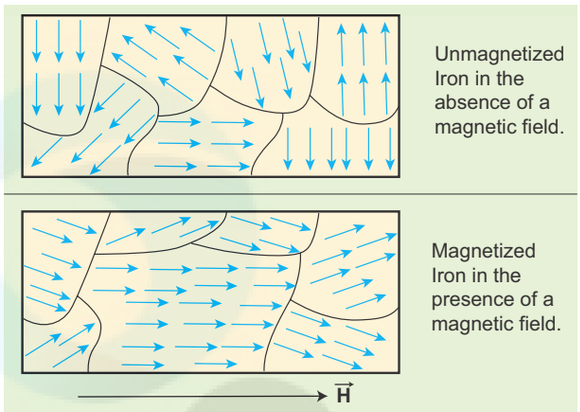


Figure 3.21 Processes of domain magnetization

When placed in a non-uniform magnetic field, the ferromagnetic materials will have a strong tendency to move from weaker to stronger part of the field. Materials which exhibit strong magnetism in the direction of applied field are called ferromagnetic materials. Examples: Iron, Nickel and Cobalt.

The properties of ferromagnetic materials are:



Magnetism plays interesting role in various aspects of life. It has connection with archeological place Keezhadi too. To find whether any archeological structure exists under the surface of a given place, well established technique called 'magnetometer surveying' is used.

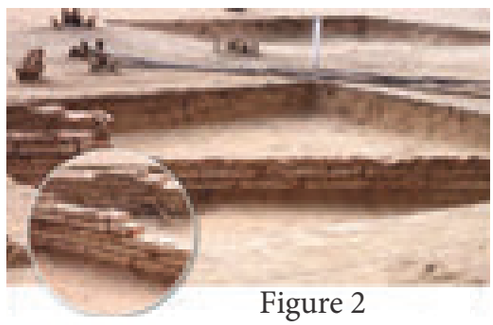


Figure 2

In this technique, the variation of the magnetic field in comparison with the neighbouring place is studied. The magnetic field variation is due to the presence of magnetic mineral magnetite and its related minerals present in the archeological structures like buried wall, pottery, bricks, buried tombs, monuments and inhabited sites. Those minerals are either diamagnetic or paramagnetic or ferromagnetic in nature and each type has different range of magnetic susceptibilities.

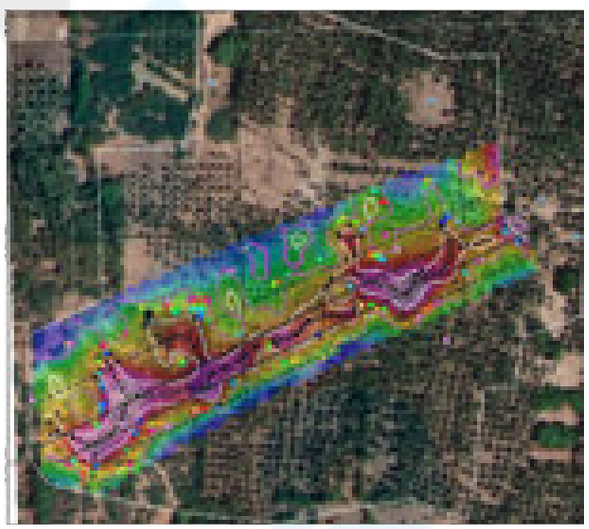


Figure 1

Indian Institute of Geomagnetism (IIG), Mumbai conducted magnetometer survey on Keezhadi site and found out that there were archeological structures like wall, pottery etc. From the picture (Figure 1), there was magnetic field variation in the range of 10 to 100nT over the particular area (coloured portion). In fact, the existence of massive brick structures at Keezhadi has been revealed through magnetism (Figure 2).

UNIT-3(XII-Physics_Vol-1).indd 146 14-02-2020 20:03:22

i) Magnetic susceptibility is positive and large.

ii) Relative permeability is large.

iii) The magnetic field lines are strongly attracted into the ferromagnetic materials when placed in a magnetic field.

iv) Susceptibility is inversely proportional to temperature.

Curie-Weiss law

As temperature increases, the ferromagnetism decreases due to the increased thermal agitation of the atomic dipoles. At a particular temperature, ferromagnetic material becomes paramagnetic. This temperature is known as Curie temperature T_C. The susceptibility of the material above the Curie temperature is given by

$$\chi_m = \frac{C}{T - T_C}$$

This relation is called Curie-Weiss law. The constant C is called Curie constant and temperature T is in kelvin scale. A plot of magnetic susceptibility with temperature is as shown in Figure 3.22.

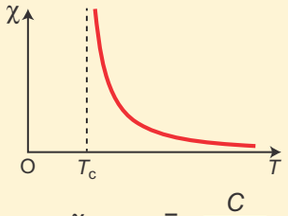
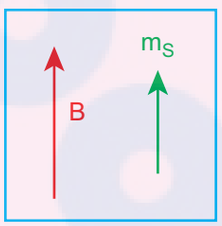


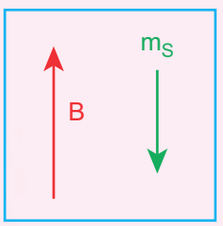
Figure 3.22 Curie-Weiss law – susceptibility vs temperature

Spin

Like mass and charge for particles, spin is also another important attribute for an elementary particle. Spin is a quantum mechanical phenomenon which is responsible for magnetic properties of the material. Spin in quantum mechanics is entirely different from spin we encounter in classical mechanics. Spin in quantum mechanics does not mean rotation; it is intrinsic angular momentum which does not have classical analogue. For historical reason, the name spin is retained. Spin of a particle takes only positive values but the orientation of the spin vector takes plus or minus values in an external magnetic field. For an example, electron has spin $s = \frac{1}{2}$. In the presence of magnetic field, the spin will orient either parallel or anti-parallel to the direction of magnetic field.



Spin is parallel to the magnetic field direction (Spin up)



Spin is anti-parallel to the magnetic field direction (Spin down)

This implies that the magnetic spin m_s takes two values for an electron, such as $m_s = \frac{1}{2}$ (spin up) and $m_s = -\frac{1}{2}$ (spin down). Spin for proton and neutron is $s = \frac{1}{2}$. For photon, spin $s = 1$.

UNIT-3(XII-Physics_Vol-1).indd 147 14-02-2020 20:03:24

Comparison of Types of Magnetism (NOT FOR EXAMINATION)

Type of magnetism	Magnetising field is absent (H = 0)	Magnetising field is present ($H \neq 0$)	Magnetisation of the material	Susceptibility	Relative permeability
Diamagnetism	(Zero magnetic moment)	(Aligned opposite to the field)	M, O, H	Negative	Less than unity
Paramagnetism	(Net magnetic moment but random alignment)	(Aligned with the field)	M, O, H	Positive and small	Greater than unity
Ferromagnetism	(Net magnetic moment in a domain but they are randomly aligned)	(Aligned with the field)	M, O, H	Positive and large	Very large

3.6 HYSTERESIS

When a ferromagnetic material is kept in a magnetising field, the material gets magnetised by induction. An important characteristic of ferromagnetic material is that the variation of magnetic induction $\vec{B}$ with magnetising field $\vec{H}$ is not linear. It means that the ratio $\frac{B}{H} = \mu$ is not a constant. Let us study this behaviour in detail.

A ferromagnetic material (example, Iron) is magnetised slowly by a magnetising field $\vec{H}$. The magnetic induction $\vec{B}$ of the material increases from point A with the magnitude of the magnetising field and then attains a saturation level. This response of the material is depicted by the path AC as shown in Figure 3.23. Saturation magnetization is defined as the maximum point up to which the material can be magnetised by applying the magnetising field.

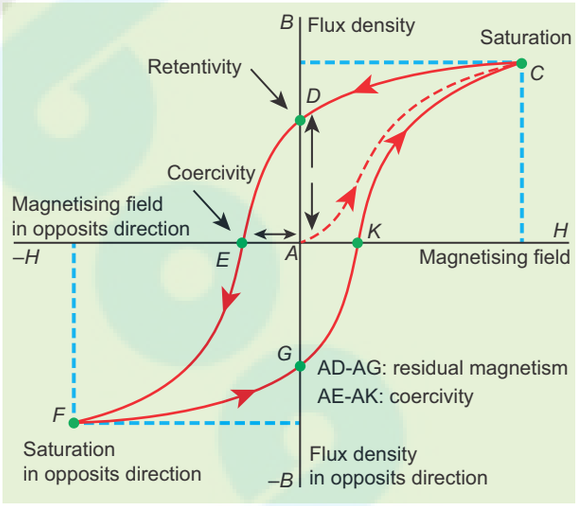


Figure 3.23 Hysteresis – plot for B vs H

UNIT-3(XII-Physics_Vol-1).indd 148 14-02-2020 20:03:25

If the magnetising field is now reduced, the magnetic induction also decreases but does not retrace the original path CA. It takes different path CD. When the magnetising field is zero, the magnetic induction is not zero and it has positive value. This implies that some magnetism is left in the specimen even when $H = 0$. The residual magnetism AD present in the specimen is called remanence or retentivity. **Remanence is defined as the ability of the materials to retain the magnetism in them even after the magnetising field disappears.**

In order to demagnetise the material, the magnetising field is gradually increased in the reverse direction. Now the magnetic induction decreases along DE and becomes zero at E. The magnetising field AE in the reverse direction is required to bring residual magnetism to zero. **The magnitude of the reverse magnetising field for which the residual magnetism of the material vanishes is called its coercivity.**

Further increase of $\vec{H}$ in the reverse direction causes the magnetic induction to increase along EF until it reaches saturation at F in the reverse direction. If magnetising field is decreased and then increased with direction reversed, the magnetic induction traces the path FGKC. This closed curve ACDEFGKC is called hysteresis loop and it corresponds to one cycle of magnetisation.

In the entire cycle, the magnetic induction B lags behind the magnetising field H. This **phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.** Hysteresis means 'lagging behind'.

Hysteresis loss

During the magnetisation of the specimen through a cycle, there is loss of energy in the form of heat. This loss is attributed to the rotation and orientation of molecular magnets in various directions. It is found that the energy lost (or dissipated) per unit volume of the material when it is carried through one cycle of magnetisation is equal to the area of the hysteresis loop.

Hard and soft magnetic materials

Based on the shape and size of the hysteresis loop, ferromagnetic materials are classified as soft magnetic materials with smaller area and hard magnetic materials with larger area. The comparison of the hysteresis loops for two magnetic materials is shown in Figure 3.24. Properties of soft and hard magnetic materials are compared in Table 3.2.

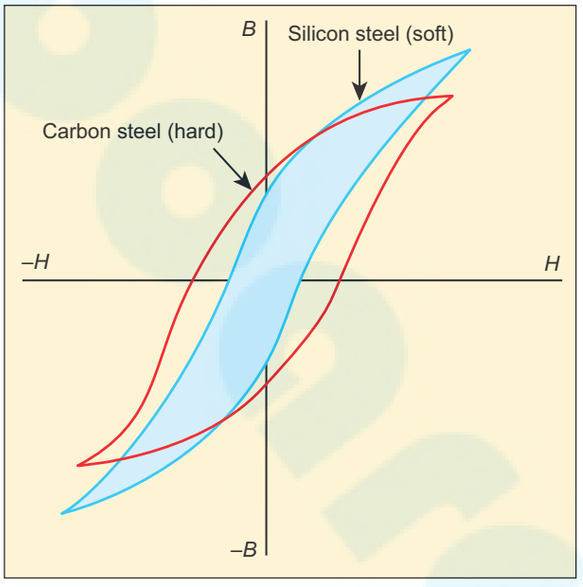


Figure 3.24 Comparison of two ferromagnetic materials based on hysteresis loop

UNIT-3(XII-Physics_Vol-1).indd 149 14-02-2020 20:03:26

Table 3.2 Differences between soft and hard ferromagnetic materials

S.No.	Properties	Soft ferromagnetic materials	Hard ferromagnetic materials
1	When external field is removed	Magnetisation disappears	Magnetisation persists
2	Area of the loop	Small	Large
3	Retentivity	Low	High
4	Coercivity	Low	High
5	Susceptibility and magnetic permeability	High	Low
6	Hysteresis loss	Less	More
7	Uses	Solenoid core, transformer core and electromagnets	Permanent magnets
8	Examples	Soft iron, Mumetal, Stalloy etc.	Carbon steel, Alnico, Lodestone etc.

Applications of hysteresis loop

The significance of hysteresis loop is that it provides information such as retentivity, coercivity, permeability, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material. Therefore, the study of hysteresis loop will help us in selecting proper and suitable material for a given purpose. Some examples:

i) Permanent magnets:

The materials with high retentivity, high coercivity and low permeability are suitable for making permanent magnets.
Examples: Carbon steel and Alnico

ii) Electromagnets:

The materials with high initial permeability, low retentivity, low coercivity and thin hysteresis loop with smaller area are preferred to make electromagnets.

Examples: Soft iron and Mumetal (Nickel Iron alloy).

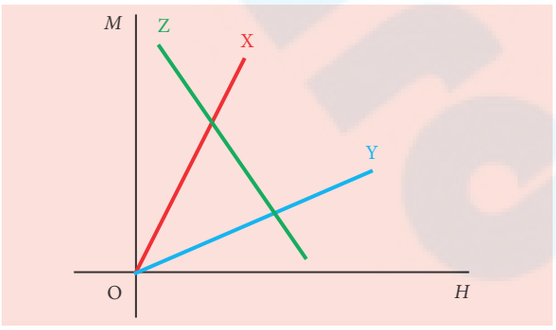
iii) Core of the transformer:

The materials with high initial permeability, large magnetic induction and thin hysteresis loop with smaller area are needed to design transformer cores.

Examples: Soft iron

EXAMPLE 3.11

The following figure shows the variation of intensity of magnetisation with the applied magnetic field intensity for three magnetic materials X, Y and Z. Identify the materials X,Y and Z.



UNIT-3(XII-Physics_Vol-1).indd 150 14-02-2020 20:03:26

Solution

The slope of *M-H* graph is a measure of the magnetic susceptibility, which is given by

$$\chi_m = \frac{M}{H}$$

Material X: Slope is positive and larger value. So, it is a ferromagnetic material.

Material Y: Slope is positive and lesser value than X. So, it could be a paramagnetic material.

Material Z: Slope is negative and hence, it is a diamagnetic material.

3.7 MAGNETIC EFFECTS OF CURRENT

3.7.1 Oersted experiment

In 1820 Hans Christian Oersted, while preparing for his lecture in physics, noticed that electric current passing through a wire deflects the nearby magnetic needle in the compass. By proper investigation, he observed that the deflection of magnetic needle is due to the change in magnetic field produced around current carrying conductor (Figure 3.25). When the direction of current is reversed, the magnetic needle is deflected in the opposite direction. This lead to the development of the theory 'electromagnetism' which unifies two branches in physics namely, electricity and magnetism.

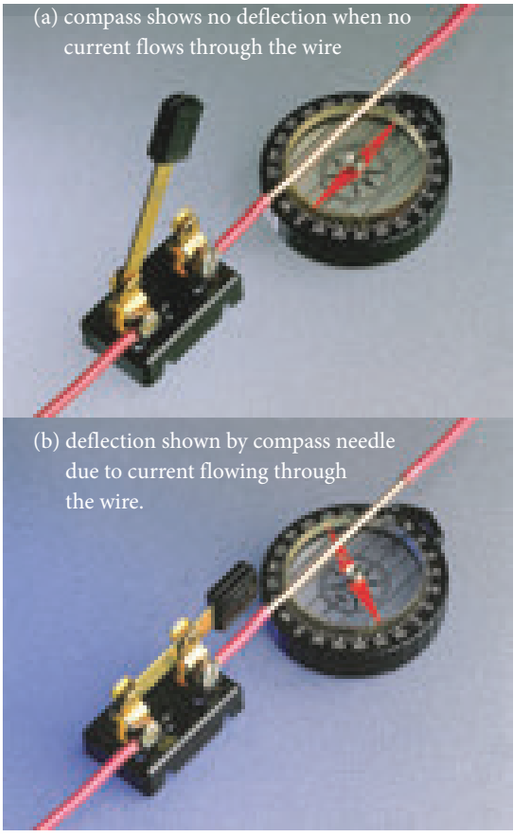


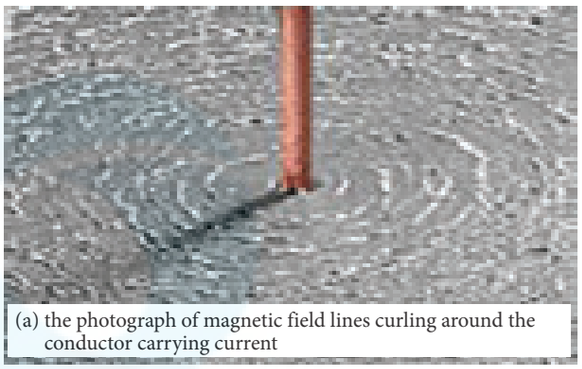
Figure 3.25 Oersted's experiment - current carrying wire and deflection of magnetic needle

3.7.2 Magnetic field around a straight current-carrying conductor and circular loop

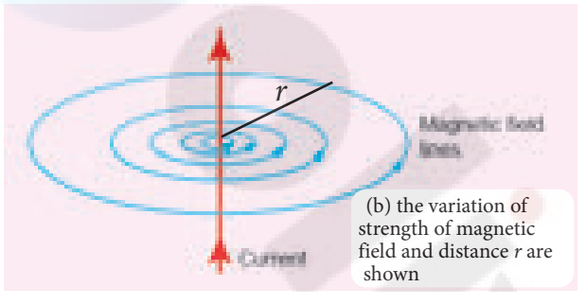
(a) Current carrying straight conductor:

Suppose we keep a magnetic compass near a current-carrying straight conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. Tracing out the direction shown by magnetic needle , we can draw the magnetic field lines at a distance. For a straight current-carrying conductor, the nature of magnetic field is like concentric circles having their common centre on the axis of the conductor as shown in Figure 3.26 (a).

UNIT-3(XII-Physics_Vol-1).indd 151 14-02-2020 20:03:26



(a) the photograph of magnetic field lines curling around the conductor carrying current



(b) the variation of strength of magnetic field and distance r are shown

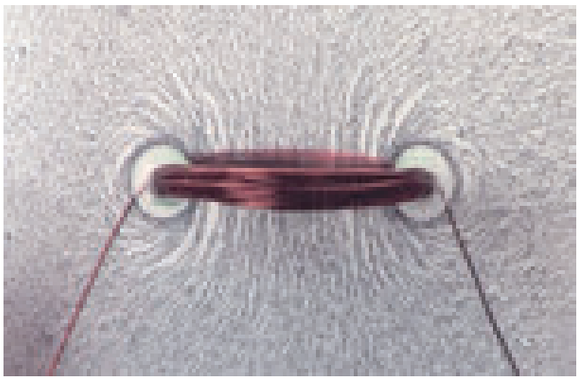
Figure 3.26 Magnetic field lines around straight, long wire carrying current

The direction of circular magnetic field lines will be clockwise or anticlockwise depending on the direction of current in the conductor. If the strength (or magnitude) of the current is increased then the density of the magnetic field will also increase. The strength of the magnetic field (B) decreases as the distance (r) from the conductor increases (Figure 3.26 (b)).

(b) Circular coil carrying current

Suppose we keep a magnetic compass near a current carrying circular conductor, then the needle of the magnetic compass experiences a torque and deflects to align in the direction of the magnetic field at that point. We can notice that at the points A and B in the vicinity of the coil, the magnetic field lines are circular. The magnetic field lines are nearly parallel to each other near the centre of the loop, indicating that the field present near the centre of the coil is almost uniform (Figure 3.27).

The strength of the magnetic field is increased if either the current in the coil or the number of turns or both are increased. The polarity (north pole or south pole) depends on the direction of current in the loop.



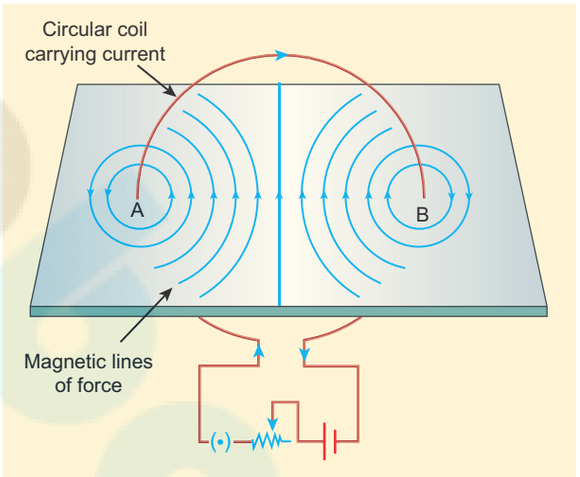


Figure 3.27 The magnetic field lines curling around the circular coil carrying current.

3.7.3 Right hand thumb rule

The right hand rule is used to find the direction of magnetic field when the direction of current in a conductor is known.

UNIT-3(XII-Physics_Vol-1).indd 152 14-02-2020 20:03:27

Assume that we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the fingers encircling the conductor point in the direction of the magnetic field lines produced.

The Figure 3.28 shows the right hand rule for current carrying straight conductor and circular coil.

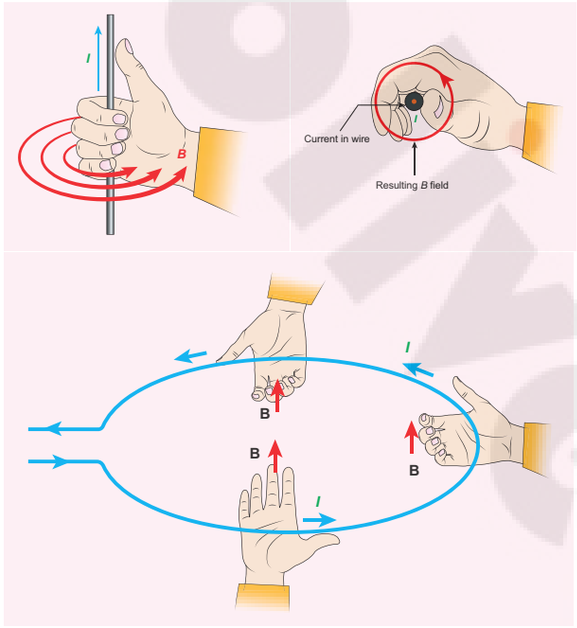


Figure 3.28 Right hand rule – straight conductor and circular loop

3.7.4 Maxwell's right hand cork screw rule

This rule can also be used to find the direction of the magnetic field around the current-carrying conductor. If we rotate a right-handed screw using a screw driver, then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field. (Figure 3.29)

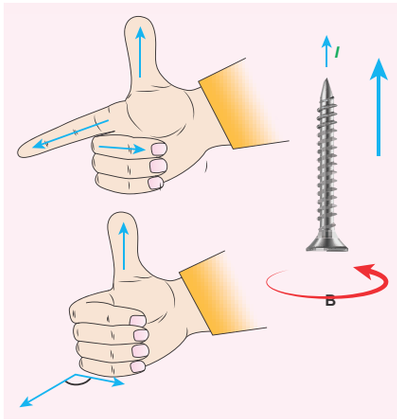
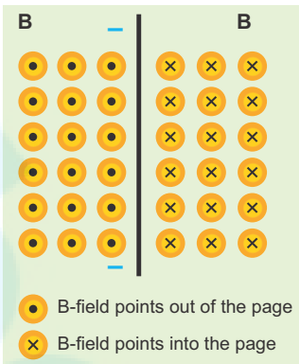


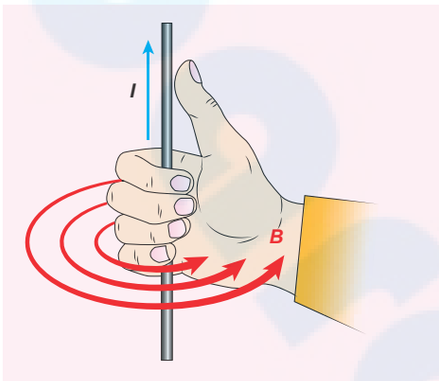
Figure 3.29 Maxwell's right hand cork screw rule

EXAMPLE 3.12

The magnetic field shown in the figure is due to the current carrying wire. In which direction does the current flow in the wire?.



Solution



Using right hand rule, current flows upwards.

UNIT-3(XII-Physics_Vol-1).indd 153 14-02-2020 20:03:28

3.8 BIOT - SAVART LAW

Soon after Oersted's discovery, both Jean-Baptiste Biot and Felix Savart in 1819 did quantitative experiments on the force experienced by a magnet kept near current carrying wire and arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the magnetic field. This is true for any shape of the conductor.

3.8.1 Definition and explanation of Biot- Savart law

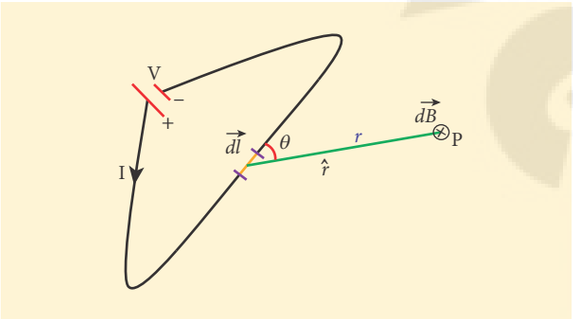


Figure 3.30 Magnetic field at a point P due to current carrying conductor

Biot and Savart experimentally observed that the magnitude of magnetic field $d\vec{B}$ at a point P (Figure 3.30) at a distance r from the small elemental length taken on a conductor carrying current varies

(i) directly as the strength of the current I

(ii) directly as the magnitude of the length element $d\vec{l}$

(iii) directly as the sine of the angle θ between $d\vec{l}$ and $\hat{r}$.

(iv) inversely as the square of the distance r between the point P and length element $d\vec{l}$.

This is expressed as

$$dB \propto \frac{Idl}{r^2}\sin\theta$$

$$dB = k\frac{I\,dl}{r^2}\sin\theta$$

where $k = \frac{\mu_\circ}{4\pi}$ in SI units.

In vector notation,

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \qquad (3.34)$$

Here vector $d\vec{B}$ is perpendicular to both $Id\vec{l}$ (pointing the direction of current flow) and the unit vector $\hat{r}$ directed from $d\vec{l}$ toward point P (Figure 3.31).

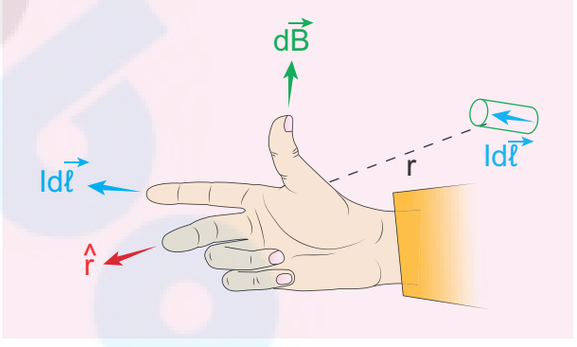


Figure 3.31 The direction of magnetic field using right hand rule

The equation (3.34) is used to compute the magnetic field only due to a small elemental length dl of the conductor. The net magnetic field at P due to the conductor is obtained from principle of superposition by considering the contribution from all current elements $I\,d\vec{l}$. Hence integrating equation (3.34), we get

UNIT-3(XII-Physics_Vol-1).indd 154 14-02-2020 20:03:32

$$\vec{B} = \int d\vec{B} = \frac{\mu_\circ I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} \qquad (3.35)$$

where the integral is taken over the entire current distribution.

Cases

1. If the point P lies on the conductor, then $\theta = 0^\circ$. Therefore, $|d\vec{B}|$ is zero.
2. If the point lies perpendicular to the conductor, then $\theta = 90^\circ$. Therefore, $d\vec{B}$ is maximum and is given by $d\vec{B} = \frac{\mu_\circ}{4\pi} \frac{I\,dl}{r^2} \hat{n}$ where $\hat{n}$ is the unit vector perpendicular to both $I\,d\vec{l}$ and $\hat{r}$

Note

Electric current is not a vector quantity. It is a scalar quantity. But electric current in a conductor has direction of flow. Therefore, the electric current flowing in a small elemental conductor can be taken as vector quantity i.e. $Id\vec{l}$

Similarities between electric field (from Coulomb's law) and magnetic field (from Biot-Savart's law)

Electric and magnetic fields

- obey inverse square law, so they are long range fields.
- obey the principle of superposition and are linear with respect to source. In magnitude,

$E \propto q$

$B \propto Idl$

Differences between electric field (from Coulomb's law) and magnetic field (from Biot-Savart's law)

S. No.	Electric field	Magnetic field
1	Produced by a scalar source i.e., an electric charge q	Produced by a vector source i.e., current element $I\,d\vec{l}$
2	It is directed along the position vector joining the source and the point at which the field is calculated	It is directed perpendicular to the position vector $\vec{r}$ and the current element $Id\vec{l}$
3	Does not depend on angle	Depends on the angle between the position vector $\vec{r}$ and the current element $Id\vec{l}$

Note

The exponent of charge q (source) and exponent of electric field E is unity. Similarly, the exponent of current element Idl (source) and exponent of magnetic field B is unity. In other words, electric field $\vec{E}$ is proportional only to charge (source) and not on higher powers of charge (q^2, q^3, etc). Similarly, magnetic field $\vec{B}$ is proportional to current element $Id\vec{l}$ (source) and not on square or cube or higher powers of current element. The cause and effect have linear relationship.

UNIT-3(XII-Physics_Vol-1).indd 155 14-02-2020 20:03:35

3.8.2 Magnetic field due to long straight conductor carrying current

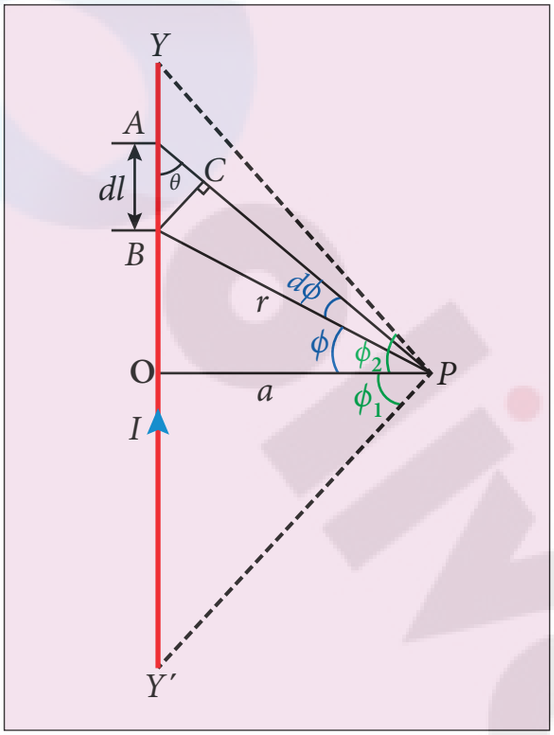


Figure 3.32 Magnetic field due to a long straight current carrying conductor

Let YY′ be an infinitely long straight conductor and I be the steady current through the conductor as shown in Figure 3.32. In order to calculate magnetic field at a point P which is at a distance a from the wire, let us consider a small line element dl (segment AB).

The magnetic field at a point P due to current element Idl can be calculated from Biot-Savart's law, which is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}\hat{n}$$

where $\hat{n}$ is the unit vector which points into the page at P, θ is the angle between current element Idl and line joining dl and the point P. Let r be the distance between line element at A to the point P.

To apply trigonometry, draw a perpendicular AC to the line BP as shown in Figure 3.32.

In triangle ΔABC, $\sin\theta = \frac{AC}{AB}$

$$\Rightarrow AC = AB\sin\theta$$

But $AB = dl \Rightarrow AC = dl\sin\theta$

Let $d\phi$ be the angle subtended between AP and BP

i.e., $\angle APB = \angle BPC = d\phi$

In a triangle $\Delta APC, \sin(d\phi) = \frac{AC}{AP}$

Since, $d\phi$ is very small, $\sin(d\phi) \simeq d\phi$

But $AP = r \Rightarrow AC = rd\phi$

$$\therefore AC = dl\sin\theta = rd\phi$$

$$\therefore d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}(rd\phi)\hat{n} = \frac{\mu_0}{4\pi}\frac{Id\phi}{r}\hat{n}$$

Let ϕ be the angle between BP and OP

In a ΔOPA, $\cos\phi = \frac{OP}{BP} = \frac{a}{r}$

$$\Rightarrow r = \frac{a}{\cos\phi}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a/\cos\phi}d\phi\hat{n}$$

$$\Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi a}\cos\phi d\phi\hat{n}$$

The total magnetic field at P due to the conductor YY′ is

$$\vec{B} = \int_{-\phi_1}^{\phi_2} d\vec{B} = \int_{-\phi 1}^{\phi_2}\frac{\mu_0 I}{4\pi a}\cos\phi d\phi\hat{n}$$

$$= \frac{\mu_0 I}{4\pi a}\left[\sin\phi\right]_{-\phi_1}^{\phi_2}\hat{n}$$

UNIT-3(XII-Physics_Vol-1).indd 156 14-02-2020 20:03:39

$$= \vec{B} = \frac{\mu_0 I}{4\pi a}\left(\sin\phi_1 + \sin\phi_2\right)\hat{n}$$

For infinitely long conductor,

$\phi_1 = \phi_2 = 90°$

$$\therefore \vec{B} = \frac{\mu_0 I}{4\pi a} \times 2\,\hat{n} \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi a}\,\hat{n} \qquad (3.36)$$

3.8.3 Magnetic field produced along the axis of the current-carrying circular coil

Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction as shown in Figure 3.33.

The magnetic field at a point P on the axis of the circular coil at a distance z from the centre of the coil O is computed by taking two diametrically opposite line elements of the coil each of length $d\vec{l}$ at C and D. Let $\vec{r}$ be the vector joining the current element ($I\,d\vec{l}$) at C and the point P.

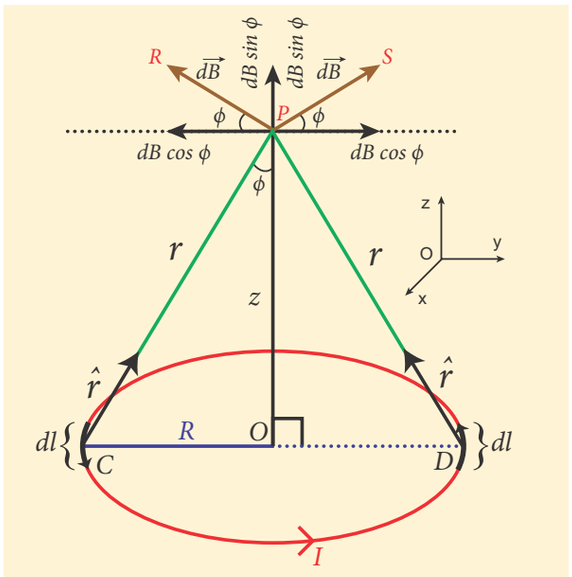


Figure 3.33 Magnetic field due to current-carrying circular loop

According to Biot-Savart's law, the magnetic field at P due to the current element at C is

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

The magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_\circ}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_\circ}{4\pi}\frac{I\,dl}{r^2}$$

where θ is the angle between $Id\vec{l}$ and $\vec{r}$. Here $\theta = 90°$.

The direction of $d\vec{B}$ is perpendicular to the current element $Id\vec{l}$ and CP. It is therefore along PR perpendicular to CP.

The magnitude of magnetic field at P due to current element at D is same as that for the element at C because of equal distances from the coil. But its direction is along PS.

The magnetic field $d\vec{B}$ due to each current element is resolved into two components; $dB\cos\phi$ along y-direction and $dB\sin\phi$ along z-direction. The horizontal components cancel out while the vertical components ($dB\sin\phi\,\hat{k}$) alone contribute to the net magnetic field $\vec{B}$ at the point P.

$$\vec{B} = \int d\vec{B} = \int dB\sin\phi\,\hat{k}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}\sin\phi\,\hat{k}$$

From ΔOCP,

$$\sin\phi = \frac{R}{\left(R^2 + z^2\right)^{1/2}} \text{ and } r^2 = R^2 + z^2.$$

Substituting these in the above equation, we get

$$\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R}{\left(R^2 + z^2\right)^{3/2}}\hat{k}\left(\int dl\right)$$

UNIT-3(XII-Physics_Vol-1).indd 157 14-02-2020 20:03:43

If we integrate the line element from 0 to $2\pi R$, we get the net magnetic field $\vec{B}$ at point P due to the current-carrying circular loop.

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + z^2\right)^{3/2}} \hat{k}$$

If the circular coil contains N turns, then the magnetic field is

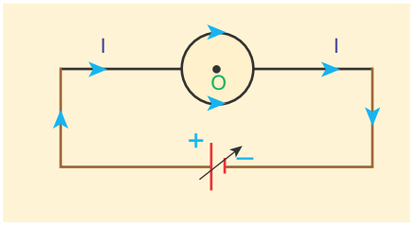
$$\vec{B} = \frac{\mu_0 NI}{2} \frac{R^2}{\left(R^2 + z^2\right)^{3/2}} \hat{k} \qquad (3.37)$$

The magnetic field at the centre of the coil is

$$\vec{B} = \frac{\mu_0 NI}{2R} \hat{k} \quad \text{since } z = 0 \qquad (3.38)$$

EXAMPLE 3.13

What is the magnetic field at the centre of the loop shown in figure?



Solution

The magnetic field due to current in the upper semicircle and lower semicircle of the circular coil are equal in magnitude but opposite in direction. Hence, the net magnetic field at the centre of the loop (at point O) is zero $\vec{B} = \vec{0}$.

3.8.4 Tangent law and Tangent Galvanometer

Tangent galvanometer is a device used to detect very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law (Figure 3.34).



Figure 3.34 Tangent Galvanometer

Tangent law

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.

Let B be the magnetic field produced by passing current through the coil of the tangent galvanometer and B_H be the horizontal component of Earth's magnetic field. Under the action of two magnetic fields, the needle comes to rest making angle θ with B_H, such that

$$B = B_H \tan\theta \qquad (3.39)$$

Construction

Tangent Galvanometer (TG) consists of copper coil of several turns wound on a non-magnetic circular frame. The frame is made up of brass or wood which is mounted vertically on a horizontal base table (turn

UNIT-3(XII-Physics_Vol-1).indd 158 14-02-2020 20:03:45

table) with three levelling screws. The TG is provided with two or more coils of different number of turns. Most of the equipments we use in laboratory, contains coils of 2 turns, 5 turns and 50 turns which are of different thickness and are used for measuring currents of different strengths.

At the centre of turn table, there is a small upright projection on which a compass box is placed. Compass box consists of a small magnetic needle which is pivoted at its centre, such that the centres of both magnetic needle and circular coil exactly coincide. A thin aluminium pointer attached perpendicular to the magnetic needle moves over a graduated circular scale. The circular scale is divided into four quadrants and they are graduated in degrees, each quadrant being numbered from 0° to 90° In order to avoid parallax error in measurement, a mirror is placed below the aluminium pointer.

Precautions

1. All the nearby magnets and magnetic materials are kept away from the instrument.
2. Using spirit level, the levelling screws at the base are adjusted so that the small magnetic needle is exactly horizontal and also coil (mounted on the frame) is exactly vertical.
3. The plane of the coil is kept parallel to the small magnetic needle by rotating the coil about its vertical axis. So that, the coil remains in magnetic meridian.
4. The compass box alone is rotated such that the aluminium pointer reads $0^o - 0^o$.

Theory

In the tangent galvanometer experiment, when no current is passed through the coil, the small magnetic needle lies along horizontal component of Earth's magnetic field. When the circuit is closed, the electric current will pass through the circular coil and produce magnetic field at the centre of the coil. Now there are two fields which are acting mutually perpendicular to each other. They are:

(1) the magnetic field (B) due to the electric current in the coil acting normal to the plane of the coil.

(2) the horizontal component of Earth's magnetic field (B_H)

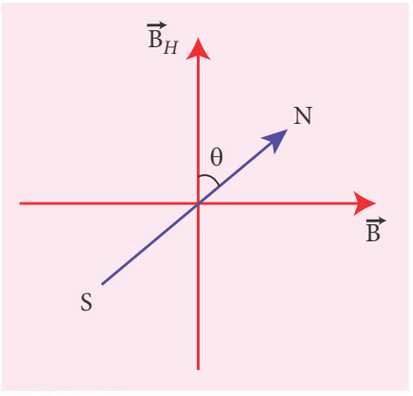


Figure 3.35 Resultant position of pivoted needle

Because of these crossed fields, the pivoted magnetic needle deflects through an angle θ. From tangent law (equation 3.39),

$$B = B_H \tan\theta$$

When an electric current is passed through a circular coil of radius R having N turns, the magnitude of magnetic field at the centre is (from equation (3.38))

$$B = \mu_\circ \frac{NI}{2R} \qquad (3.40)$$

From equation (3.39) and equation (3.40), we get

$$\mu_\circ \frac{NI}{2R} = B_H \tan\theta$$

The horizontal component of Earth's magnetic field is given by

UNIT-3(XII-Physics_Vol-1).indd 159 14-02-2020 20:03:46

$$B_H = \frac{\mu_\circ N}{2R}\frac{I}{\tan q} \qquad (3.41)$$

EXAMPLE 3.14

A coil of a tangent galvanometer of diameter 0.24 m has 100 turns. If the horizontal component of Earth's magnetic field is 25×10^{-6} T then, calculate the current which gives a deflection of 60°.

Solution

The diameter of the coil is 0.24 m. Therefore, radius of the coil is 0.12 m.

Number of turns is 100 turns.

Earth's magnetic field is 25×10^{-6} T

Deflection is

$$\theta = 60^\circ \Rightarrow \tan 60^\circ = \sqrt{3} = 1.732$$

$$I = \frac{2RB_H}{\mu_\circ N}\tan\theta$$

$$= \frac{2\times 0.12\times 25\times 10^{-6}}{4\times 10^{-7}\times 3.14\times 100}\times 1.732 = 0.82\times 10^{-1}\text{A}.$$

$$I = 0.082 \text{ A}$$

3.8.5 Current loop as a magnetic dipole

The magnetic field at a point on the axis of the current-carrying circular loop of radius R at a distance z from its centre is given by

$$\vec{B} = \frac{\mu_\circ I}{2}\frac{R^2}{\left(R^2+z^2\right)^{\frac{3}{2}}}\hat{k} \quad \text{(From eqn. 3.37)}$$

At larger distance $z >> R$, therefore $R^2 + z^2 \approx z^2$, we have

$$\vec{B} = \frac{\mu_\circ I}{2}\frac{R^2}{z^3}\hat{k} \text{ or } \vec{B} = \frac{\mu_\circ I}{2\pi}\frac{\pi R^2}{z^3}\hat{k} \qquad (3.42)$$

Let A be the area of the circular loop $A = \pi R^2$. So rewriting the equation (3.42) in terms of area of the loop, we have

$$\vec{B} = \frac{\mu_\circ I}{2\pi}\frac{A}{z^3}\hat{k}$$

(or)

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{2IA}{z^3}\hat{k} \qquad (3.43)$$

Comparing equation (3.43) with equation (3.14) dimensionally, we get

$$p_m = I\,A$$

where p_m is called magnetic dipole moment. In vector notation,

$$\vec{p}_m = I\,\vec{A} \qquad (3.44)$$

This implies that a current carrying circular loop behaves as a magnetic dipole of magnetic moment $\vec{p}_m$. So, **the magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.**

Right hand thumb rule

In order to determine the direction of magnetic moment, we use right hand thumb rule which states that

If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.

Table 3.3 End rule – polarity with direction of current in circular loop

Current in circular loop	Polarity	Picture
Anti-clockwise current	North Pole	 Anti-clockwise current Polarity: North Pole
Clockwise current	South Pole	 Clockwise current Polarity: South Pole

UNIT-3(XII-Physics_Vol-1).indd 160 14-02-2020 20:03:52

3.8.6 Magnetic dipole moment of revolving electron

Suppose an electron undergoes circular motion around the nucleus as shown in Figure 3.36. The circulating electron in a loop is like current in a circular loop (since flow of charge constitutes current). The magnetic dipole moment due to current carrying circular loop is

$$\vec{\mu}_L = I\vec{A} \qquad (3.45)$$

In magnitude,

$$\mu_L = IA$$

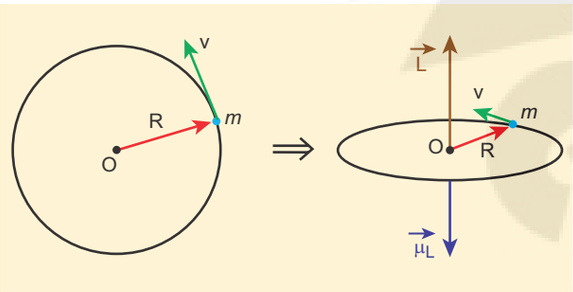


Figure 3.36 (a) Electron revolving in a circular orbit (b) Direction of magnetic dipole moment vector and orbital angular momentum vector are opposite

If T is the time period of revolution of an electron, the current due to circular motion of the electron is

$$I = \frac{-e}{T} \qquad (3.46)$$

where $-e$ is the charge of an electron. If R is the radius of the circular orbit and υ is the velocity of the electron in the circular orbit, then

$$T = \frac{2\pi R}{\upsilon} \qquad (3.47)$$

Using equation (3.46) and equation (3.47) in equation (3.45), we get

$$\mu_L = -\frac{e}{\frac{2\pi R}{\upsilon}}\pi R^2 = -\frac{e\upsilon R}{2} \qquad (3.48)$$

where $A = \pi R^2$ is the area of the circular loop. By definition, angular momentum of the electron about O is

$$\vec{L} = \vec{R} \times \vec{p}$$

In magnitude,

$$L = Rp = m\upsilon R \qquad (3.49)$$

Using equation (3.48) and equation (3.49), we get

$$\frac{\mu_L}{L} = -\frac{e\upsilon R/2}{m\upsilon R} = -\frac{e}{2m} \Rightarrow \vec{\mu}_L = -\frac{e}{2m}\vec{L} \qquad (3.50)$$

The negative sign indicates that the magnetic moment and angular momentum are in opposite direction.

In magnitude,

$$\frac{\mu_L}{L} = \frac{e}{2m} = \frac{1.60\times10^{-19}}{2\times9.11\times10^{-31}} = 0.0878\times10^{12}\ \text{C kg}^{-1}$$

$$\frac{\mu_L}{L} = 8.78\times10^{10}\ \text{C kg}^{-1} = \text{constant}$$

The ratio $\frac{\mu_L}{L}$ is a constant known as gyro-magnetic ratio $\left(\frac{e}{2m}\right)$. It must be noted that the gyro-magnetic ratio is a constant of proportionality which connects angular momentum of the electron and the magnetic moment of the electron.

UNIT-3(XII-Physics_Vol-1).indd 161 14-02-2020 20:03:55

According to Neil's Bohr quantization rule, the angular momentum of an electron moving in a stationary orbit is quantized which means

$$L = n\hbar = n\frac{h}{2\pi}$$

where h is the Planck's constant (h = 6.63 x 10^{-34} J s) and number n is the orbit number, i.e., n = 1,2,3,..... Hence,

$$\mu_L = \frac{e}{2m}L = n\frac{eh}{4\pi m}$$

$$\mu_L = n \times \frac{(1.60\times10^{-19})h}{4\pi m}\,\mathrm{A\,m^2}$$

$$= n \times \frac{(1.60\times10^{-19})(6.63\times10^{-34})}{4\times3.14\times(9.11\times10^{-31})}$$

$$\mu_L = n\times9.27\times10^{-24}\,\mathrm{A\,m^2}$$

The minimum value of magnetic moment can be obtained by substituting n = 1,

$$\mu_L = 9.27\times10^{-24}\,\mathrm{A\,m^2} = 9.27\times10^{-24}\,\mathrm{J\,T^{-1}}$$
$$= (\mu_L)_{\min} = \mu_B$$

where $\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\,\mathrm{A\,m^2}$ is called Bohr magneton which is used to measure atomic magnetic moments.

3.9 AMPÈRE'S CIRCUITAL LAW

Ampère's circuital law is used to calculate magnetic field at a point whenever there is a symmetry in the problem. This is similar to Gauss's law in electrostatics.

3.9.1 Ampère's circuital law

Ampère's law: The line integral of magnetic field over a closed loop is μ0 times net current enclosed by the loop.

$$\oint_C \vec{B}.\vec{dl} = \mu_\circ I_{enclosed} \qquad (3.51)$$

where $I_{enclosed}$ is the net current linked by the closed loop C. Note that the line integral does not depend on the shape of the path or the position of the conductor with the magnetic field.

Note

Line integral means integral over a line or curve, symbol used is $\int_c$

Closed line integral means integral over a closed curve (or line), symbol is $\oint$ *or* $\oint_c$

3.9.2 Magnetic field due to the current carrying wire of infinite length using Ampère's law

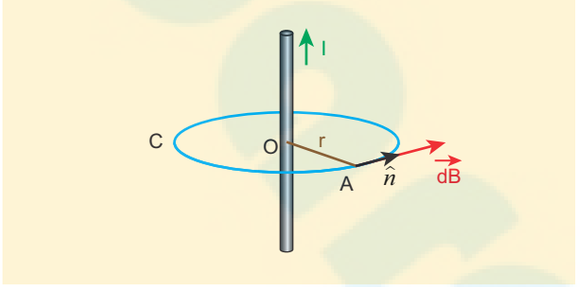


Figure 3.37 Ampèrian loop for current carrying straight wire

Consider a straight conductor of infinite length carrying current I and

UNIT-3(XII-Physics_Vol-1).indd 162 14-02-2020 20:03:57

the direction of magnetic field lines is shown in Figure 3.37. Since the wire is geometrically cylindrical in shape and symmetrical about its axis, we construct an Ampèrian loop in the form of a circular shape at a distance r from the centre of the conductor as shown in Figure 3.37. From the Ampère's law, we get

$$\oint_C \vec{B}.d\vec{l} = \mu_\circ I$$

where $d\vec{l}$ is the line element along the Amperian loop (tangent to the circular loop). Hence, the angle between magnetic field vector and line element is zero. Therefore,

$$\oint_C B\,dl = \mu_\circ I$$

where I is the current enclosed by the Ampèrian loop. Due to the symmetry, the magnitude of the magnetic field is uniform over the Ampèrian loop. Hence

$$B\oint_C dl = \mu_\circ I$$

For a circular loop, the circumference is 2πr, which implies,

$$B\int_0^{2\pi r} dl = \mu_\circ I$$

$$B\cdot 2\pi r = \mu_\circ I$$

$$B = \frac{\mu_\circ I}{2\pi r}$$

In vector form, the magnetic field is

$$\vec{B} = \frac{\mu_\circ I}{2\pi r}\hat{n}$$

where $\hat{n}$ is the unit vector along the tangent to the Ampèrian loop as shown in the Figure 3.37.

EXAMPLE 3.15

Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1 A at distance of 1m from it. Compare it with Earth's magnetic field.

Solution

Given that I = 1 A and radius r = 1 m

$$B_{straightwire} = \frac{\mu_\circ I}{2\pi r} = \frac{4\pi\times10^{-7}\times1}{2\pi\times1} = 2\times10^{-7}\ \text{T}$$

But the Earth's magnetic field is $B_{Earth} \sim 10^{-5}$ T

So, $B_{straightwire}$ is one hundred times smaller than B_{Earth}.

Solenoid

A solenoid is a long coil of wire closely wound in the form of helix as shown in Figure 3.38. When electric current is passed through the solenoid, the magnetic field is produced. The magnetic field of the solenoid is due to the superposition of magnetic fields of each turn of the solenoid. The direction of magnetic field due to solenoid is given by right hand palm-rule.

Inside the solenoid, the magnetic field is nearly uniform and parallel to its axis whereas, outside the solenoid the field is negligibly small. Based on the direction of the current, one end of the solenoid behaves like North Pole and the other end behaves like South Pole.

The current carrying solenoid is held in right hand. If the fingers curl in the direction of current, then extended thumb gives the direction of magnetic field of current carrying solenoid. It is shown in

UNIT-3(XII-Physics_Vol-1).indd 163 14-02-2020 20:04:01



Figure 3.38 Solenoid

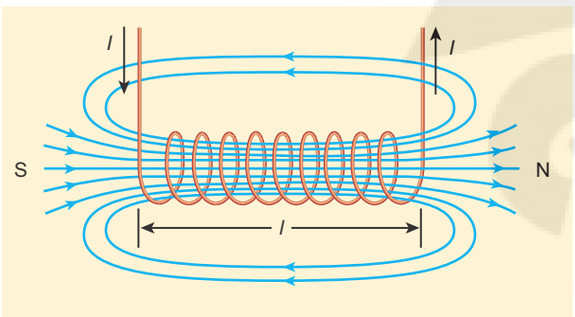


Figure 3.39 Solenoid as a bar magnet

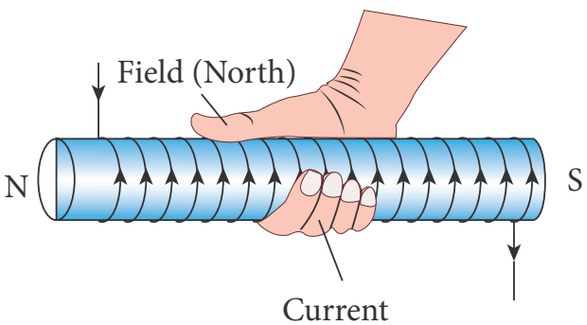


Figure 3.40 The direction of magnetic field of solenoid

Figure 3.39. Hence, the magnetic field of a solenoid looks like the magnetic field of a bar magnet.

The solenoid is assumed to be long which means that the length of the solenoid is large when compared to its diameter. The winding need not to be always circular, it can also be in other shapes. We consider here only circularly wound solenoid as shown in Figure 3.40.

3.9.3 Magnetic field due to a long current carrying solenoid

Consider a solenoid of length L having N turns. The diameter of the solenoid is assumed to be much smaller when compared to its length and the coil is wound very closely.

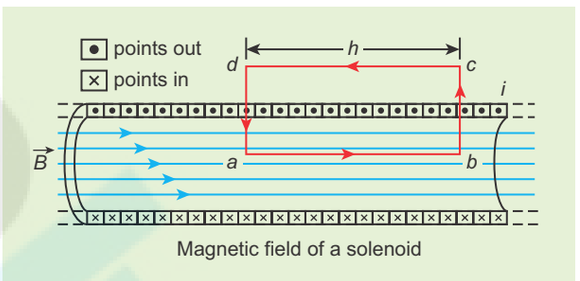


Figure 3.41 Amperian loop for solenoid

In order to calculate the magnetic field at any point inside the solenoid, we use Ampere's circuital law. Consider a rectangular loop abcd as shown in Figure 3.41. Then from Ampère's circuital law,

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_\circ I_{enclosed}$$

$$= \mu_\circ \times (\text{total current enclosed by Amperian loop})$$

The left hand side of the equation is

$$\oint_C \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}.d\vec{l} + \int_b^c \vec{B}.d\vec{l} + \int_c^d \vec{B}.d\vec{l} + \int_d^a \vec{B}.d\vec{l}$$

Since the elemental lengths along bc and da are perpendicular to the magnetic field

UNIT-3(XII-Physics_Vol-1).indd 164 14-02-2020 20:04:03

which is along the axis of the solenoid, the integrals

$$\int_b^c \vec{B}.d\vec{l} = \int_b^c \left|\vec{B}\right|\left|d\vec{l}\right| \cos 90^\circ = 0$$

Similarly

$$\int_d^a \vec{B}.d\vec{l} = 0$$

Since the magnetic field outside the solenoid is zero, the integral $\int_c^d \vec{B}.d\vec{l} = 0$

For the path along ab, the integral is

$$\int_a^b \vec{B}.d\vec{l} = B\int_a^b dl \cos 0^\circ = B\int_a^b dl$$

where the length of the loop ab as shown in the Figure 3.41 is h. But the choice of length of the loop ab is arbitrary. We can take very large loop such that it is equal to the length of the solenoid L. Therefore the integral is

$$\int_a^b \vec{B}.d\vec{l} = BL$$

Let I be the current passing through the solenoid of N turns, then

$$\int_a^b \vec{B}.d\vec{l} = BL = \mu_\circ NI \Rightarrow B = \mu_\circ \frac{NI}{L}$$

The number of turns per unit length is given by $\frac{N}{L} = n$, Then

$$B = \mu_\circ \frac{nLI}{L} = \mu_\circ nI \qquad (3.52)$$

Since n is a constant for a given solenoid and μ_0 is also constant. For a fixed current I, the magnetic field inside the solenoid is also a constant.

Note

Solenoid can be used as electromagnet. It produces strong magnetic field that can be turned ON or OFF. This is not possible in case of permanent magnet. Further the strength of the magnetic field can be increased by keeping iron bar inside the solenoid. This is because the magnetic field of the solenoid magnetizes the iron bar and hence the net magnetic field is the sum of magnetic field of the solenoid and magnetic field of magnetised iron. Because of these properties, solenoids are useful in designing variety of electrical appliances.

EXAMPLE 3.16

Calculate the magnetic field inside a solenoid, when

(a) the length of the solenoid becomes twice with fixed number of turns

(b) both the length of the solenoid and number of turns are doubled

(c) the number of turns becomes twice for the fixed length of the solenoid

Compare the results.

Solution

The magnetic field of a solenoid (inside) is

$$B_{L,N} = \mu_\circ \frac{NI}{L}$$

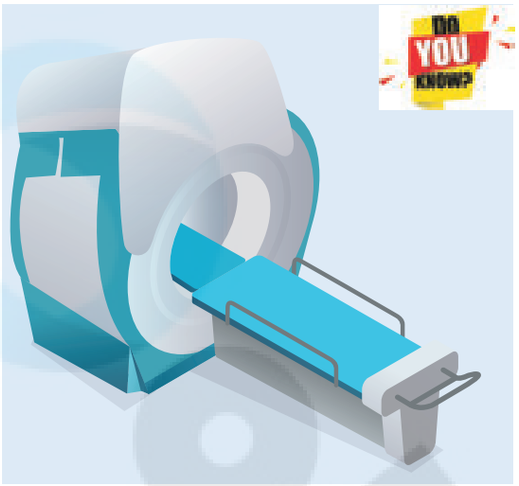
(a) length of the solenoid becomes twice with fixed number of turns

$L \rightarrow 2L$ (length becomes twice)

$N \rightarrow N$ (number of turns remains constant)

The magnetic field is

UNIT-3(XII-Physics_Vol-1).indd 165 14-02-2020 20:04:05



MRI is Magnetic Resonance Imaging which helps the physicians to diagonise or monitor treatment for a variety of abnormal conditions happening within the head, chest, abdomen and pelvis. It is a non invasive medical test. The patient is placed in a circular opening (actually interior of a solenoid which is made up of superconducting wire) and large current is sent through the superconducting wire to produce a strong magnetic field. So, it uses more powerful magnet, radio frequency pulses and a computer to produce pictures of organs which helps the physicians to examine various parts of the body.

$$B_{2L,N} = \mu_\circ \frac{NI}{2L} = \frac{1}{2} B_{L,N}$$

(b) both the length of the solenoid and number of turns are doubled

L→2L (length becomes twice)

N→2N (number of turns becomes twice)

The magnetic field is

$$B_{2L,2N} = \mu_\circ \frac{2NI}{2L} = B_{L,N}$$

(c) the number of turns becomes twice but the length of the solenoid remains same

L→L (length is fixed)

N→2N (number of turns becomes twice)

The magnetic field is

$$B_{L,2N} = \mu_\circ \frac{2NI}{L} = 2B_{L,N}$$

From the above results,

$$B_{L,2N} > B_{2L,2N} > B_{2L,N}$$

Thus, strength of the magnetic field is increased when we pack more loops into the same length for a given current.

3.9.5 Toroid

A solenoid is bent in such a way its ends are joined together to form a closed ring shape, is called a toroid which is shown in Figure 3.42. The magnetic field has constant magnitude inside the toroid whereas in the interior region (say, at point P) and exterior region (say, at point Q), the magnetic field is zero.



Figure 3.42 Toroid

(a) Open space interior to the toroid

Let us calculate the magnetic field B_P at point P. We construct an Amperian loop 1 of radius r_1 around the point P as shown in

UNIT-3(XII-Physics_Vol-1).indd 166 14-02-2020 20:04:07

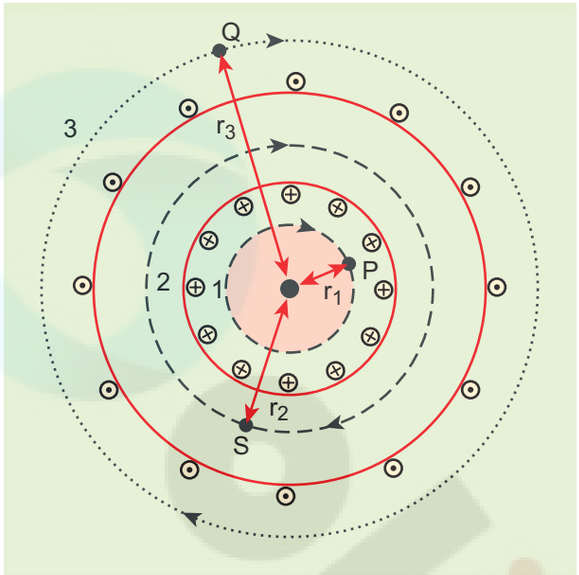


Figure 3.43 Toroid – Amperian loop

Figure 3.43. For simplicity, we take circular loop so that the length of the loop is its circumference.

$$L_1 = 2\pi r_1$$

Ampère's circuital law for the loop 1 is

$$\oint_{loop1} \vec{B}_P . \vec{dl} = \mu_\circ I_{enclosed}$$

Since the loop 1 encloses no current, $I_{enclosed} = 0$

$$\oint_{loop1} \vec{B}_P . \vec{dl} = 0$$

This is possible only if the magnetic field at point P vanishes i.e.

$$\vec{B}_P = 0$$

(b) Open space exterior to the toroid

Let us calculate the magnetic field B_Q at point Q. We construct an Amperian loop 3 of radius r_3 around the point Q as shown in Figure 3.43. The length of the loop is

$$L_3 = 2\pi r_3$$

Ampère's circuital law for the loop 3 is

$$\oint_{loop3} \vec{B}_Q . \vec{dl} = \mu_\circ I_{enclosed}$$

Since in each turn of the toroid loop, current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I_{enclosed} = 0$

$$\oint_{loop3} \vec{B}_Q . \vec{dl} = 0$$

This is possible only if the magnetic field at point Q vanishes i.e.

$$\vec{B}_Q = 0$$

(c) Inside the toroid

Let us calculate the magnetic field B_S at point S by constructing an Amperian loop 2 of radius r_2 around the point S as shown in Figure 3.43. The length of the loop is

$$L_2 = 2\pi r_2$$

Ampere's circuital law for the loop 2 is

$$\oint_{loop2} \vec{B}_S . \vec{dl} = \mu_\circ I_{enclosed}$$

Let I be the current passing through the toroid and N be the number of turns of the toroid, then

$$I_{enclosed} = NI$$

Therefore,

$$\oint_{loop2} \vec{B}_S . \vec{dl} = \oint_{loop2} B_S \, dl \cos\theta = B_S 2\pi r_2$$

$$\oint_{loop2} \vec{B}_S . \vec{dl} = \mu_\circ NI$$

$$B_S = \mu_\circ \frac{NI}{2\pi r_2}$$

If the number of turns per unit length $n = \frac{N}{2\pi r_2}$, then the magnetic field at point S is

$$B_S = \mu_\circ nI \qquad (3.53)$$

UNIT-3(XII-Physics_Vol-1).indd 167 14-02-2020 20:04:12

3.10 LORENTZ FORCE

When an electric charge q is kept at rest in a magnetic field, no force acts on it. At the same time, if the charge moves in the magnetic field, it experiences a force. This force is different from Coulomb force, studied in unit 1. This force is known as magnetic force. It is given by the equation

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \qquad (3.54)$$

In general, if the charge is moving in both the electric and magnetic fields, the total force experienced by the charge is given by $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$. It is known as Lorentz force.

3.10.1 Force on a moving charge in a magnetic field

When an electric charge q is moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$, it experiences a force, called magnetic force $\vec{F}_m$. After careful experiments, Lorentz deduced the force experienced by a moving charge in the magnetic field $\vec{F}_m$

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right) \qquad (3.55)$$

In magnitude, $F_m = qvB\sin\theta$ (3.56)

The equations (3.55) and (3.56) imply

1. $\vec{F}_m$ is directly proportional to the magnetic field $\vec{B}$
2. $\vec{F}_m$ is directly proportional to the velocity $\vec{v}$ of the moving charge
3. $\vec{F}_m$ is directly proportional to sine of the angle between the velocity and magnetic field
4. $\vec{F}_m$ is directly proportional to the magnitude of the charge q
5. The direction of $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$ as $\vec{F}_m$ is the cross product of $\vec{v}$ and $\vec{B}$

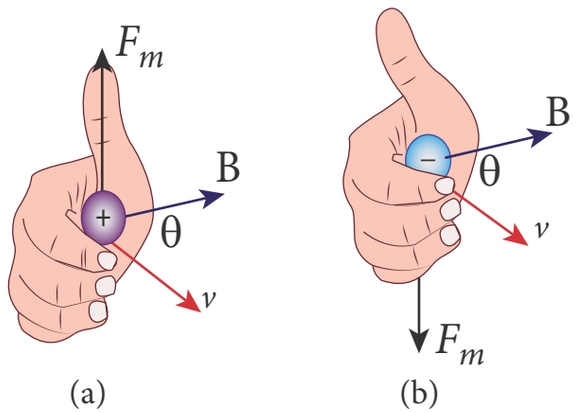


Figure 3.44 Direction of the Lorentz force on (a) positive charge (b) negative charge

6. The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge provided other factors are identical as shown Figure 3.44 (b)
7. If velocity $\vec{v}$ of the charge q is along magnetic field $\vec{B}$ then, $\vec{F}_m$ is zero

Definition of tesla

The strength of the magnetic field is one tesla if a unit charge moving in it with unit velocity experiences unit force.

$$1\ \mathrm{T} = \frac{1\,\mathrm{N\,s}}{\mathrm{C\,m}} = 1\frac{\mathrm{N}}{\mathrm{A\,m}} = 1\,\mathrm{N\,A^{-1}m^{-1}}$$

EXAMPLE 3.17

A particle of charge q moves with velocity $\vec{v}$ along positive y - direction in a magnetic field $\vec{B}$. Compute the Lorentz force experienced by the particle (a) when

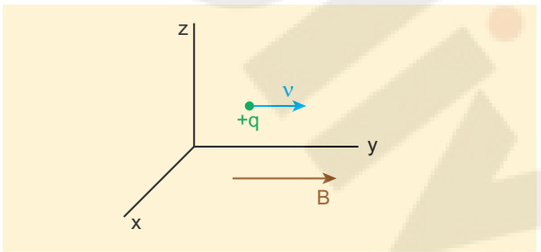
UNIT-3(XII-Physics_Vol-1).indd 168 14-02-2020 20:04:17

magnetic field is along positive y-direction (b) when magnetic field points in positive z - direction (c) when magnetic field is in zy - plane and making an angle θ with velocity of the particle. Mark the direction of magnetic force in each case.

Solution

Velocity of the particle is $\vec{v} = v\,\hat{j}$

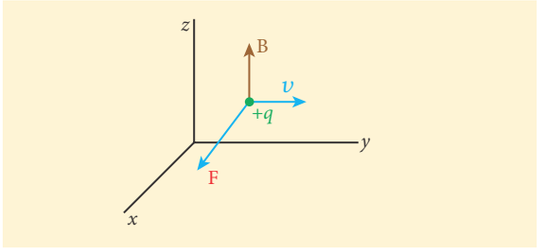
(a) Magnetic field is along positive y - direction, this implies, $\vec{B} = B\hat{j}$



From Lorentz force, $\vec{F}_m = q\left(v\,\hat{j} \times B\,\hat{j}\right) = \vec{0}$

So, no force acts on the particle when it moves along the direction of magnetic field.

(b) Since the magnetic field points in positive z - direction, this implies, $\vec{B} = B\,\hat{k}$

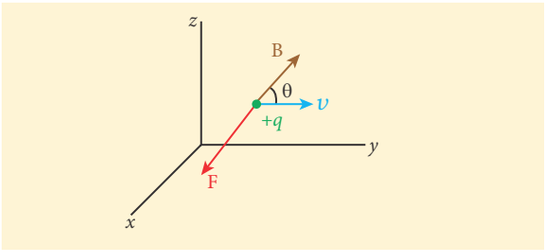


From Lorentz force, $\vec{F}_m = q(v\,\hat{j} \times B\hat{k})$

$= qvB\,\hat{i}$

Therefore, the magnitude of the Lorentz force is qvB and direction is along positive x - direction.

(c) Magnetic field is in zy - plane and making an angle θ with the velocity of the particle, which implies $\vec{B} = B\cos\theta\ \hat{j} + B\sin\theta\ \hat{k}$



From Lorentz force,

$$\vec{F}_m = q\left(v\,\hat{j}\right) \times (B\cos\theta\hat{j} + B\sin\theta\hat{k})$$
$$= q\,v\,B\sin\theta\ \hat{i}$$

EXAMPLE 3.18

Compute the work done and power delivered by the Lorentz force on the particle of charge q moving with velocity $\vec{v}$. Calculate the angle between Lorentz force and velocity of the charged particle and also interpret the result.

Solution

For a charged particle moving on a magnetic field, $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$

The work done by the magnetic field is

$$W = \int \vec{F}.d\vec{r} = \int \vec{F}.\vec{v}\,dt$$

$$W = q\int \left(\vec{v} \times \vec{B}\right).\vec{v}\,dt = 0$$

Since $\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$ and hence $\left(\vec{v} \times \vec{B}\right)\cdot\vec{v} = \vec{0}$ This means that Lorentz force does no work on the particle. From work-kinetic energy theorem, (Refer section 4.2.6, XI th standard Volume I)

$$\frac{dW}{dt} = P = 0$$

UNIT-3(XII-Physics_Vol-1).indd 169 14-02-2020 20:04:20

Since, $\vec{F}.\vec{v} = 0 \Rightarrow \vec{F}$ and $\vec{v}$ are perpendicular to each other. The angle between Lorentz force and velocity of the charged particle is 90°. Thus Lorentz force changes the direction of the velocity but not the magnitude of the velocity. Hence Lorentz force does no work and also does not alter kinetic energy of the particle.

3.10.2 Motion of a charged particle in a uniform magnetic field

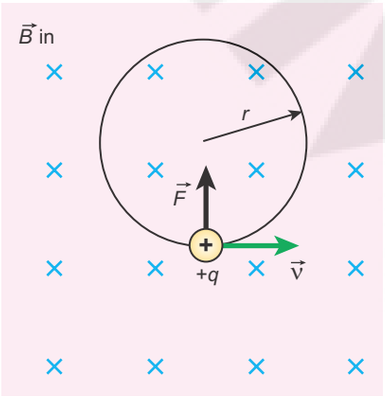


Figure 3.45 Circular motion of a charged particle in a perpendicular uniform magnetic field

Consider a charged particle of charge q having mass m entering into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that velocity is perpendicular to the magnetic field. As soon as the particle enters into the field, Lorentz force acts on it in a direction perpendicular to both magnetic field $\vec{B}$ and velocity $\vec{v}$.

As a result, the charged particle moves in a circular orbit as shown in Figure 3.45. The Lorentz force on the charged particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Since Lorentz force alone acts on the particle, the magnitude of the net force on the particle is

$$\sum_i F_i = F_m = qvB$$

This Lorentz force acts as centripetal force for the particle causing it to execute circular motion. Therefore,

$$qvB = m\frac{v^2}{r}$$

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{p}{qB} \qquad (3.57)$$

where $p = mv$ is the magnitude of the linear momentum of the particle. Let T be the time taken by the particle to finish one complete circular motion, then

$$T = \frac{2\pi r}{v} \qquad (3.58)$$

Hence substituting (3.57) in (3.58), we get

$$T = \frac{2\pi m}{qB} \qquad (3.59)$$

Equation (3.59) is called the **cyclotron period**. The reciprocal of time period is the frequency f, which is

$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi m} \qquad (3.60)$$

In terms of angular frequency ω,

$$\omega = 2\pi f = \frac{q}{m}B \qquad (3.61)$$

UNIT-3(XII-Physics_Vol-1).indd 170 14-02-2020 20:04:24

Equations (3.60) and (3.61) are called as **cyclotron frequency or gyro-frequency**.

From equations (3.59), (3.60) and (3.61), we infer that time period and frequency depend only on charge-to-mass ratio (specific charge) but not on velocity or the radius of the circular path.

If a charged particle moves in a region of uniform magnetic field such that its velocity is not perpendicular to the magnetic field, then the velocity of the particle is split up into two components; one component is parallel to the field while the other component perpendicular to the field. The component of velocity parallel to field remains unchanged and the component perpendicular to the field keeps changing due to Lorentz force. Hence the path of the particle is not a circle; it is a helical around the field lines as shown in Figure 3.46.

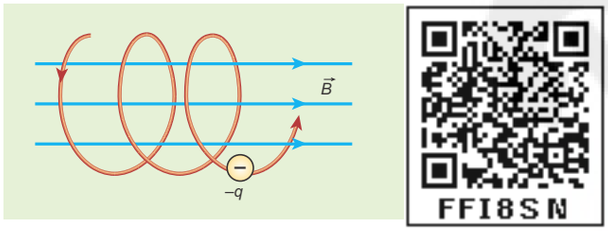


Figure 3.46 Helical path of the electron in a uniform magnetic field

For an example, the helical path of an electron when it moves in a magnetic field is shown in Figure 3.47. Inside the particle detector called cloud chamber, the path is made visible by the condensation of water droplets.

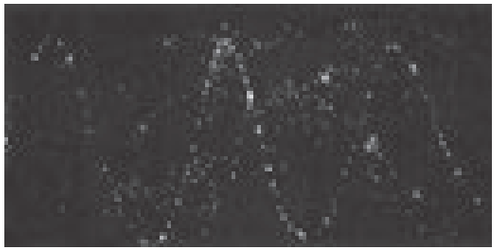


Figure 3.47 Helical path of the electron inside the cloud chamber

EXAMPLE 3.19

An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.50 mm. What is the speed of electron?

Solution

Charge of an electron $q = -1.60 \times 10^{-19}$ C

$\Rightarrow |q| = 1.60 \times 10^{-19}$ C

Magnitude of magnetic field $B = 0.500$ T

Mass of the electron, $m = 9.11 \times 10^{-31}$ kg

Radius of the orbit, $r = 2.50$ mm $= 2.50 \times 10^{-3}$ m

Speed of the electron, $v = |q|\dfrac{rB}{m}$

$$v = 1.60 \times 10^{-19} \times \frac{2.50 \times 10^{-3} \times 0.500}{9.11 \times 10^{-31}}$$

$$v = 2.195 \times 10^{8}\ \text{ms}^{-1}$$

EXAMPLE 3.20

A proton moves in a uniform magnetic field of strength 0.500 T magnetic field is directed along the x-axis. At initial time, $t = 0\ s$, the proton has velocity $\vec{v} = \left(1.95 \times 10^{5}\,\hat{i} + 2.00 \times 10^{5}\hat{k}\right)\text{ms}^{-1}$. Find

(a) At initial time, what is the acceleration of the proton.

(b) Is the path circular or helical? If helical, calculate the radius of helical trajectory and also calculate the pitch of the helix (Note: Pitch of the helix is the distance travelled along the helix axis per revolution).

Solution

Magnetic field $\vec{B} = 0.500\hat{i}$ T

UNIT-3(XII-Physics_Vol-1).indd 171 14-02-2020 20:04:26

Velocity of the particle

$\vec{v} = \left(1.95\times10^{5}\,\hat{i} + 2.00\times10^{5}\,\hat{k}\right)\text{ms}^{-1}$

Charge of the proton $q = 1.60\times10^{-19}\text{C}$

Mass of the proton $m = 1.67\times10^{-27}\text{kg}$

(a) The force experienced by the proton is

$$\vec{F} = q(\vec{v}\times\vec{B})$$
$$= 1.60\times10^{-19}\times\left(\left(1.95\times10^{5}\,\hat{i} + 2.00\times10^{5}\,\hat{k}\right)\times\left(0.500\,\hat{i}\right)\right)$$
$$\vec{F} = 1.60\times10^{-14}\,\hat{j}\ \text{N}$$

Therefore, from Newton's second law,

$$\vec{a} = \frac{1}{m}\vec{F} = \frac{1}{1.67\times10^{-27}}\left(1.60\times10^{-14}\right)\hat{j}$$
$$= 9.58\times10^{12}\,\hat{j}\,\text{m s}^{-2}$$

(b) Trajectory is helical

Radius of helical path is

$$R = \frac{mv_z}{|q|B} = \frac{1.67\times10^{-27}\times2.00\times10^{5}}{1.60\times10^{-19}\times0.500}$$
$$= 4.175\times10^{-3}\,\text{m} = 4.18\,\text{mm}$$

Pitch of the helix is the distance travelled along x-axis in a time T, which is $P = v_x\,T$

But time,

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\times3.14\times1.67\times10^{-27}}{1.60\times10^{-19}\times0.500}$$
$$= 13.1\times10^{-8}\,\text{s}$$

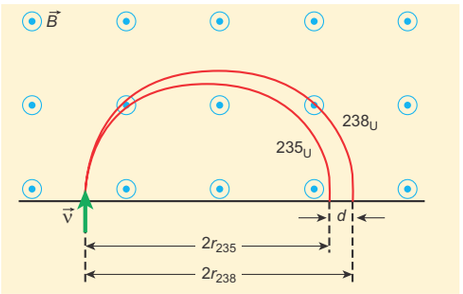
Hence, pitch of the helix is

$$P = v_x T = \left(1.95\times10^{5}\right)\left(13.1\times10^{-8}\right)$$
$$= 25.5\times10^{-3}\,\text{m} = 25.5\ \text{mm}$$

The proton experiences appreciable acceleration in the magnetic field, hence the pitch of the helix is almost six times greater than the radius of the helix.

EXAMPLE 3.21

Two singly ionized isotopes of uranium $^{235}_{92}U$ and $^{238}_{92}U$ (isotopes have same atomic number but different mass number) are sent with velocity 1.00×10^{5} m s^{-1} into a magnetic field of strength 0.500 T normally. Compute the distance between the two isotopes after they complete a semi-circle. Also compute the time taken by each isotope to complete one semi-circular path. (Given: masses of the isotopes: $m_{235} = 3.90\times10^{-25}$ kg and $m_{238} = 3.95\times10^{-25}$ kg)



Solution

Since isotopes are singly ionized, they have equal charge which is equal to the charge of an electron, $q = -1.6\times10^{-19}$ C. Mass of uranium $^{235}_{92}U$ and $^{238}_{92}U$ are 3.90×10^{-25} kg and 3.95×10^{-25} kg respectively. Magnetic field applied, $B = 0.500$ T. Velocity of the electron is 1.00×10^{5} ms^{-1}, then

(a) the radius of the path of $^{235}_{92}U$ is r_{235}

$$r_{235} = \frac{m_{235}v}{|q|B} = \frac{3.90\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$
$$= 48.8\times10^{-2}\,m$$
$$r_{235} = 48.8\ \text{cm}$$

The diameter of the semi-circle due to $^{235}_{92}U$ is $d_{235} = 2r_{235} = 97.6$ cm

The radius of the path of $^{238}_{92}U$ is r_{238} then

$$r_{238} = \frac{m_{238}v}{|q|B} = \frac{3.95\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$
$$= 49.4\times10^{-2}\,m$$
$$r_{238} = 49.4\ \text{cm}$$

The diameter of the semi-circle due to $^{238}_{92}U$ is $d_{238} = 2r_{238} = 98.8$ cm

UNIT-3(XII-Physics_Vol-1).indd 172 14-02-2020 20:04:31

Therefore the separation distance between the isotopes is $\Delta d = d_{238} - d_{235} = 1.2\,\text{cm}$

(b) The time taken by each isotope to complete one semi-circular path are

$$t_{235} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{97.6\times10^{-2}}{1.00\times10^{5}} = 9.76\times10^{-6}\,\text{s} = 9.76\,\mu\text{s}$$

$$t_{238} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{98.8\times10^{-2}}{1.00\times10^{5}} = 9.88\times10^{-6}\,\text{s} = 9.88\,\mu\text{s}$$

Note that even though the difference between mass of two isotopes are very small, this arrangement helps us to convert this small difference into an easily measurable distance of separation. This arrangement is known as mass spectrometer. A mass spectrometer is used in many areas in sciences, especially in medicine, in space science, in geology etc. For example, in medicine, anaesthesiologists use it to measure the respiratory gases and biologist use it to determine the reaction mechanisms in photosynthesis.

3.10.3 Motion of a charged particle under crossed electric and magnetic field (velocity selector)

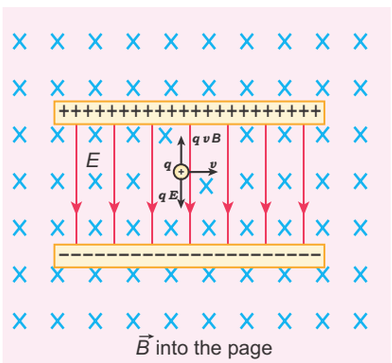


Figure 3.48 Velocity selector

Let us consider an experimental arrangement to illustrate velocity selector as shown in Figure 3.48. In the region of space between the parallel plates of a capacitor which produce uniform electric field $\vec{E}$, a uniform magnetic field $\vec{B}$ is maintained perpendicular to the direction of electric field.

Suppose a charged particle with charge q enters the space from left side with a velocity $\vec{v}$, the net force on the particle is

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

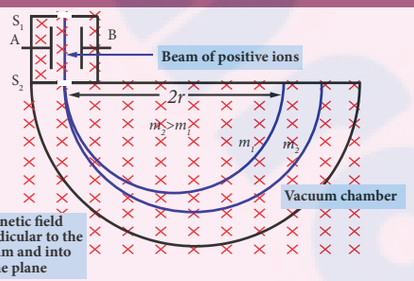
For a positive charge, the electric force on the charge acts in downward direction whereas the Lorentz force acts upwards. When these two forces balance each other, then

$$qE = qv_{\circ}B$$

$$\Rightarrow v_{\circ} = \frac{E}{B} \qquad (3.62)$$

Note This principle is used in Bainbridge mass spectrograph to separate the isotopes. This concept is explained in Example (3.21).

Schematic diagram of Bainbridge mass spectrometer



UNIT-3(XII-Physics_Vol-1).indd 173 14-02-2020 20:04:34

This means, for a given magnitude of $\vec{E}$- field and $\vec{B}$- field, the forces act only on the particle moving with particular speed $v_{\circ} = \frac{E}{B}$. This speed is independent of mass and charge.

By proper choice of electric and magnetic fields, the particle with particular speed can be selected. Such an arrangement of fields is called a **velocity selector**.

SC53HJ

EXAMPLE 3.22

Let E be the electric field of magnitude 6.0×10^6 N C^{-1} and B be the magnetic field magnitude 0.83 T. Suppose an electron is accelerated with a potential of 200 V, will it show zero deflection?. If not, at what potential will it show zero deflection.

Solution:

Electric field, $E = 6.0 \times 10^6$ N C^{-1} and magnetic field, $B = 0.83$ T.

Then

$$v = \frac{E}{B} = \frac{6.0\times10^6}{0.83}$$
$$= 7.23\times10^6 \text{ ms}^{-1}$$

When an electron goes with this velocity, it shows null deflection. Since the accelerating potential is 200 V, the electron acquires kinetic energy because of this accelerating potential. Hence,

$$\frac{1}{2}mv^2 = eV$$
$$v = \sqrt{\frac{2eV}{m}}$$

Since the mass of the electron, $m = 9.1\times10^{-31}$ kg and charge of an electron, $|q| = e = 1.6\times10^{-19}$ C. The velocity acquired by the electron due to accelerating potential of 200 V is

$$v_{200} = \sqrt{\frac{2\left(1.6\times10^{-19}\right)(200)}{\left(9.1\times10^{-31}\right)}} = 8.39\times10^6 \text{ m s}^{-1}$$

Since the speed $v_{200} > v$, the electron is deflected towards direction of Lorentz force. So, in order to have null deflection, the potential, we have to supply is

$$V = \frac{1}{2}\frac{mv^2}{e} = \frac{\left(9.1\times10^{-31}\right)\times\left(7.23\times10^6\right)^2}{2\times\left(1.6\times10^{-19}\right)}$$
$$V = 148.65 \text{ V}$$

3.10.4 Cyclotron

Cyclotron (Figure 3.49) is a device used to accelerate the charged particles to gain large kinetic energy. It is also called as high energy accelerator. It was invented by Lawrence and Livingston in 1934.

Principle

When a charged particle moves perpendicular to the magnetic field, it experiences magnetic Lorentz force.

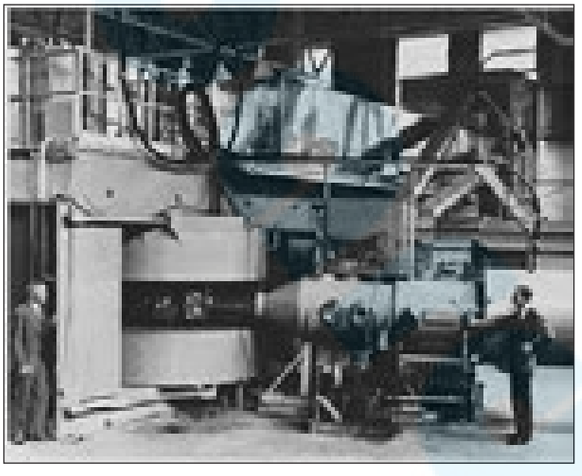


Figure 3.49 Cyclotron invented by Lawrence and Livingston

UNIT-3(XII-Physics_Vol-1).indd 174 14-02-2020 20:04:37

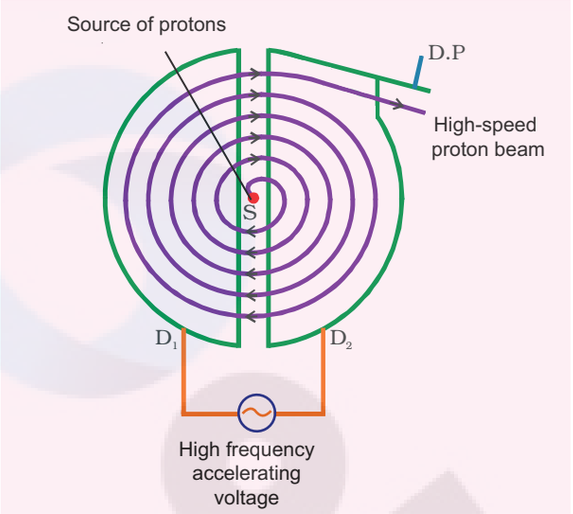


Figure 3.50 Working of cyclotron

Construction

The schematic diagram of a cyclotron is shown in Figure 3.50. The particles are allowed to move in between two semi-circular metal containers called Dees (hollow D - shaped objects). Dees are enclosed in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an electromagnet. The direction of magnetic field is normal to the plane of the Dees. The two Dees are kept separated with a gap and the source S (which ejects the particle to be accelerated) is placed at the centre in the gap between the Dees. Dees are connected to high frequency alternating potential difference.

Working

Let us assume that the ion ejected from source S is positively charged. As soon as ion is ejected, it is accelerated towards a Dee (say, Dee – 1) which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees, the ion moves in a circular path. After one semi-circular path inside Dee-1, the ion reaches the gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee-2 with a greater velocity. For this circular motion, the centripetal force on the charged particle q is provided by Lorentz force.

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{m}{qB}v$$

$$\Rightarrow r \propto v \qquad (3.63)$$

From the equation (3.63), the increase in velocity increases the radius of circular path. This process continues and hence the particle moves in spiral path of increasing radius. Once it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T.

The important condition in cyclotron operation is that when the frequency f at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator f_{osc}. This is called resonance condition.

From equation (3.60), we have

$$f_{osc} = \frac{qB}{2\pi m}$$

The time period of oscillation is

$$T = \frac{2\pi m}{qB}$$

The kinetic energy of the charged particle is

$$KE = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m} \qquad (3.64)$$

Limitations of cyclotron

(a) the speed of the ion is limited
(b) electron cannot be accelerated
(c) uncharged particles cannot be accelerated

UNIT-3(XII-Physics_Vol-1).indd 175 14-02-2020 20:04:38

Note

Deutrons (bundles of one proton and one neutron) can be accelerated because it has same charge as that of proton. But neutron (electrically neutral particle) cannot be accelerated by the cyclotron. When a deutron is bombarded with a beryllium target, a beam of high energy neutrons are produced. These high-energy neutrons are sent into the patient's cancerous region to break the bonds in the DNA of the cancer cells (killing the cells). This is used in treatment of fast-neutron cancer therapy.

EXAMPLE 3.23

Suppose a cyclotron is operated to accelerate protons with a magnetic field of strength 1 T. Calculate the frequency in which the electric field between two Dees could be reversed.

Solution

Magnetic field $B = 1$ T

Mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Charge of the proton, $q = 1.60 \times 10^{-19}$ C

$$f = \frac{qB}{2\pi m_p} = \frac{\left(1.60 \times 10^{-19}\right)(1)}{2(3.14)(1.67 \times 10^{-27})}$$

$$= 15.3 \times 10^{6}\,\text{Hz} = 15.3\,\text{MHz}$$

3.10.5 Force on a current carrying conductor placed in a magnetic field

When a current carrying conductor is placed in a magnetic field, the force experienced by the conductor is equal to the sum of Lorentz forces on the individual charge carriers in the conductor. Consider a small segment of conductor of length *dl*, with cross-sectional area *A*

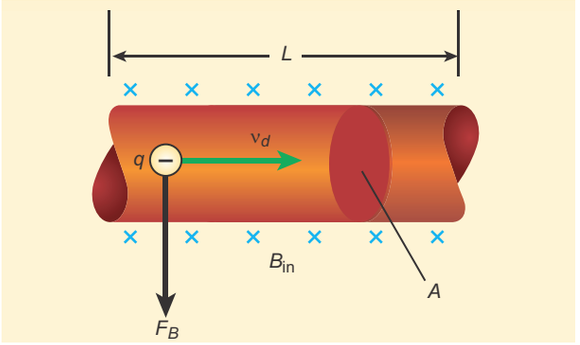


Figure 3.51 Current carrying conductor in a magnetic field

and current *I* as shown in Figure 3.51. The free electrons drift opposite to the direction of current. So the relation between current *I* and magnitude of drift velocity v_d (Refer Unit 2) is

$$I = neAv_d \qquad (3.65)$$

If the conductor is kept in a magnetic field $\vec{B}$, then average force experienced by the charge (electron) in the conductor is

$$\vec{f} = -e\left(\vec{v}_d \times \vec{B}\right)$$

If *n* is the number of free electrons present in unit volume, then

$$n = \frac{N}{V}$$

where *N* is the number of free electrons in the small element of volume $V = Adl$.

Hence Lorentz force on the elementary section of length *dl* is the product of the number of the electrons ($N = nAdl$) and the force acting on each electron.

$$\vec{F} = -enAdl\left(\vec{v}_d \times \vec{B}\right)$$

The current element in the conductor is $I\vec{dl} = -enA\vec{v}_d dl$. Therefore the force on the small elemental section of the current-carrying conductor is

$$d\vec{F} = \left(I\,\vec{dl} \times \vec{B}\right) \qquad (3.66)$$

UNIT-3(XII-Physics_Vol-1).indd 176 14-02-2020 20:04:40

Thus the force on a straight current carrying conductor of length l placed in a uniform magnetic field is

$$\vec{F}_{\text{total}} = \left(I\vec{l} \times \vec{B}\right) \qquad (3.67)$$

In magnitude,

$$F_{\text{total}} = BIl\sin\theta$$

(a) If the conductor is placed along the direction of the magnetic field, the angle then $\theta = 0^{\circ}$. Hence, the force experienced by the conductor is zero.

(b) If the conductor is placed perpendicular to the magnetic field, then the angle $\theta = 90^{\circ}$ Hence, the force experienced by the conductor is maximum, which is $F_{\text{total}} = BIl$.

Fleming's left hand rule

When a current carrying conductor is placed in a magnetic field, the direction of the force experienced by it is given by Fleming's Left Hand Rule (FLHR) as shown in Figure 3.52.

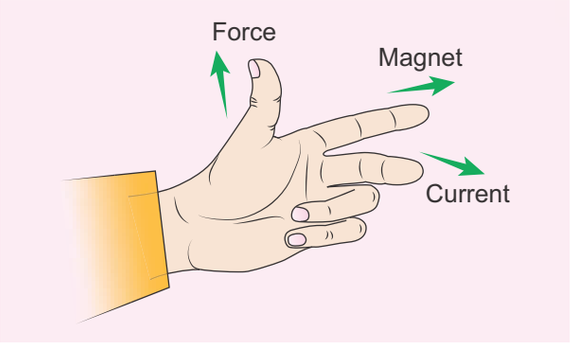
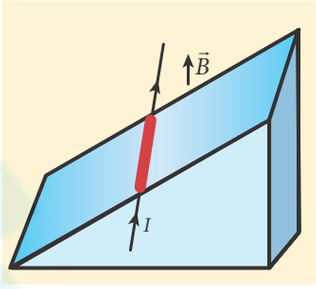


Figure 3.52 Fleming's Left Hand Rule (FLHR)

Stretch out forefinger, the middle finger and the thumb of the left hand such that they are in three mutually perpendicular directions. If the forefinger points in the direction of magnetic field, the middle finger in the direction of the electric current, then thumb will point in the direction of the force experienced by the conductor.

EXAMPLE 3.24

A metallic rod of linear density is 0.25 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25 T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.

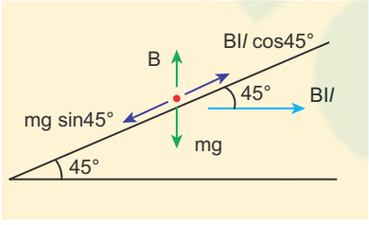


Solution

The linear density of the rod i.e., mass per unit length of the rod is 0.25 kg m^{-1}

$$\Rightarrow \frac{m}{l} = 0.25 \text{ kg m}^{-1}$$

Let I be the current flowing in the metallic rod. The direction of electric current is into the plane of the paper. The direction of magnetic force IBl is given by Fleming's left hand rule.



UNIT-3(XII-Physics_Vol-1).indd 177 14-02-2020 20:04:42

For equilibrium of the rod,

$$mg\sin 45^\circ = IBl\cos 45^\circ$$

$$\Rightarrow I = \frac{1}{B}\frac{m}{l}g\tan 45^\circ$$

$$= \frac{0.25\,\text{kg m}^{-1}}{0.25\,T}\times 1\times 9.8\ \text{m s}^{-2}$$

$$\Rightarrow I = 9.8\,\text{A}$$

So, we need to supply current of 9.8 A to keep the metallic rod stationary.

3.10.6 Force between two long parallel current carrying conductors

Let two long straight parallel current carrying conductors separated by a distance r be kept in air medium as shown in Figure 3.53. Let I_1 and I_2 be the electric currents passing through the conductors A and B in same direction (i.e. along z - direction) respectively. The net magnetic field at a distance r due to current I_1 in conductor A is

$$\vec{B}_1 = \frac{\mu_\circ I_1}{2\pi r}(-\hat{i}) = -\frac{\mu_\circ I_1}{2\pi r}\hat{i}$$

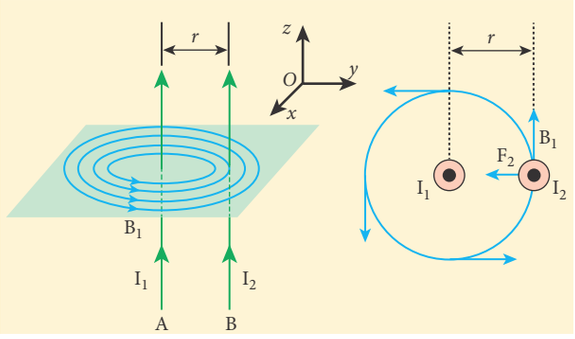


Figure 3.53 Two long straight parallel wires

From thumb rule, the direction of magnetic field is perpendicular to the plane of the paper and inwards (arrow into the page ⊗) i.e. along negative $\hat{i}$ direction.

Let us consider a small elemental length dl in conductor B at which the magnetic field $\vec{B}_1$ is present. From equation 3.66, Lorentz force on the element dl of conductor B is

$$d\vec{F} = \left(I_2 d\vec{l}\times\vec{B}_1\right) = -I_2 dl\frac{\mu_\circ I_1}{2\pi r}(\hat{k}\times\hat{i})$$

$$= -\frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j}$$

Therefore the force on dl of the wire B is directed towards the conductor A. So the element of length dl in B is attracted towards the conductor A. Hence the force per unit length of the conductor B due to current in the conductor A is

$$\frac{\vec{F}}{l} = -\frac{\mu_\circ I_1 I_2}{2\pi r}\hat{j}$$

Similarly, the net magnetic induction due to current I_2 (in conductor B) at a distance r in the elemental length dl of conductor A is

$$\vec{B}_2 = \frac{\mu_\circ I_2}{2\pi r}\hat{i}$$

From the thumb rule, direction of magnetic field is perpendicular to the plane of the paper and outwards (arrow out of the page ⊙) i.e., along positive $\hat{i}$ direction. Hence, the magnetic force acting on element dl of the conductor A is

$$d\vec{F} = \left(I_1 d\vec{l}\times\vec{B}_2\right) = I_1 dl\frac{\mu_\circ I_2}{2\pi r}(\hat{k}\times\hat{i})$$

$$= \frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j} \qquad (3.68)$$

Therefore the force on dl of conductor A is directed towards the conductor B. So the length dl is attracted towards the conductor B as shown in Figure (3.54).

UNIT-3(XII-Physics_Vol-1).indd 178 14-02-2020 20:04:46

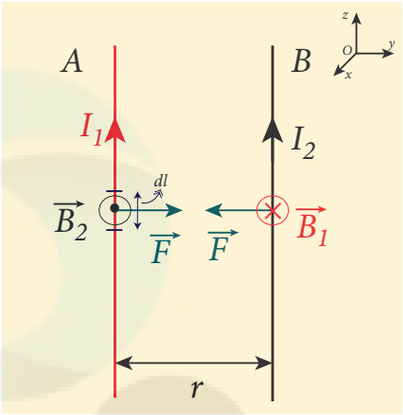


Figure 3.54 Current in both the conductors are in the same direction - attracts each other

The force acting per unit length of the conductor A due to the current in conductor B is

$$\frac{\vec{F}}{l} = \frac{\mu_\circ I_1 I_2}{2\pi r}\hat{j}$$

Thus the force between two parallel current carrying conductors is attractive if they carry current in the same direction. (Figure 3.55)

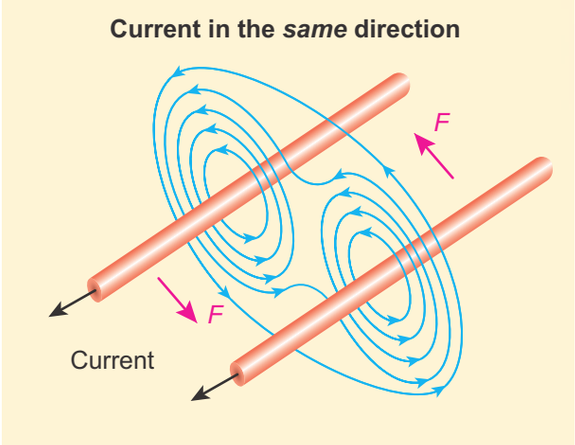


Figure 3.55 Two parallel conductors carrying current in same direction experience an attractive force

The force between two parallel current carrying conductors is repulsive if they carry current in opposite directions (Figure 3.56).

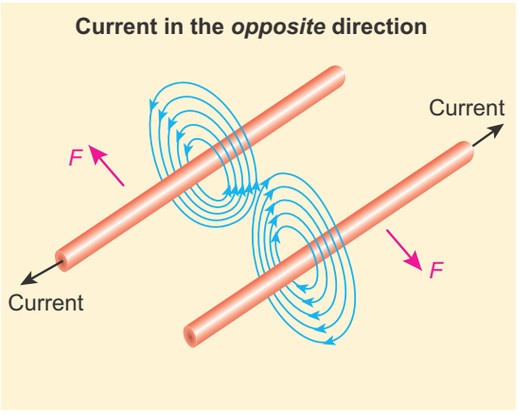


Figure 3.56 Two parallel conductors carrying current in opposite direction experience a repulsive force

Definition of ampere

One ampere is defined as that constant current which when passed through each of the two infinitely long parallel straight conductors kept side by side parallelly at a distance of one metre apart in air or vacuum causes each conductor to experience a force of 2×10^{-7} newton per metre length of conductor.

3.11 TORQUE ON A CURRENT LOOP

The force on a current carrying wire in a magnetic field is responsible for the motor operation.

3.11.1 Torque on a current loop placed in a magnetic field

Consider a rectangular loop PQRS carrying current I is placed in a uniform magnetic field $\vec{B}$. Let a and b be the length and breadth of rectangular loop respectively. The unit vector $\hat{n}$ normal to the plane of the

UNIT-3(XII-Physics_Vol-1).indd 179 14-02-2020 20:04:47

loop makes an angle θ with the magnetic field as shown in Figure 3.57.

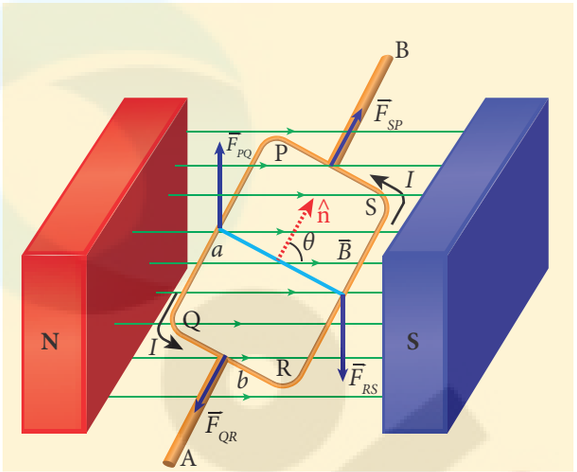


Figure 3.57 Rectangular coil placed in a magnetic field

The magnitude of the magnetic force acting on the current-carrying arm PQ is $F_{PQ} = IaB\sin\left(\pi/2\right) = IaB$. The direction of the force is found using right hand cork screw rule and its direction is upwards.

The magnitude of the force on the arm QR is $F_{QR} = IbB\sin\left(\pi/2 - \theta\right) = IbB\cos\theta$ and its direction is as shown in Figure 3.57.

The magnitude of the force on the arm RS is $F_{RS} = IaB\sin\left(\pi/2\right) = IaB$ and its direction is downwards.

The magnitude of the force acting on the arm SP is $F_{SP} = IbB\sin\left(\pi/2 + \theta\right) = IbB\cos\theta$ and its direction is also as shown in the Figure 3.57.

Since the forces F_{QR} and F_{SP} are equal, opposite and collinear, they cancel each other. But the forces F_{PQ} and F_{RS} , which are equal in magnitude and opposite in direction, are not acting along same straight line. Therefore, F_{PQ} and F_{RS} constitute a couple which exerts a torque on the loop.

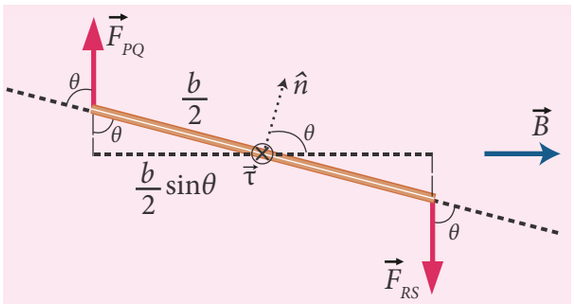


Figure 3.58 Side view of current loop

The magnitude of torque acting on the arm PQ about AB is $\tau_{PQ} = \left(\frac{b}{2}\sin\theta\right)IaB$ and it points in the direction of AB. The magnitude of the torque acting on the arm RS about AB is $\tau_{RS} = \left(\frac{b}{2}\sin\theta\right)IaB$ and points also in the same direction AB as shown in Figure 3.58.

58QX7F

The total torque acting on the entire loop about an axis AB is given by

$$\tau = \left(\frac{b}{2}\sin\theta\right)F_{PQ} + \left(\frac{b}{2}\sin\theta\right)F_{RS}$$
$$= Ia(b\sin\theta)B$$

$\tau = IAB\sin\theta$ along the direction AB

In vector form,

$$\vec{\tau} = \left(I\vec{A}\right)\times\vec{B}$$

The above equation can also be written in terms of magnetic dipole moment

$$\vec{\tau} = \vec{p}_m \times \vec{B} \text{ where } \vec{p}_m = I\vec{A}$$

The tendency of the torque is to rotate the loop so as to align its normal vector with the direction of the magnetic field.

If there are N turns in the rectangular loop, then the torque is given by

$$\tau = NIAB\sin\theta$$

UNIT-3(XII-Physics_Vol-1).indd 180 14-02-2020 20:04:52

Special cases:

(a) When $\theta = 90°$or the plane of the loop is parallel to the magnetic field, the torque on the current loop is maximum.

$$\tau_{max} = IAB$$

(b) When $\theta = 0°/180°$ or the plane of the loop is perpendicular to the magnetic field, the torque on the current loop is zero.

3.11.2 Moving coil galvanometer

Moving coil galvanometer is a device which is used to detect the flow of current in an electrical circuit.

Principle When a current carrying loop is placed in a uniform magnetic field, it experiences a torque.

Construction

A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns wound over a light metallic frame. A cylindrical soft-iron core is placed symmetrically inside the coil as shown in Figure 3.59. The rectangular coil is suspended freely between two pole pieces of a horse-shoe magnet.

The upper end of the rectangular coil is attached to one end of fine strip of phosphor bronze and the lower end of the coil is connected to a hair spring which is also made up of phosphor bronze. In a fine suspension strip, a small plane mirror is attached in order to measure the deflection of the coil with the help of lamp and scale arrangement. The other end of the mirror is connected to a torsion head. In order to pass electric current through the galvanometer, the suspension strip and the spring S are connected to terminals.

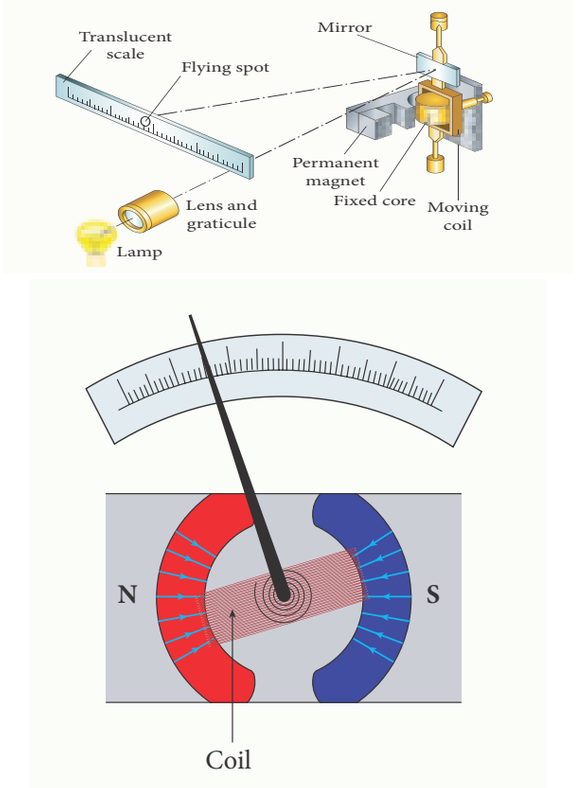


Figure 3.59 Moving coil galvanometer – its parts

Working

Consider a single turn of the rectangular coil PQRS whose length is l and breadth b. PQ = RS = l and QR = SP = b. Let I be the electric current flowing through the rectangular coil PQRS as shown in Figure 3.60. The horse-shoe magnet has hemi - spherical magnetic poles which produces a radial magnetic field. Due to this radial field, the sides QR and SP are always parallel to the magnetic field B and experience no force. The sides PQ and RS are always perpendicular to the magnetic field and experience equal forces in opposite directions. Due to this, torque is produced.

For single turn, the deflecting torque is

$$\tau = bF = bBIl = (lb)\, BI = ABI$$

since area of the coil, $A = lb$

For coil with N turns, we get

UNIT-3(XII-Physics_Vol-1).indd 181 14-02-2020 20:04:53

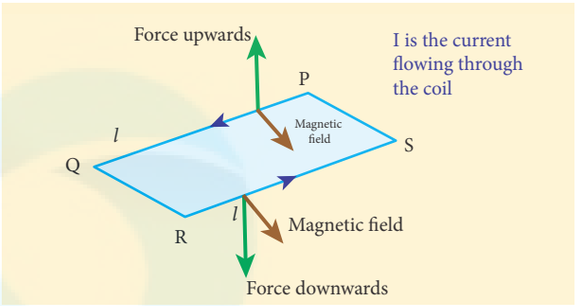


Figure 3.60 Force acting on current carrying coil

$$\tau = NABI \qquad (3.69)$$

Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed. Hence the moment of the restoring torque is proportional to the amount of twist θ.Thus

$$\tau = K\theta \qquad (3.70)$$

where K is the restoring couple per unit twist.

At equilibrium, the deflecting couple must be equal to the restoring couple. Therefore we get,

$$NABI = K\theta$$

$$\Rightarrow I = \frac{K}{NAB}\theta \qquad (3.71)$$

(or) $I = G\theta$

where $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

Since the suspended moving coil galvanometer is very sensitive, we have to handle with high care while doing experiments. Most of the galvanometer we use are pointer type moving coil galvanometer.

Figure of merit of a galvanometer

It is defined as the current required to produce a deflection of one scale division in the galvanometer.

Sensitivity of a galvanometer

The galvanometer is said to be sensitive if it shows large scale deflection even for a small current passed through it or a small voltage applied across it.

Current sensitivity

It is defined as **the deflection produced per unit current flowing through galvanometer**.

$$I_S = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_S = \frac{1}{G} \qquad (3.72)$$

The current sensitivity of a galvanometer can be increased by

(i) increasing the number of turns, N
(ii) increasing the magnetic induction, B
(iii) increasing the area of the coil, A
(iv) decreasing the couple per unit twist of the suspension wire, K.

Phosphor - bronze wire is used as the suspension wire because the couple per unit twist is very small.

Voltage sensitivity

It is defined as **the deflection produced per unit voltage applied across galvanometer**.

$$V_S = \frac{\theta}{V}$$

$$V_S = \frac{\theta}{IR_g} = \frac{NAB}{KR_g} \quad \text{(or)}$$

$$V_S = \frac{1}{GR_g} = \frac{I_S}{R_g} \qquad (3.73)$$

where R_g is the resistance of galvanometer.

UNIT-3(XII-Physics_Vol-1).indd 182 14-02-2020 20:04:55

EXAMPLE 3.25

The coil of a moving coil galvanometer has 5 turns and each turn has an effective area of 2×10^{-2} m^2. It is suspended in a magnetic field whose strength is 4×10^{-2} Wb m^{-2}. If the torsional constant K of the suspension fibre is 4×10^{-9} N m deg^{-1}.

(a) Find its current sensitivity in division per micro - ampere.

(b) Calculate the voltage sensitivity of the galvanometer for it to have full scale deflection of 50 divisions for 25 mV.

(c) Compute the resistance of the galvanometer.

Solution

$N = 5$ turns

$A = 2 \times 10^{-2}$ m^2

$B = 4 \times 10^{-2}$ Wbm^{-2}

$K = 4 \times 10^{-9}$ N m deg^{-1}

(a) Current sensitivity

$$I_S = \frac{NAB}{K} = \frac{5 \times 2 \times 10^{-2} \times 4 \times 10^{-2}}{4 \times 10^{-9}}$$

$= 10^6$ divisions per ampere

$1\mu A = 1 \text{micro ampere} = 10^{-6} \text{ampere}$

Therefore,

$$I_S = 10^6 \frac{\text{div}}{\text{A}} = 1 \frac{\text{div}}{10^{-6}\text{A}} = 1 \frac{\text{div}}{\mu\text{A}}$$

$$I_S = 1 \text{ div} (\mu\text{A})^{-1}$$

(b) Voltage sensitivity

$$V_S = \frac{\theta}{V} = \frac{50 \text{ div}}{25 \text{ mV}} = 2 \times 10^3 \text{ div V}^{-1}$$

(c) The resistance of the galvanometer is

$$R_g = \frac{I_S}{V_S} = \frac{10^6 \frac{\text{div}}{\text{A}}}{2 \times 10^3 \frac{\text{div}}{\text{V}}} = 0.5 \times 10^3 \frac{\text{V}}{\text{A}} = 0.5 \text{ k}\Omega$$

EXAMPLE 3.26

The resistance of a moving coil galvanometer is made twice its original value in order to increase current sensitivity by 50%. Find the percentage change in voltage sensitivity.

Solution

Voltage sensitivity is $V_S = \frac{I_S}{R_g}$

When the resistance is doubled, then new resistance is $R_g' = 2R_g$

Increase in current sensitivity is

$$I_S' = \left(1 + \frac{50}{100}\right) I_S = \frac{3}{2} I_S$$

The new voltage sensitivity is

$$V_S' = \frac{\frac{3}{2} I_S}{2R_g} = \frac{3}{4} V_S$$

Hence the voltage sensitivity decreases. The percentage decrease in voltage sensitivity is

$$\frac{V_S - V_S'}{V_S} \times 100\% = 25\%$$

Conversion of galvanometer into ammeter and voltmeter

A galvanometer is very sensitive instrument to detect the current. It can be easily converted into ammeter and voltmeter.

Galvanometer to an Ammeter

Ammeter is an instrument used to measure current flowing in the electrical circuit. The ammeter must offer low resistance such that it will not change the current passing through it. So ammeter is connected in series to measure the circuit current.

UNIT-3(XII-Physics_Vol-1).indd 183 14-02-2020 20:04:58

A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer. This low resistance is called shunt resistance *S*. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.

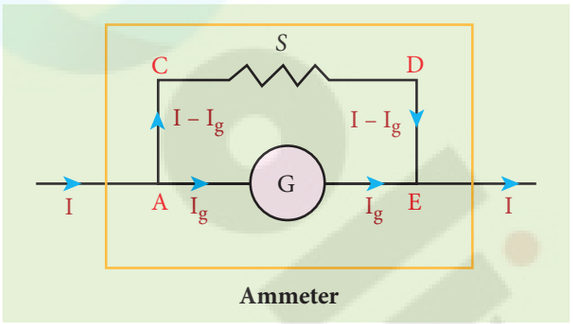


Figure 3.61 Shunt resistance connected in parallel

Let I be the current passing through the circuit as shown in Figure 3.61. When current I reaches the junction A, it divides into two components. Let I_g be the current passing through the galvanometer of resistance R_g through a path AGE and the remaining current $(I - I_g)$ passes along the path ACDE through shunt resistance S. The value of shunt resistance is so adjusted that current I_g produces full scale deflection in the galvanometer. The potential difference across galvanometer is same as the potential difference across shunt resistance.

$$V_{galvanometer} = V_{shunt}$$

$$\Rightarrow I_g R_g = (I - I_g) S$$

$$S = \frac{I_g}{(I - I_g)} R_g \text{ or}$$

$$I_g = \frac{S}{S + R_g} I$$

Since, the deflection in the galvanometer is proportional to the current passing through it,

$$\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I \text{ So,}$$

the deflection produced in the galvanometer is a measure of the current I passing through the circuit.

Shunt resistance is connected in parallel to galvanometer. Therefore, resistance of ammeter (R_a) can be determined by computing the effective resistance, which is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S} = R_a$$

Since, the shunt resistance is a very low resistance and the ratio $\frac{S}{R_g}$ is also small. This means, R_a is also small, i.e., the resistance offered by the ammeter is small. So, when we connect ammeter in series, the ammeter will not change appreciably the current in the circuit. For an ideal ammeter, the resistance must be equal to zero. But in reality, the reading in ammeter is always less than the actual current in the circuit. Let I_{ideal} be the current measured by ideal ammeter and I_{actual} be the actual current in the circuit. Then, the percentage error in measuring a current through an ammeter is

$$\frac{\Delta I}{I} \times 100\% = \frac{I_{ideal} - I_{actual}}{I_{ideal}} \times 100\%$$

Key points

1. An ammeter is a low resistance instrument and it is always connected in series to the circuit
2. An ideal ammeter has zero resistance

UNIT-3(XII-Physics_Vol-1).indd 184 14-02-2020 20:05:01

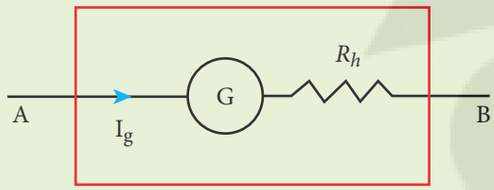
3. In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is

$$S = \frac{R_g}{n-1}$$

Galvanometer to a voltmeter

A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits. It should not draw any current from the circuit otherwise the value of potential difference to be measured will change.

Voltmeter must have high resistance and when it is connected in parallel, it will not draw appreciable current so that it will indicate the true potential difference.



Voltmeter

Figure 3.62 High resistance connected in series

A galvanometer is converted into a voltmeter by connecting high resistance R_h in series with galvanometer as shown in Figure 3.62. The scale is now calibrated in volt and the range of voltmeter depends on the values of the resistance R_h connected in series i.e. the value of resistance is so adjusted so that current I_g produces full scale deflection in the galvanometer.

Let R_g be the resistance of galvanometer and I_g be the current with which the galvanometer produces full scale deflection. Since the galvanometer is connected in series with high resistance, the current in the electrical circuit is same as the current passing through the galvanometer.

$$I = I_g$$

$$I = I_g \Rightarrow I_g = \frac{\text{potential difference}}{\text{total resistance}}$$

Since the galvanometer and high resistance are connected in series, the total resistance or effective resistance in the circuit is the sum of their resistances. This gives the resistance of voltmeter. Thus the voltmeter resistance is

$$R_v = R_g + R_h$$

Therefore,

$$I_g = \frac{V}{R_g + R_h}$$

$$\Rightarrow R_h = \frac{V}{I_g} - R_g$$

Note that $I_g \propto V$

The deflection in the galvanometer is proportional to current I_g. But current I_g is proportional to the potential difference. Hence the deflection in the galvanometer is a measure of potential difference. Since the resistance of voltmeter is very large, a voltmeter connected in parallel in an electrical circuit will draw least current in the circuit. An ideal voltmeter is one which has infinite resistance.

Key points

1. Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which the potential difference is to be measured.
2. An ideal voltmeter has infinite resistance
3. In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is

$$R_h = (n\text{-}1)\, R_g$$

UNIT-3(XII-Physics_Vol-1).indd 185 14-02-2020 20:05:03

SUMMARY

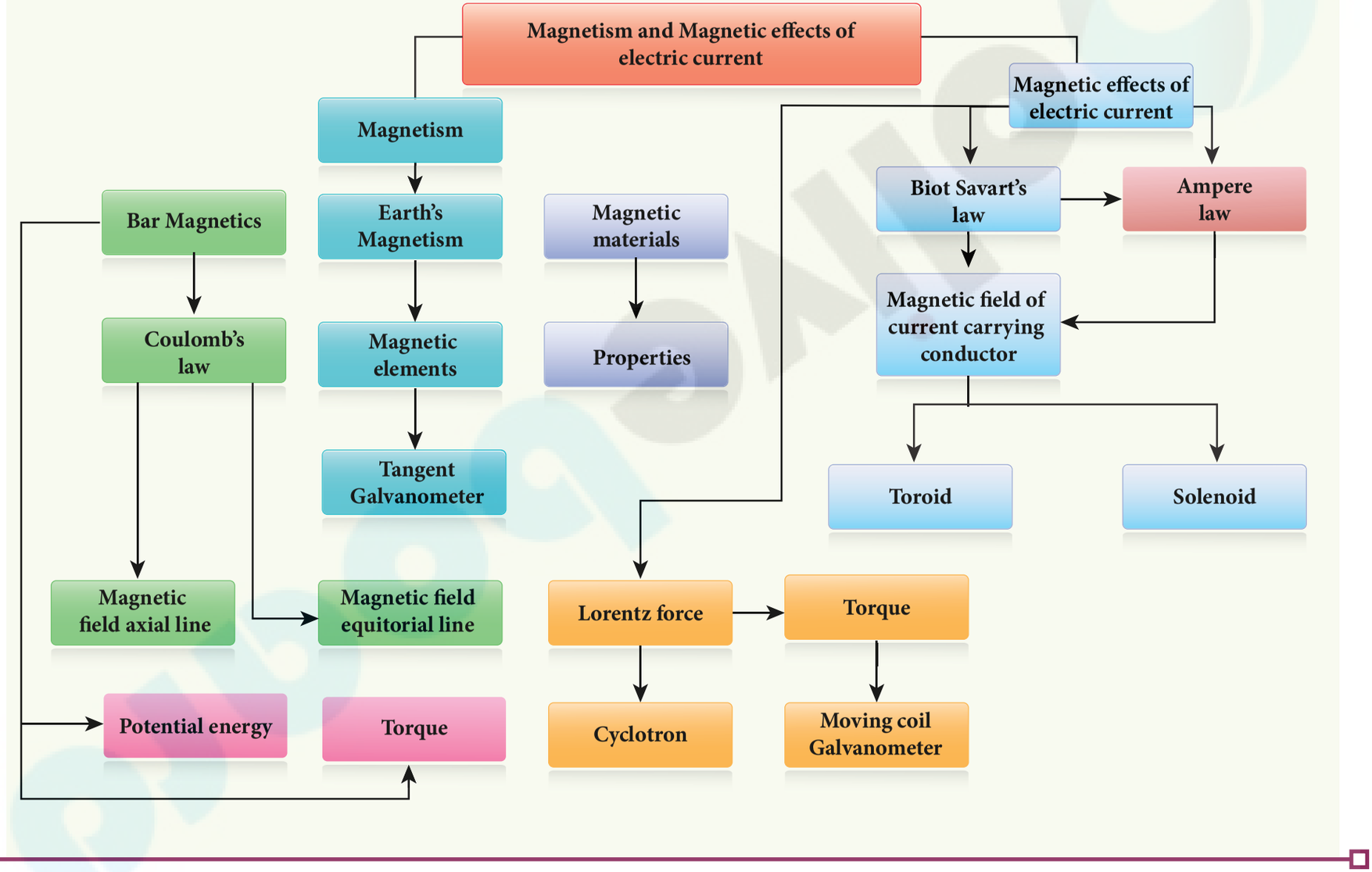
- A vertical plane passing through geographic axis is called geographic meridian.
- A vertical plane passing through magnetic axis is called magnetic meridian.
- The angle between magnetic meridian at a point with the geographic meridian is called the declination or magnetic declination.
- The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination at that point.
- The magnetic moment is defined as the product of its pole strength and magnetic length. It is a vector quantity, denoted by $\vec{P}_m$.
- The region surrounding magnet where magnetic pole of strength unity experiences a force is known as magnetic field. It is a vector quantity and denoted by $\vec{B}$. Its unit is N A^{-1} m^{-1}.
- The number of magnetic field lines crossing normally through a given area is called magnetic flux Φ_B. It is a scalar quantity. In SI unit, magnetic flux Φ_B is weber, symbol Wb.
- Statement of Coulomb's law in magnetism "The force of attraction or repulsion between two magnetic poles is proportional to the product of their pole strengths and inversely proportional to the square of distance between them".
- Magnetic dipole kept in a uniform magnetic field experiences torque.
- Tangent galvanometer is a device used to measure very small currents. It is a moving magnet type galvanometer. Its working is based on tangent law.
- Tangent law is $B = B_H \tan \theta$.
- The magnetic field which is used to magnetize a sample or specimen is called the magnetising field. It is a vector quantity and denoted by $\vec{H}$ and its unit is A m^{-1}.
- The measure of ability of the material to allow the passage of magnetic lines of force through it is known as magnetic permeability.
- The net magnetic moment per unit volume of material is known as intensity of magnetisation.
- Magnetic susceptibility is defined as the ratio of the intensity of magnetisation $\left(\vec{I}\right)$ induced in the material due to the magnetising field $\left(\vec{H}\right)$.
- Magnetic materials are classified into three categories: diamagnetic, paramagnetic and ferromagnetic materials.
- The lagging of magnetic induction $\vec{B}$ behind the cyclic variation in magnetising field $\vec{H}$ is defined as "Hysteresis", which means "lagging behind".
- The right hand thumb rule "Assume that we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the rest of the fingers encircling the wire point in the direction of the magnetic field lines produced".
- Maxwell right hand cork screw rule "If we rotate a screw by a screw driver, then the direction of current is same as the direction in which screw advances, and the direction of rotation of the screw will determine the direction of the magnetic field".

UNIT-3(XII-Physics_Vol-1).indd 186 14-02-2020 20:05:04

- Ampère's circuital law is $\oint_C \vec{B} \cdot d\vec{l} = \mu_\circ I_{enclosed}$.
- Magnetic field inside the solenoid is $B = \mu_\circ nI$, where n is the number of turns per unit length.
- Magnetic field interior to the toroid is $B = \mu_\circ nI$, where n is the number of turns per unit length.
- Lorentz force is $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$.
- Charged particle moving in a uniform magnetic field will undergo circular motion.
- Fleming's Left Hand Rule: Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If we keep the forefinger in the direction of magnetic field, the middle finger in the direction of the electric current, then the thumb points in the direction the force experienced by the conductors.
- One ampere is defined as that constant current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per metre length of the conductor.
- When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but net torque is not zero. The magnitude of net torque is $\tau = NABI \sin\theta$.
- Moving coil galvanometer is an instrument used for the detection and measurement of small currents.
- In moving coil galvanometer, current passing through the galvanometer is directly proportional to the deflection. Mathematically, $I = G\theta$, where $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.
- Current sensitivity is defined as the deflection produced per unit current flowing through it, $I_s = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_s = \frac{1}{G}$.
- Voltage sensitivity is defined as the deflection produced per unit voltage which is applied across it, $V_s = \frac{\theta}{V} = \frac{1}{GR_g} = \frac{I_S}{R_g}$, where, R_g is the resistance of galvanometer.
- Ammeter is an instrument used to measure current in an electrical circuit.
- A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer.
- An ideal ammeter has zero resistance.
- Voltmeter is an instrument used to measure potential difference across any element in an electrical circuit.
- A galvanometer can be converted into suitable voltmeter of given range by connecting a suitable high resistance R_h in series with the given galvanometer.
- An ideal voltmeter has infinite resistance.

UNIT-3(XII-Physics_Vol-1).indd 187 14-02-2020 20:05:06

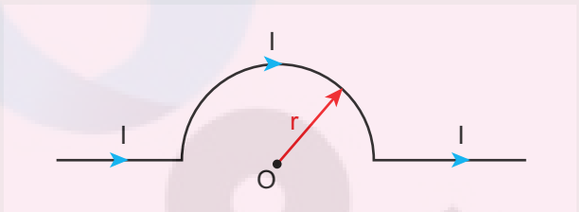
CONCEPT MAP



UNIT-3(XII-Physics_Vol-1).indd 188 14-02-2020 20:05:06

I Multiple choice questions

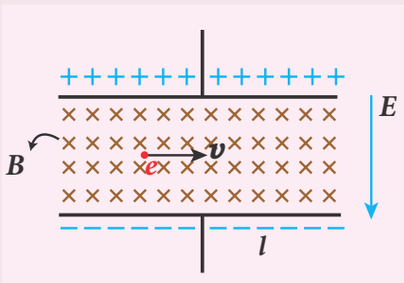
1. The magnetic field at the centre O of the following current loop is



(a) $\frac{\mu_\circ I}{4r}\otimes$ (b) $\frac{\mu_\circ I}{4r}\odot$

(c) $\frac{\mu_\circ I}{2r}\otimes$ (d) $\frac{\mu_\circ I}{2r}\odot$

2. An electron moves in a straight line inside a charged parallel plate capacitor of uniform charge density σ. The time taken by the electron to cross the parallel plate capacitor undeflected when the plates of the capacitor are kept under constant magnetic field of induction $\vec{B}$ is



H5CRWB

(a) $\varepsilon_\circ \frac{elB}{\sigma}$ (b) $\varepsilon_\circ \frac{lB}{\sigma l}$

(c) $\varepsilon_\circ \frac{lB}{e\sigma}$ (d) $\varepsilon_\circ \frac{lB}{\sigma}$

3. A particle having mass m and charge q accelerated through a potential difference V. Find the force experienced when it is kept under perpendicular magnetic field $\vec{B}$.

(a) $\sqrt{\frac{2q^3BV}{m}}$ (b) $\sqrt{\frac{q^3B^2V}{2m}}$

(c) $\sqrt{\frac{2q^3B^2V}{m}}$ (d) $\sqrt{\frac{2q^3BV}{m^3}}$

4. A circular coil of radius 5 cm and 50 turns carries a current of 3 ampere. The magnetic dipole moment of the coil is nearly

(a) 1.0 A m² (b) 1.2 A m²

(c) 0.5 A m² (d) 0.8 A m²

5. A thin insulated wire forms a plane spiral of $N = 100$ tight turns carrying a current $I = 8$ m A (milli ampere). The radii of inside and outside turns are $a = 50$ mm and $b = 100$ mm respectively. The magnetic induction at the centre of the spiral is

(a) 5 μT (b) 7 μT

(c) 8 μT (d) 10 μT

6. Three wires of equal lengths are bent in the form of loops. One of the loops is circle, another is a semi-circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque ?

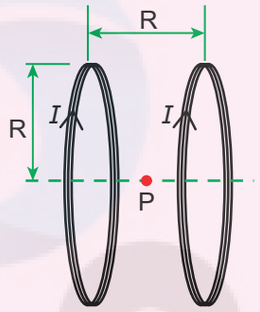
(a) Circle (b) Semi-circle

(c) Square (d) All of them

7. Two identical coils, each with N turns and radius R are placed coaxially at a distance R as shown in the figure. If I is the current passing through the loops in the same direction, then the

UNIT-3(XII-Physics_Vol-1).indd 189 14-02-2020 20:05:11

magnetic field at a point P at a distance of $R/2$ from the centre of each coil is



(a) $\frac{8N\mu_{\circ}I}{\sqrt{5}R}$ (b) $\frac{8N\mu_{\circ}I}{5^{3/2}R}$

(c) $\frac{8N\mu_{\circ}I}{5R}$ (d) $\frac{4N\mu_{\circ}I}{\sqrt{5}R}$

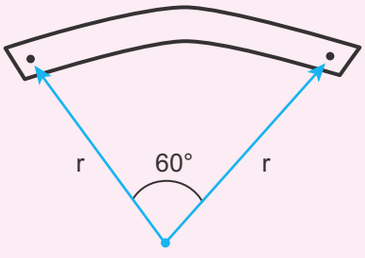
8. A wire of length l carrying a current I along the Y direction is kept in a magnetic field given by $\vec{B}=\frac{\beta}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})T$. The magnitude of Lorentz force acting on the wire is

(a) $\sqrt{\frac{2}{3}}\beta Il$ (b) $\sqrt{\frac{1}{3}}\beta Il$

(c) $\sqrt{2}\beta Il$ (d) $\sqrt{\frac{1}{2}}\beta Il$

9. A bar magnet of length l and magnetic moment p_m is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be

(NEET 2013)



(a) p_m (b) $\frac{3}{\pi}p_m$

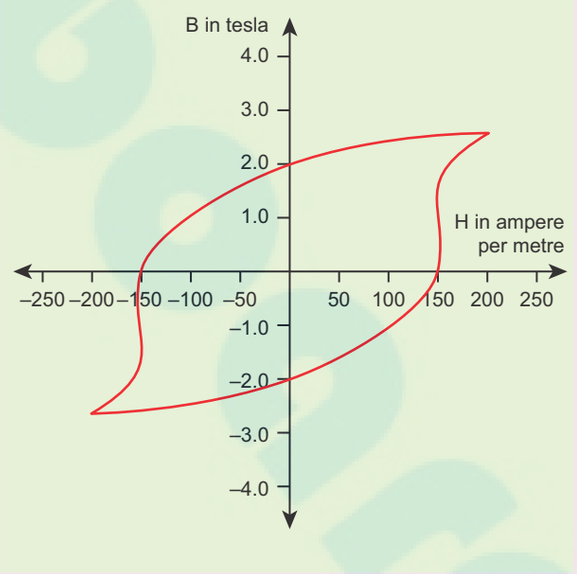
(c) $\frac{2}{\pi}p_m$ (d) $\frac{1}{2}p_m$

10. A non-conducting charged ring carrying a charge of q, mass m and radius r is rotated about its axis with constant angular speed ω. Find the ratio of its magnetic moment with angular momentum is

(a) $\frac{q}{m}$ (b) $\frac{2q}{m}$

(c) $\frac{q}{2m}$ (d) $\frac{q}{4m}$

11. The BH curve for a ferromagnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns/cm. The current that should be passed in the solenonid to demagnetize the ferromagnet completely is



(a) 1.00 m A (b) 1.25 mA

(c) 1.50 mA (d) 1.75 mA

UNIT-3(XII-Physics_Vol-1).indd 190 14-02-2020 20:05:16

12. Two short bar magnets have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is (Horizontal components of Earth's magnetic induction is 3.6×10^{-5} Wb m^{-2})

(NSEP 2000-2001)

(a) 3.60×10^{-5} Wb m^{-2}

(b) 3.5×10^{-5} Wb m^{-2}

(c) 2.56×10^{-4} Wb m^{-2}

(d) 2.2×10^{-4} Wb m^{-2}

13. The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?

(a) 30° (b) 45°

(c) 60° (d) 90°

14. A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an axis perpendicular to its plane passing through the centre with angular velocity ω. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic field whose strength is B which is directed perpendicular to the axis of rotation

(a) $\frac{1}{4}\sigma\omega\pi BR$ (b) $\frac{1}{2}\sigma\omega\pi BR^2$

(c) $\frac{1}{4}\sigma\omega\pi BR^3$ (d) $\frac{1}{4}\sigma\omega\pi BR^4$

15. The potential energy of magnetic dipole whose dipole moment is $\vec{p}_m = (-0.5\hat{i} + 0.4\hat{j})$ Am^2 kept in uniform magnetic field $\vec{B} = 0.2\,\hat{i}$ T

(a) –0.1 J (b) –0.8 J

(c) 0.1 J (d) 0.8 J

Answers

1) a	**2)** d	**3)** c	**4)** b	**5)** b
6) a	**7)** b	**8)** a	**9)** b	**10)** c
11) c	**12)** c	**13)** b	**14)** d	**15)** c

II Short answer questions:

1. What is meant by magnetic induction?

2. Define magnetic flux.

3. Define magnetic dipole moment.

4. State Coulomb's inverse law.

5. What is magnetic susceptibility?

6. State Biot-Savart's law.

7. What is magnetic permeability?

8. State Ampere's circuital law.

9. Compare dia, para and ferro-magnetism.

10. What is meant by hysteresis?

11. Define magnetic declination and inclination.

12. What is resonance condition in cyclotron?

13. Define ampere.

14. State Fleming's left hand rule.

15. Is an ammeter connected in series or parallel in a circuit? Why?

16. Explain the concept of velocity selector.

17. Why is the path of a charged particle not a circle when its velocity is not perpendicular to the magnetic field?

18. Give the properties of dia / para / ferromagnetic materials.

UNIT-3(XII-Physics_Vol-1).indd 191 14-02-2020 20:05:18

19. What happens to the domains in a ferromagnetic material in the presence of external magnetic field?

20. How is a galvanometer converted into (i) an ammeter and (ii) a voltmeter?

III Long answer questions

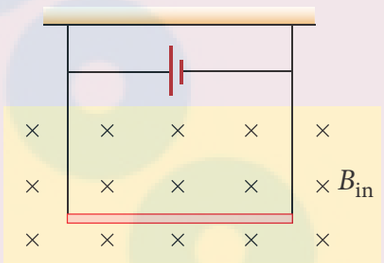
1. Discuss Earth's magnetic field in detail.
2. Deduce the relation for the magnetic field at a point due to an infinitely long straight conductor carrying current.
3. Obtain a relation for the magnetic field at a point along the axis of a circular coil carrying current.
4. Compute the torque experienced by a magnetic needle in a uniform magnetic field.
5. Calculate the magnetic field at a point on the axial line of a bar magnet.
6. Obtain the magnetic field at a point on the equatorial line of a bar magnet.
7. Find the magnetic field due to a long straight conductor using Ampere's circuital law.
8. Discuss the working of cyclotron in detail.
9. What is tangent law? Discuss in detail.
10. Derive the expression for the torque on a current-carrying coil in a magnetic field.
11. Discuss the conversion of galvanometer into an ammeter and also a voltmeter.
12. Calculate the magnetic field inside and outside of the long solenoid using Ampere's circuital law.
13. Derive the expression for the force between two parallel, current-carrying conductors.
14. Give an account of magnetic Lorentz force.
15. Compare the properties of soft and hard ferromagnetic materials.
16. Derive the expression for the force on a current-carrying conductor in a magnetic field.

IV. Numerical problems

1. A bar magnet having a magnetic moment $\vec{p}_m$ is cut into four pieces i.e., first cut into two pieces along the axis of the magnet and each piece is further cut along the axis into two pieces. Compute the magnetic moment of each piece.

   Answer $\vec{p}_{m_{new}} = \frac{1}{4}\vec{p}_m$

2. A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume $g = 10$ m s^{-2}



   Answer 2 mA

3. A circular coil with cross-sectional area 0.1 cm^2 is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate

   (a) total torque on the coil

UNIT-3(XII-Physics_Vol-1).indd 192 14-02-2020 20:05:19

(b) total force on the coil

(c) average force on each electron in the coil due to the magnetic field. (The free electron density for the material of the wire is 10^{28} m^{-3}).

Answer (a) zero (b) zero (c) 0.6×10^{-23} N

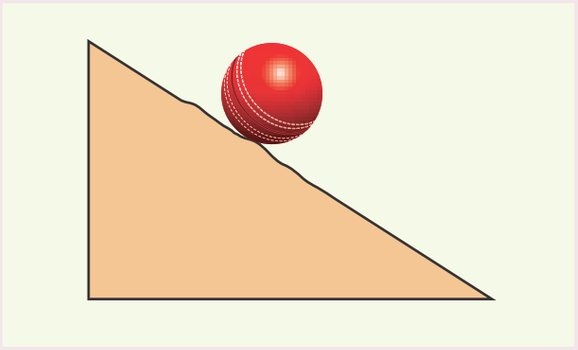
4. A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T. If the bar magnet is oriented at an angle 30° with the external field experiences a torque of 0.2 Nm. Calculate:

(i) the magnetic moment of the magnet

(ii) the work done by the magnetic field in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

Answer (i) 0.5 A m^2 (ii) W = 0.8 J and W_{mag} = -0.8 J

5. A non - conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current I required to rest the sphere in equilibrium. Answer $\frac{2}{\pi}A$



6. Calculate the magnetic field at the centre of a square loop which carries a current of 1.5 A, length of each side being 50 cm. Answer 3.4×10^{-6} T

BOOKS FOR REFERENCE

1. H. C. Verma, *Concepts of Physics – Volume 2*, Bharati Bhawan Publisher.
2. Halliday, Resnick and Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition.
3. Serway and Jewett, *Physics for scientist and engineers with modern physics*, Brook/Coole publishers, Eighth edition.
4. David J. Griffiths, *Introduction to electrodynamics*, Pearson publishers.
5. Rita John, *Solid State Physics (Magnetism chapter)*, McGraw Hill Education (India) Pvt. Ltd.
6. Paul Tipler and Gene Mosca, *Physics for scientist and engineers with modern physics*, Sixth edition, W.H. Freeman and Company.

UNIT-3(XII-Physics_Vol-1).indd 193 14-02-2020 20:05:20

ICT CORNER

Magnetism

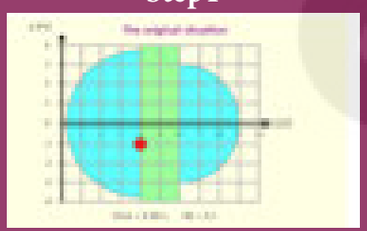
Topic: Cyclotron

In this activity you will be able to visualize and understand the working of cyclotron.

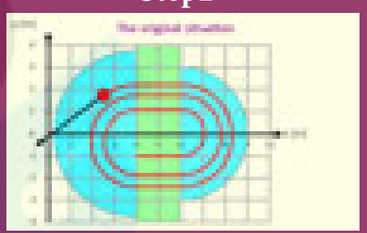
STEPS:

- Open the browser and type 'physics.bu.edu/~duffy/HTML5/cyclotron.html' in the address bar.
- Click 'play' to release the positively charged particle between the D-shaped sections.
- Observe trajectory of positively charged particle under the magnetic field between D-shaped sections.
- Note the kinetic energy of the particle after some time (say t = 20 s)

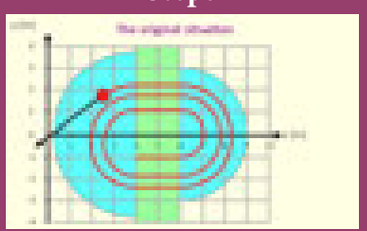
Step1



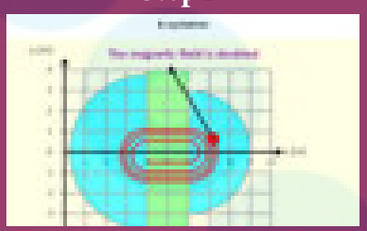
Step2



Step3



Step4



Double the electric and magnetic fields by clicking corresponding buttons and observe the change in kinetic energy for a particular given time t.

URL:

http://physics.bu.edu/~duffy/HTML5/cyclotron.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT-3(XII-Physics_Vol-1).indd 194 14-02-2020 20:05:23

UNIT 4

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

"Nature is our kindest friend and best critic in experimental science if we only allow her intimations to fall unbiased on our minds" — Michael Faraday

Learning Objectives

In this unit, the student is exposed to

U1YLL7

- the phenomenon of electromagnetic induction
- the application of Lenz's law to find the direction of induced emf
- the concept of Eddy current and its uses
- the phenomenon of self-induction and mutual-induction
- the various methods of producing induced emfs
- the construction and working of AC generators
- the principle of transformers and its role in long distance power communication
- the notion of root mean square value of alternating current
- the idea of phasors and phase relationships in different AC circuits
- the insight about power in an AC circuit and wattless current
- the understanding of energy conservation during LC oscillations

4.1 ELECTROMAGNETIC INDUCTION

4.1.1 Introduction

In the previous chapter, we have learnt that whenever an electric current flows through a conductor, it produces a magnetic field around it. This was discovered by Christian Oersted. Later, Ampere proved that a current-carrying loop behaves like a bar magnet. These are the magnetic effects produced by the electric current.

Physicists then began to think of the converse effect. Is it possible to produce an electric current with the help of a magnetic field? A series of experiments were conducted to establish the converse effect. These experiments were done by Michael Faraday of UK and Joseph Henry of USA, almost simultaneously and independently. These attempts became successful and led to the discovery of the phenomenon, called Electromagnetic Induction. Michael Faraday is credited with the discovery of electromagnetic induction in 1831.

UNIT-4(XII-Physics_Vol-1).indd 195 17-02-2020 13:17:30

In this chapter, let us see a few experiments of Faraday, the results and the phenomenon of Electromagnetic Induction. Before that, we will recollect the concept of magnetic flux linked with a surface area.

An anecdote!

Michael Faraday was enormously popular for his lectures as well. In one of his lectures, he demonstrated his experiments which led to the discovery of electromagnetic induction.

At the end of the lecture, one member of the audience approached Faraday and said, "Mr. Faraday, the behaviour of the magnet and the coil of wire was interesting, but what is the use of it?" Faraday answered politely, "Sir, what is the use of a newborn baby?"

Note: We will soon see the greatness of 'that little child' who has now grown as an adult to cater to the energy needs.

4.1.2 Magnetic Flux (Φ_B)

The magnetic flux Φ_B through an area A in a magnetic field is defined as the number of magnetic field lines passing through that area normally and is given by the equation (Figure 4.1(a)).

$$\Phi_B = \int_A \vec{B}.\, d\vec{A} \qquad (4.1)$$

where the integral is taken over the area A and θ is the angle between the direction of the magnetic field and the outward normal to the area.

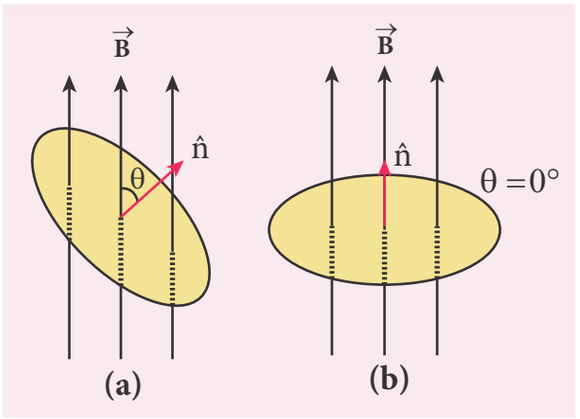


Figure 4.1 Magnetic flux

If the magnetic field $\vec{B}$ is uniform over the area A and is perpendicular to the area as shown in Figure 4.1(b), then the above equation becomes

$$\Phi_B = \int_A \vec{B}.\, d\vec{A} = BA\cos\theta$$
$$= BA \qquad \text{since } \theta = 0°,\ \cos 0° = 1$$

The SI unit of magnetic flux is T m^2. It is also measured in weber or Wb.

1 Wb = 1 T m^2

EXAMPLE 4.1

A circular antenna of area 3 m^2 is installed at a place in Madurai. The plane of the area of antenna is inclined at 47° with the direction of Earth's magnetic field. If the magnitude of Earth's field at that place is 4.1×10^{-5} T find the magnetic flux linked with the antenna.

Solution

$B = 4.1 \times 10^{-5}$ T; $\theta = 90° - 47° = 43°$; $A = 3\text{m}^2$

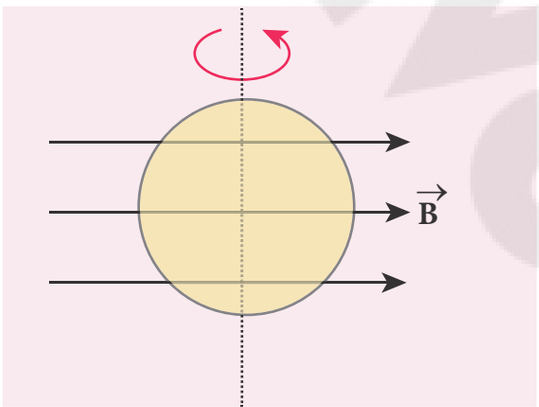
UNIT-4(XII-Physics_Vol-1).indd 196 17-02-2020 13:17:31

We know that $\Phi_B = BA\cos\theta$

$$\Phi_B = 4.1 \times 10^{-5} \times 3 \times \cos 43^\circ$$
$$= 4.1 \times 10^{-5} \times 3 \times 0.7314$$
$$= 89.96\ \mu\text{Wb}$$

EXAMPLE 4.2

A circular loop of area $5 \times 10^{-2}\ m^2$ rotates in a uniform magnetic field of 0.2 T. If the loop rotates about its diameter which is perpendicular to the magnetic field as shown in figure. Find the magnetic flux linked with the loop when its plane is (i) normal to the field (ii) inclined 60° to the field and (iii) parallel to the field.



Solution

$A = 5 \times 10^{-2}\ m^2$; $B = 0.2$ T

(i) $\theta = 0^\circ$;

$$\Phi_B = BA\cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 0^\circ$$
$$\Phi_B = 1 \times 10^{-2}\ \text{Wb}$$

(ii) $\theta = 90^\circ - 60^\circ = 30^\circ$;

$$\Phi_B = BA\cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 30^\circ$$
$$\Phi_B = 1 \times 10^{-2} \times \frac{\sqrt{3}}{2} = 8.66 \times 10^{-3}\ \text{Wb}$$

(iii) $\theta = 90^\circ$;

$$\Phi_B = BA\cos 90^\circ = 0$$

4.1.3 Faraday's Experiments on Electromagnetic Induction

First Experiment

Consider a closed circuit consisting of a coil *C* of insulated wire and a galvanometer *G* (Figure 4.2(a)). The galvanometer does not indicate deflection as there is no electric current in the circuit.

When a bar magnet is inserted into the stationary coil, with its north pole facing the coil, there is a momentary deflection in the galvanometer. This indicates that an electric current is set up in the coil (Figure 4.2(b)). If the magnet is kept stationary inside the

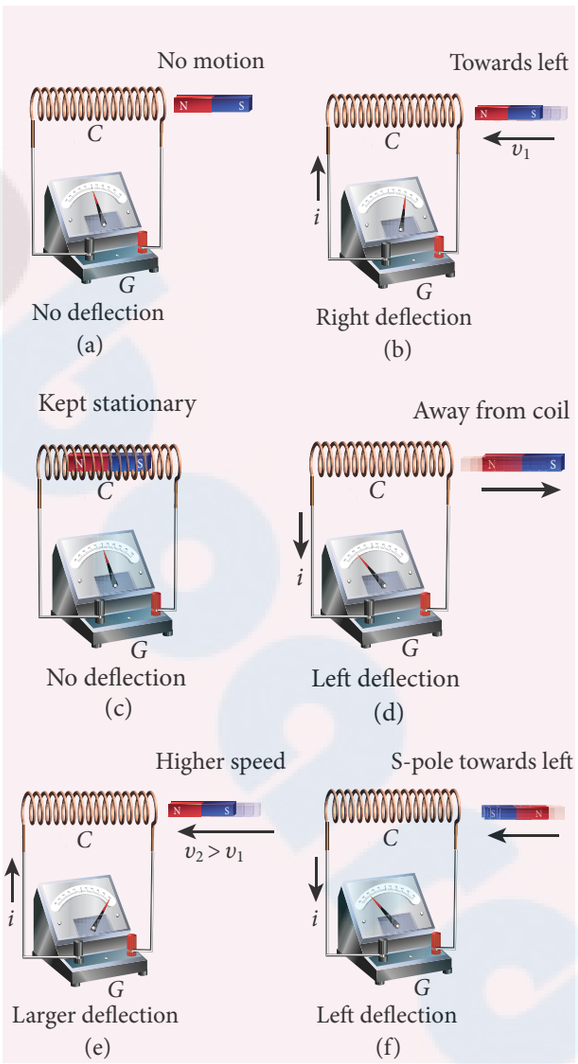


Figure 4.2 Faraday's first experiment

UNIT-4(XII-Physics_Vol-1).indd 197 17-02-2020 13:17:35

coil, the galvanometer does not indicate deflection (Figure 4.2(c)).

The bar magnet is now withdrawn from the coil, the galvanometer again gives a momentary deflection but in the opposite direction. So the electric current flows in opposite direction (Figure 4.2(d)). Now if the magnet is moved faster, it gives a larger deflection due to a greater current in the circuit (Figure 4.2(e))

The bar magnet is reversed i.e., the south pole now faces the coil. When the above experiment is repeated, the deflections are opposite to that obtained in the case of north pole (Figure 4.2(f)).

If the magnet is kept stationary and the coil is moved towards or away from the coil, similar results are obtained. It is concluded that whenever there is a relative motion between the coil and the magnet, there is deflection in the galvanometer, indicating the electric current setup in the coil.

Second Experiment

Consider two closed circuits as shown in Figure 4.3(a). The circuit consisting of a coil *P*, a battery *B* and a key *K* is called as primary circuit while the circuit with a coil *S* and a galvanometer *G* is known as secondary circuit. The coils *P* and *S* are kept at rest in close proximity with respect to one another.

If the primary circuit is closed, electric current starts flowing in the primary circuit. At that time, the galvanometer gives a momentary deflection (Figure 4.3(a)).

After that, when the electric current reaches a certain steady value, no deflection is observed in the galvanometer.

Likewise if the primary circuit is broken, the electric current starts decreasing and there is again a sudden deflection but in the opposite direction (Figure 4.3(b)).

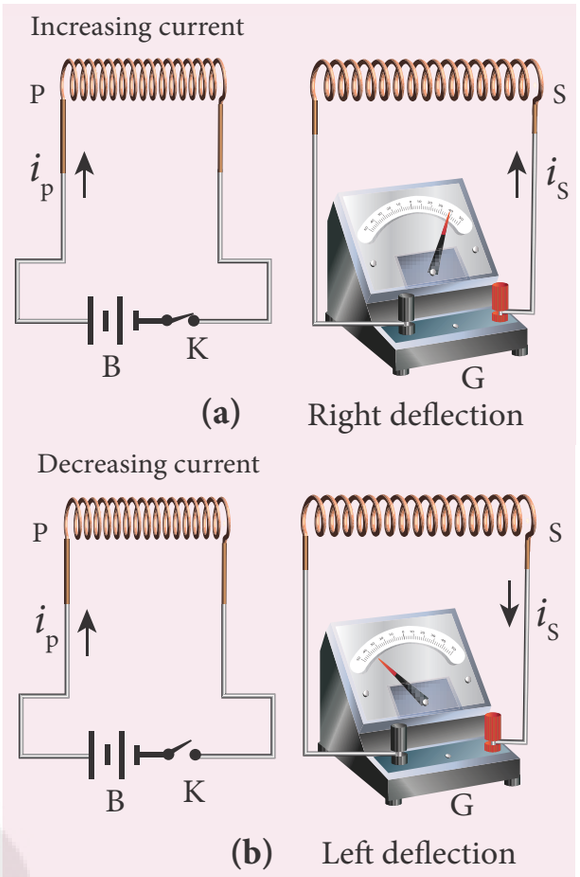


Figure 4.3 Faraday's second experiment

When the electric current becomes zero, the galvanometer shows no deflection.

From the above observations, it is concluded that whenever the electric current in the primary circuit changes, the galvanometer shows a deflection.

Faraday's Law of Electromagnetic Induction

From the results of his experiments, Faraday realized that

whenever the magnetic flux linked with a closed coil changes, an emf (electromotive force) is induced and hence an electric current flows in the circuit. This current is called an induced current and the emf giving rise to such current is called an induced emf. This phenomenon is known as electromagnetic induction.

UNIT-4(XII-Physics_Vol-1).indd 198 17-02-2020 13:17:36

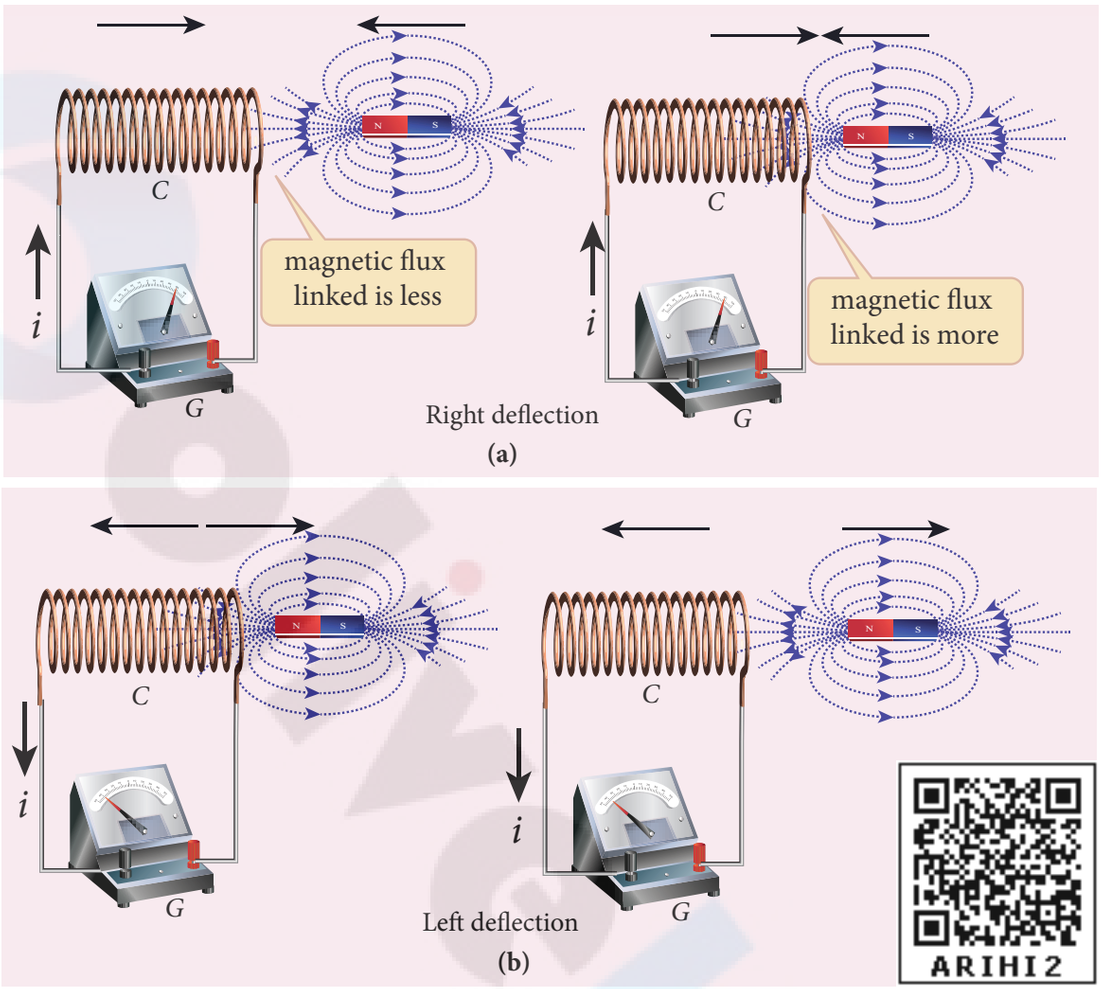


Figure 4.4 Explanation of Faraday's first experiment

Based on this idea, Faraday's experiments are understood in the following way. In the first experiment, when a bar magnet is placed close to a coil, some of the magnetic field lines of the bar magnet pass through the coil i.e., the magnetic flux is linked with the coil. When the bar magnet and the coil approach each other, the magnetic flux linked with the coil increases. So this increase in magnetic flux induces an emf and hence a transient electric current flows in the circuit in one direction (Figure 4.4(a)).

At the same time, when they recede away from one another, the magnetic flux linked with the coil decreases. The decrease in magnetic flux again induces an emf in opposite direction and hence an electric current flows in opposite direction (Figure 4.4(b)). So there is deflection in the galvanometer when there is a relative motion between the coil and the magnet.

In the second experiment, when the primary coil *P* carries an electric current, a magnetic field is established around it. The magnetic lines of this field pass through itself and the neighbouring secondary coil *S*.

When the primary circuit is open, no electric current flows in it and hence the magnetic flux linked with the secondary coil is zero (Figure 4.5(a)).

However, when the primary circuit is closed, the increasing current builds up a

UNIT-4(XII-Physics_Vol-1).indd 199 17-02-2020 13:17:36

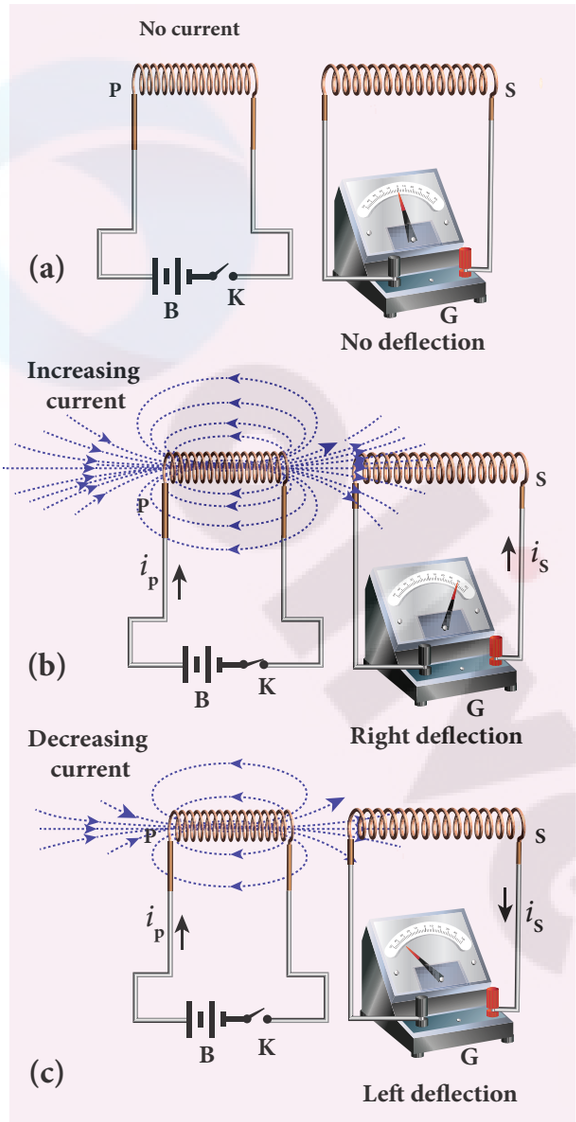


Figure 4.5 Explanation of Faraday's second experiment

magnetic field around the primary coil. Therefore, the magnetic flux linked with the secondary coil increases. This increasing flux linked induces a transient electric current in the secondary coil (Figure 4.5(b)). When the electric current in the primary coil reaches a steady value, the magnetic flux linked with the secondary coil does not change and the electric current in the secondary coil will disappear.

Similarly, when the primary circuit is broken, the decreasing primary current induces an electric current in the secondary coil, but in the opposite direction (Figure 4.5(c)). So there is deflection in the galvanometer whenever there is a change in the primary current.

The conclusions of Faraday's experiments are stated as two laws.

First law

Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit which lasts in the circuit as long as the magnetic flux is changing.

Second law

The magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.

If the magnetic flux linked with each turn of the coil changes by $d\Phi_B$ in a time dt, then the induced emf in each turn is given by

$$\varepsilon = \frac{d\Phi_B}{dt}$$

If a coil consisting of N turns is tightly wound such that each turn covers the same area, then the flux through each turn will be the same. Then total emf induced in the coil is given by

$$\varepsilon = N\frac{d(\Phi_B)}{dt} = \frac{d(N\Phi_B)}{dt} \qquad (4.2)$$

Here $N\Phi_B$ is called flux linkage, defined as the product of number of turns N of the coil and the magnetic flux linking each turn of the coil Φ_B.

UNIT-4(XII-Physics_Vol-1).indd 200 17-02-2020 13:17:37

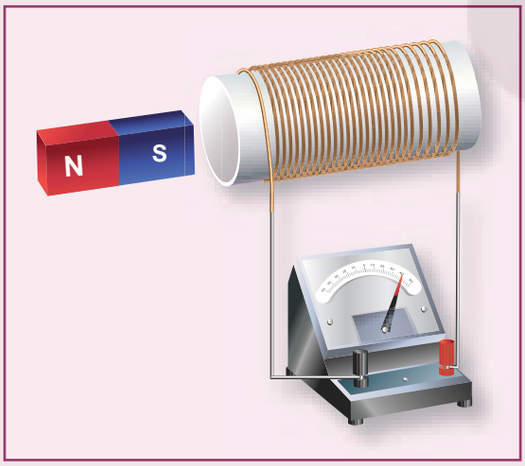
Importance of Electromagnetic Induction!

The application of the phenomenon of Electromagnetic Induction is almost everywhere in the present day life. Right from home appliances to huge factory machineries, from cellphone to computers and internet, from electric guitar to satellite communication, all need electricity for their operation. There is an ever growing demand for electric power.

All these are met with the help of electric generators and transformers which function on electromagnetic induction. The modern, sophisticated human life would not be possible without the discovery of electromagnetic induction.

ACTIVITY

Exploring Electromagnetic Induction



Make a circuit containing a coil of insulated wire wound around soft hollow core and a galvanometer as shown in Figure. It is better to use a thin wire for the coil so that we can wind many turns in the available space. Perform the steps described in first experiment of Faraday with the help of a strong bar magnet. Students will get hands-on experience about electromagnetic induction.

EXAMPLE 4.3

A cylindrical bar magnet is kept along the axis of a circular solenoid. If the magnet is rotated about its axis, find out whether an electric current is induced in the coil.

Solution

The magnetic field of a cylindrical magnet is symmetrical about its axis. As the magnet is rotated along the axis of the solenoid, there is no induced current in the solenoid because the flux linked with the solenoid does not change due to the rotation of the magnet.

EXAMPLE 4.4

A closed coil of 40 turns and of area 200 cm^2, is rotated in a magnetic field of flux density 2 Wb m^{-2}. It rotates from a position where its plane makes an angle of 30° with the field to a position perpendicular to the field in a time 0.2 s. Find the magnitude of the emf induced in the coil due to its rotation.

Solution

$N = 40$ turns; $B = 2$ Wb m^{-2}

$A = 200$ cm^2 $= 200 \times 10^{-4}$ m^2;

Initial flux, $\Phi_i = BA\cos\theta$

$= 2 \times 200 \times 10^{-4} \times \cos 60^\circ$

since $\theta = 90^\circ - 30^\circ = 60^\circ$

$\Phi_i = 2 \times 10^{-2}$ Wb

Final flux, $\Phi_f = BA\cos\theta$

$= 2 \times 200 \times 10^{-4} \times \cos 0^\circ$

since $\theta = 0^\circ$

$\Phi_f = 4 \times 10^{-2}$ Wb

The magnitude of the induced emf is

$$\varepsilon = N\frac{d\Phi_B}{dt}$$

$$= \frac{40 \times \left(4 \times 10^{-2} - 2 \times 10^{-2}\right)}{0.2} = 4\text{V}$$

UNIT-4(XII-Physics_Vol-1).indd 201 17-02-2020 13:17:40

EXAMPLE 4.5

A straight conducting wire is dropped horizontally from a certain height with its length along east – west direction. Will an emf be induced in it? Justify your answer.

Solution

Yes! An emf will be induced in the wire because it moves perpendicular to the horizontal component of Earth's magnetic field and hence it cuts the magnetic lines of Earth's magnetic filed.

4.1.4 Lenz's law

A German physicist Heinrich Lenz performed a series of experiments on electromagnetic induction and deduced a law to determine the direction of the induced current. This law is known as Lenz's law.

Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.

Faraday discovered that whenever magnetic flux linked with a coil changes, an electric current is induced in the circuit. Here the flux change is the cause while the induction of current is the effect. Lenz's law says that the effect always opposes the cause. Therefore, the induced current would flow in a direction so that it could oppose the flux change.

By incorporating Lenz's law into Faraday's law, the equation (4.2) is rewritten as

$$\varepsilon = -\frac{d(N\Phi_B)}{dt} \qquad (4.3)$$

The negative sign signifies that the direction of induced emf is such that it opposes the change in magnetic flux.

To understand Lenz's law, let us consider two illustrations in which we find the direction of the induced current in a circuit.

Illustration 1

Consider a uniform magnetic field, with its field lines perpendicular to the plane of the paper and pointing inwards. These field lines are represented by crosses (x) as shown in Figure 4.6(a). A rectangular metallic frame *ABCD* is placed in this magnetic field,

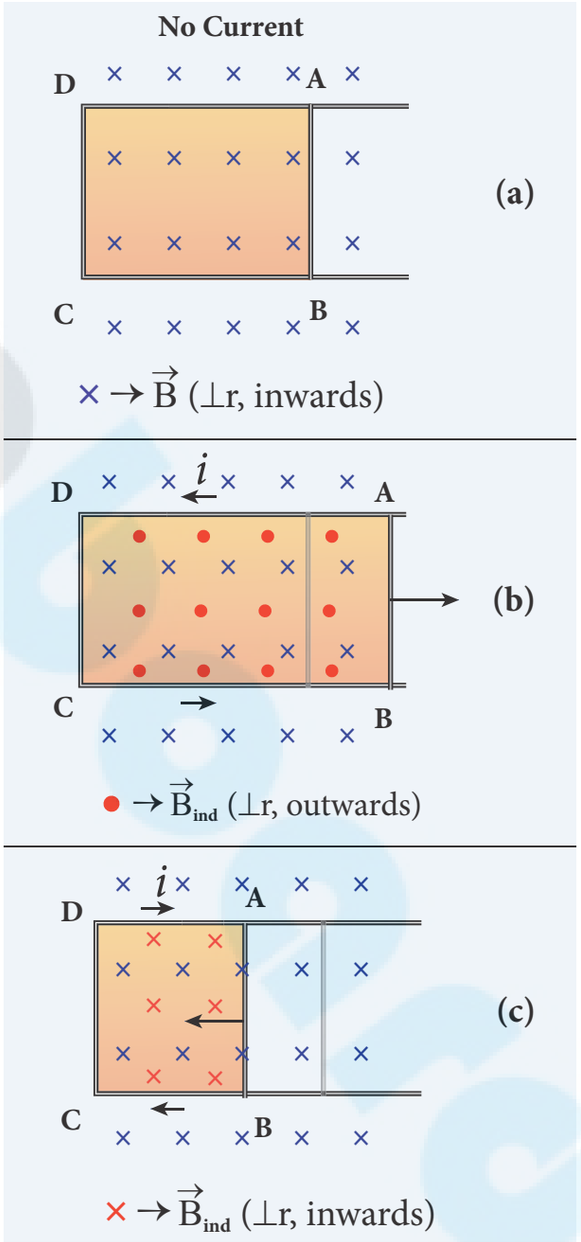


Figure 4.6 First illustration of Lenz's law

UNIT-4(XII-Physics_Vol-1).indd 202 17-02-2020 13:17:41

with its plane perpendicular to the field. The arm *AB* is movable so that it can slide towards right or left.

If the arm AB slides to our right side, the number of field lines (magnetic flux) passing through the frame *ABCD* increases and a current is induced. As suggested by Lenz's law, the induced current opposes this flux increase and it tries to reduce it by producing **another magnetic field pointing outwards i.e., opposite to the existing magnetic field.**

The magnetic lines of this induced field are represented by red-colored circles in the Figure 4.6(b). From the direction of the magnetic field thus produced, the direction of the induced current is found to be anti-clockwise by using right-hand thumb rule.

The leftward motion of arm *AB* decreases magnetic flux. The induced current, this time, produces **a magnetic field in the inward direction (red-colored crosses) i.e., in the direction of the existing magnetic field** (Figure 4.6(c)). Therefore, the flux decrease is opposed by the flow of induced current. From this, it is found that induced current flows in clockwise direction.

Illustration 2

Let us move a bar magnet towards the solenoid, with its north pole pointing the solenoid (Figure 4.7(b)). This motion increases the magnetic flux of the coil which in turn, induces an electric current. Due to the flow of induced current, the coil becomes a magnetic dipole whose two magnetic poles are on either end of the coil.

In this case, the cause producing the induced current is the movement of the magnet. According to Lenz's law, the induced

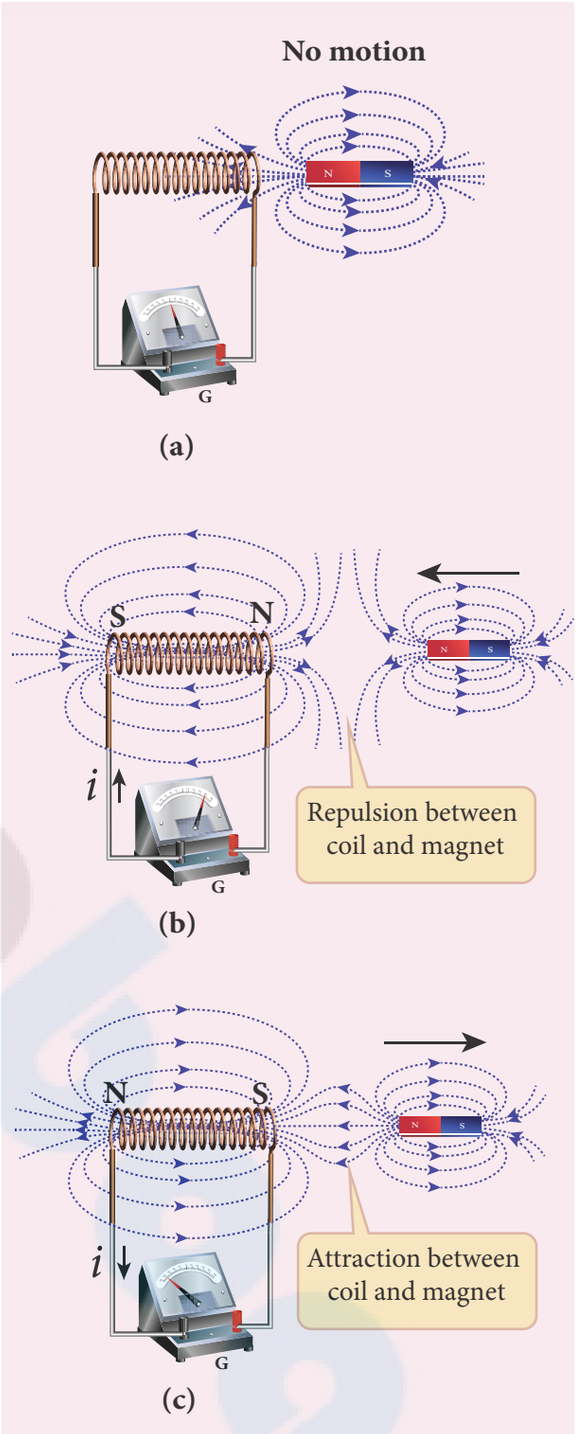


Figure 4.7 Second illustration of Lenz's law

current should flow in such a way that it opposes the movement of the north pole towards coil. It is possible if the end nearer to the magnet becomes north pole (Figure 4.7(b)). Then it repels the north pole of the

UNIT-4(XII-Physics_Vol-1).indd 203 17-02-2020 13:17:41

bar magnet and opposes the movement of the magnet. Once pole ends are known, the direction of the induced current could be found by using right hand thumb rule.

When the bar magnet is withdrawn, the nearer end becomes south pole which attracts north pole of the bar magnet, opposing the receding motion of the magnet (Figure 4.7(c)).

Thus the direction of the induced current can be found from Lenz's law.

Conservation of energy

The truth of Lenz's law can be established on the basis of the law of conservation of energy. The explanation is as follows: According to Lenz's law, when a magnet is moved either towards or away from a coil, the induced current produced opposes its motion. As a result, there will always be a resisting force on the moving magnet. Work has to be done by some external agency to move the magnet against this resisting force. Here the mechanical energy of the moving magnet is converted into the electrical energy which in turn, gets converted into Joule heat in the coil i.e., energy is converted from one form to another.

On the contrary to Lenz's law, let us assume that the induced current *helps* the cause responsible for its production. Now when we push the magnet little bit towards the coil, the induced current helps the movement of the magnet towards the coil. Then the magnet starts moving towards the coil without any expense of energy. This, then, becomes a perpetual motion machine. In practice, no such machine is possible. Therefore, the assumption that the induced current *helps* the cause is wrong. Thus Lenz's law is an excellent example of conservation of energy.

4.1.5 Fleming's right hand rule

When a conductor moves in a magnetic field, the direction of motion of the conductor, the field and the induced current are given by Fleming's right hand rule and is as follows:

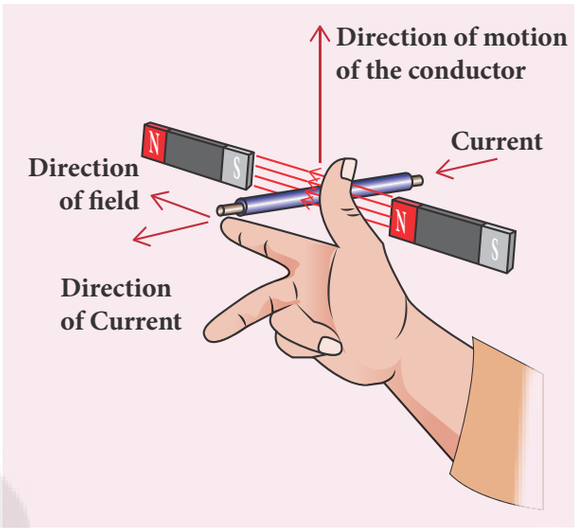


Figure 4.8 Fleming's right hand rule

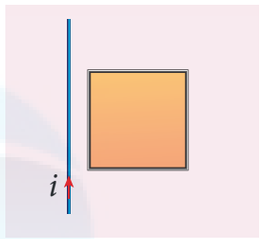
The thumb, index finger and middle finger of right hand are stretched out in mutually perpendicular directions (as shown in Figure 4.8). If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.

Fleming's right hand rule is also known as generator rule.

EXAMPLE 4.6

If the current i flowing in the straight conducting wire as shown in the figure decreases, find out the direction of induced current in the metallic square loop placed near it.

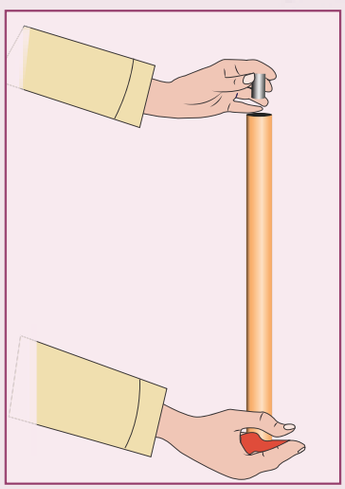
UNIT-4(XII-Physics_Vol-1).indd 204 17-02-2020 13:17:41



Solution

From right hand rule, the magnetic field by the straight wire is directed into the plane of the square loop perpendicularly and its magnetic flux is decreasing. The decrease in flux is opposed by the current induced in the loop by producing a magnetic field in the same direction as the magnetic field of the wire. Again from right hand rule, for this inward magnetic field, the direction of the induced current in the loop is clockwise.

ACTIVITY

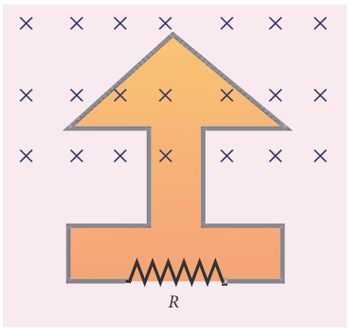


Demonstration of Lenz's law

Take a narrow copper pipe and a strongly magnetized button magnet as shown in figure. Keep the copper pipe vertical and drop the magnet into the pipe. Watch the motion of the magnet and note that magnet has become slower than its free fall. The reason is that an electric current generated by a moving magnet will always *oppose* the original motion of the magnet that produced the current.

EXAMPLE 4.7

The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = \left(2t^3 + 3t^2 + 8t + 5\right)\text{mWb}$, what is the magnitude of the induced emf in the loop when $t = 3$ s? Find out the direction of current through the circuit.



Solution

$\Phi_B = \left(2t^3 + 3t^2 + 8t + 5\right)\text{mWb}$; $N = 1$; $t = 3$ s

(i) $$\varepsilon = \frac{d\left(N\Phi_B\right)}{dt}$$

$$= \frac{d}{dt}\left(2t^3 + 3t^2 + 8t + 5\right)\times 10^{-3}$$

$$= \left(6t^2 + 6t + 8\right)\times 10^{-3}$$

At t = 3 s,

$$\varepsilon = \left[\left(6\times 9\right) + \left(6\times 3\right) + 8\right]\times 10^{-3}$$

$$= 80\times 10^{-3}\,\text{V} = 80\,\text{mV}$$

(ii) As time passes, the magnetic flux linked with the loop increases. According to Lenz's law, the direction of the induced current should be in a way so as to oppose the flux increase. So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards. Therefore, the induced current flows in anti-clockwise direction.

UNIT-4(XII-Physics_Vol-1).indd 205 17-02-2020 13:17:43

4.1.6 Motional emf from Lorentz force

Consider a straight conducting rod AB of length l in a uniform magnetic field $\vec{B}$ which is directed perpendicularly into the plane of the paper as shown in Figure 4.9(a). The length of the rod is normal to the magnetic field. Let the rod move with a constant velocity $\vec{v}$ towards right side.

When the rod moves, the free electrons present in it also move with same velocity $\vec{v}$ in $\vec{B}$. As a result, the Lorentz force acts on free electrons in the direction from B to A and is given by the relation

$$\vec{F_B} = -e\left(\vec{v} \times \vec{B}\right) \qquad (4.4)$$

The action of this Lorentz force is to accumulate the free electrons at the end A. This accumulation of free electrons produces a potential difference across the rod which in turn establishes an electric field $\vec{E}$ directed along BA (Figure 4.9(b)). Due to the electric field $\vec{E}$, the coulomb force starts acting on the free electrons along AB and is given by

$$\vec{F_E} = -e\vec{E} \qquad (4.5)$$

The magnitude of the electric field $\vec{E}$ keeps on increasing as long as accumulation of electrons at the end A continues. The force $\vec{F}_E$ also increases until equilibrium is reached. At equilibrium, the magnetic Lorentz force $\vec{F}_B$ and the coulomb force $\vec{F}_E$ balance each other and no further accumulation of free electrons at the end A takes place. i.e.,

$$\left|\vec{F_B}\right| = \left|\vec{F_E}\right|$$

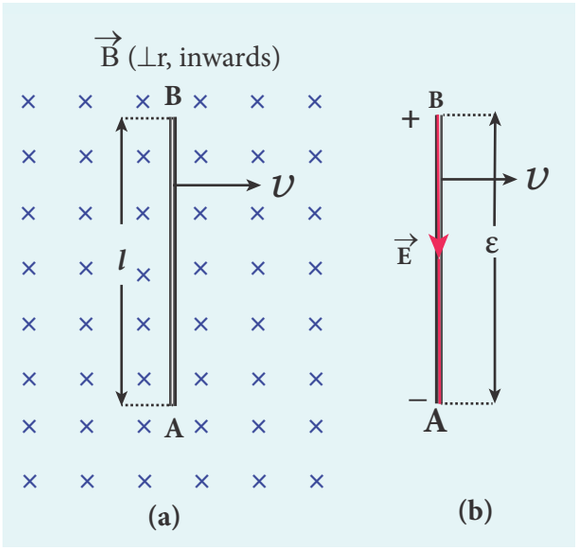


Figure 4.9 Motional emf from Lorentz force

$$\left|-e\ (\vec{v} \times \vec{B})\right| = \left|-e\vec{E}\right|$$

$$vB\sin 90° = E$$

$$vB = E \qquad (4.6)$$

The potential difference between two ends of the rod is

$$V = El$$

$$V = vBl$$

Thus the Lorentz force on the free electrons is responsible to maintain this potential difference and hence produces an emf

$$\varepsilon = Blv \qquad (4.7)$$

As this emf is produced due to the movement of the rod, it is often called as **motional emf**. If the ends A and B are connected by an external circuit of total resistance R, then current $i = \frac{\varepsilon}{R} = \frac{Blv}{R}$ flows in it. The direction of the current is found from right-hand thumb rule.

UNIT-4(XII-Physics_Vol-1).indd 206 17-02-2020 13:17:48

EXAMPLE 4.8

A conducting rod of length 0.5 m falls freely from the top of a building of height 7.2 m at a place in Chennai where the horizontal component of Earth's magnetic field is 4.04×10^{-5} T. If the length of the rod is perpendicular to Earth's horizontal magnetic field, find the emf induced across the conductor when the rod is about to touch the ground. (Assume that the rod falls down with constant acceleration of 10 m s^{-2})

Solution

$l = 0.5$ m; $h = 7.2$ m; $u = 0$ m s^{-1};
$g = 10$ m s^{-2}; $B_H = 4.04 \times 10^{-5}$ T

The final velocity of the rod is

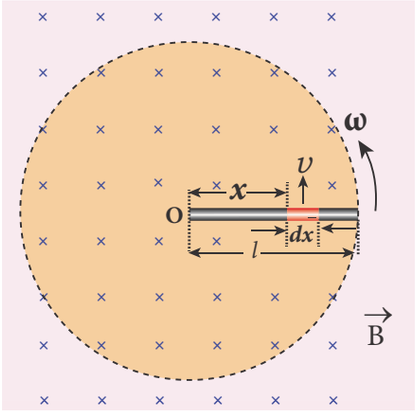
$$v^2 = u^2 + 2gh = 0 + (2 \times 10 \times 7.2) = 144$$
$$v = 12 \text{ m s}^{-1}$$

The magnitude of the induced emf when the rod is about to touch the ground is

$$\varepsilon = B_H l v = 4.04 \times 10^{-5} \times 0.5 \times 12$$
$$= 242.4 \ \mu\text{V}$$

EXAMPLE 4.9

A copper rod of length l rotates about one of its ends with an angular velocity ω in a magnetic field B as shown in the figure. The plane of rotation is perpendicular to the field. Find the emf induced between the two ends of the rod.



Solution

Consider a small element of length dx at a distance x from the centre of the circle described by the rod. As this element moves perpendicular to the field with a linear velocity $v = x\omega$, the emf developed in the element dx is

$$d\varepsilon = Bv\,dx \quad = B(x\omega)dx$$

This rod is made up of many such elements, moving perpendicular to the field. The emf developed across two ends is

$$\varepsilon = \int d\varepsilon = \int_0^l B\omega x\,dx \quad = B\omega\left[\frac{x^2}{2}\right]_0^l$$
$$\varepsilon = \frac{1}{2}B\omega l^2$$

4.2 EDDY CURRENTS

According to Faraday's law of electromagnetic induction, an emf is induced in a conductor when the magnetic flux passing through it changes. However, the conductor need not be in the form of a wire or coil.

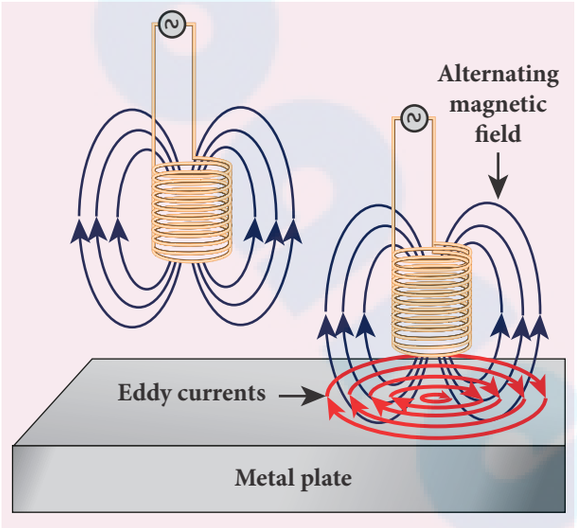


Figure 4.10 Eddy currents

UNIT-4(XII-Physics_Vol-1).indd 207 17-02-2020 13:17:51

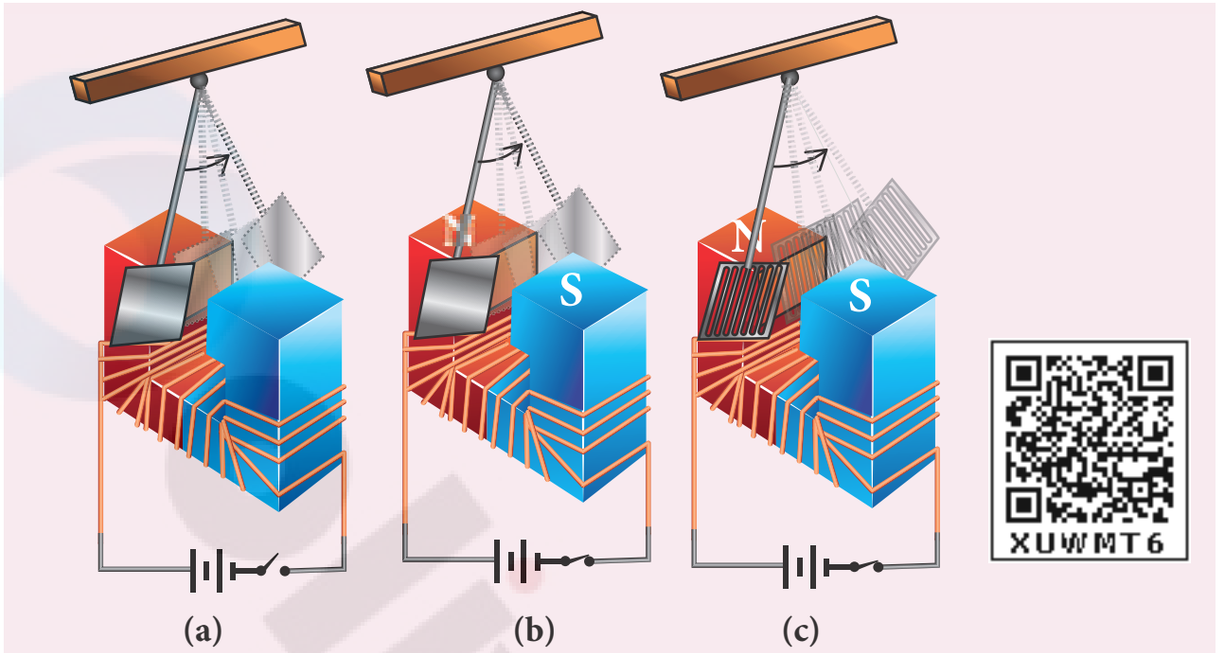


Figure 4.11 Demonstration of eddy currents

Even for a conductor in the form of a sheet or plate, an emf is induced when magnetic flux linked with it changes. But the difference is that there is no definite loop or path for induced current to flow away. As a result, the induced currents flow in concentric circular paths (Figure 4.10). As these electric currents resemble eddies of water, these are known as **Eddy currents**. They are also called **Foucault currents**.

Demonstration

Here is a simple demonstration for the production of eddy currents. Consider a pendulum that can be made to oscillate between the poles of a powerful electromagnet (Figure 4.11(a)).

First the electromagnet is switched off, the pendulum is slightly displaced and released. It begins to oscillate and it executes a large number of oscillations before stopping. The air friction is the only damping force.

When the electromagnet is switched on and the disc of the pendulum is made to oscillate, eddy currents are produced in it which will oppose the oscillation. A heavy damping force of eddy currents will bring the pendulum to rest within a few oscillations (Figure 4.11(b)).

However if some slots are cut in the disc (Figure 4.11(c)), the eddy currents are reduced. The pendulum now will execute several oscillations before coming to rest. This clearly demonstrates the production of eddy current in the disc of the pendulum.

Drawbacks of Eddy currents

When eddy currents flow in the conductor, a large amount of energy is dissipated in the form of heat. The energy loss due to the flow of eddy current is inevitable but it can be reduced to a greater extent with suitable measures.

The design of transformer core and electric motor armature is crucial in order to minimise the eddy current loss. To reduce these losses, the core of the transformer is made up of thin laminas insulated from one another (Figure 4.12 (a)) while for electric motor the winding is made up of a group

UNIT-4(XII-Physics_Vol-1).indd 208 17-02-2020 13:17:52

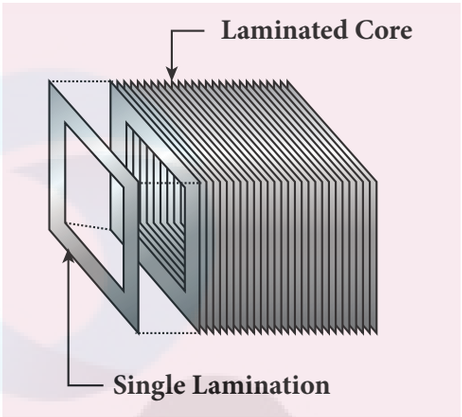


Figure 4.12 (a) Insulated laminas of the core of a transformer

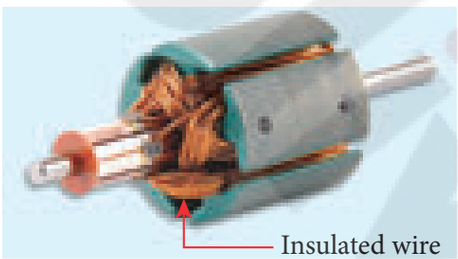


Figure 4.12 (b) Insulated winding of an electric motor

of wires insulated from one another (Figure 4.12 (b)). The insulation used does not allow huge eddy currents to flow and hence losses are minimized.

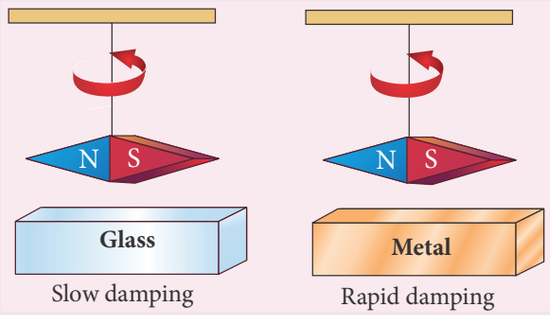
Example

A spherical stone and a spherical metallic ball of same size and mass are dropped from the same height. Which one, a stone or a metal ball, will reach the Earth's surface first? Justify your answer. Assume that there is no air friction.

Answer

The stone will reach the Earth's surface earlier than the metal ball. The reason is that when the metal ball falls through the magnetic field of Earth, the eddy currents are produced in it which opposes its motion. But in the case of stone, no eddy currents are produced and it falls freely.

ACTIVITY



Make a pendulum with a strong magnet suspended at the lower end of the suspension wire as shown in the first figure. Make it oscillate with a glass plate below it and note the time it takes to come to rest.

Next just place a metallic plate below the oscillating magnet as shown in the second figure and again note the time it takes to stop.

In the second case, the magnet stops soon because eddy currents are produced in the plate which opposes the oscillation of the magnet.

Application of eddy currents

Though the production of eddy current is undesirable in some cases, it is useful in some other cases. A few of them are

i. Induction stove
ii. Eddy current brake
iii. Eddy current testing
iv. Electromagnetic damping

i. Induction stove

Induction stove is used to cook the food quickly and safely with less energy consumption. Below the cooking zone, there is a tightly wound coil of insulated wire. The cooking pan made of suitable material, is placed over the cooking zone. When the stove is switched on, an alternating current flowing in the coil produces high frequency alternating magnetic field which induces very strong eddy

UNIT-4(XII-Physics_Vol-1).indd 209 17-02-2020 13:17:53

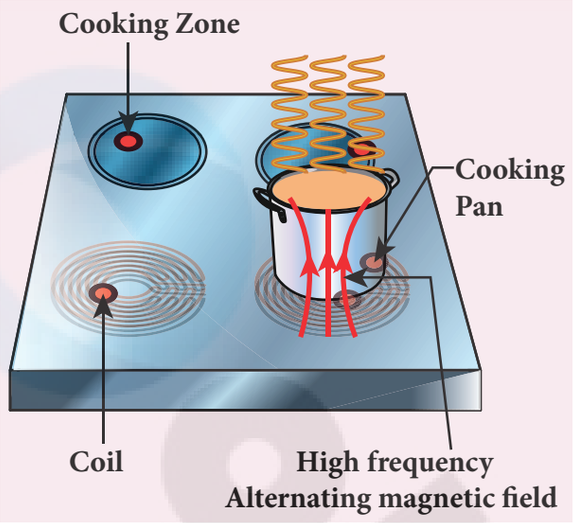


Figure 4.13 Induction stove

currents in the cooking pan. The eddy currents in the pan produce so much of heat due to Joule heating which is used to cook the food (Figure 4.13).

Note: The frequency of the domestic AC supply is increased from 50–60 Hz to around 20–40 KHz before giving it to the coil in order to produce high frequency alternating magnetic field.

ii. Eddy current brake

This eddy current braking system is generally used in high speed trains and roller coasters. Strong electromagnets are fixed just above the rails. To stop the train, electromagnets are switched on. The magnetic field of these magnets induces eddy currents in the rails which oppose or resist the movement of the train. This is Eddy current linear brake (Figure 4.14(a)).

In some cases, the circular disc, connected to the wheel of the train through a common shaft, is made to rotate in between the poles of an electromagnet. When there is a relative motion between the disc and the magnet, eddy currents are induced in the disc which stop the train. This is Eddy current circular brake (Figure 4.14(b))

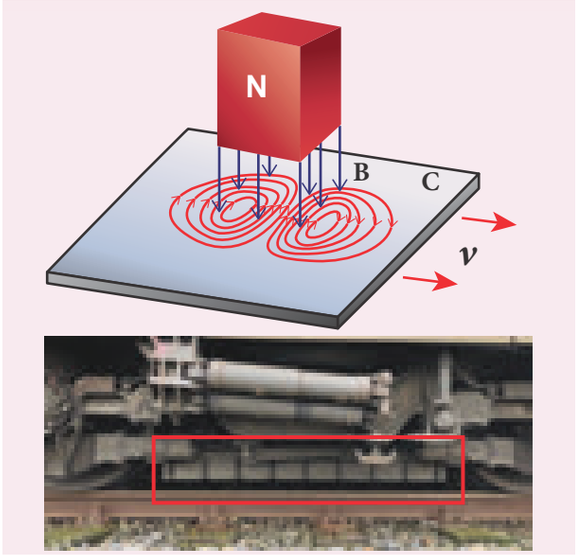


Figure 4.14(a) Linear Eddy current brake

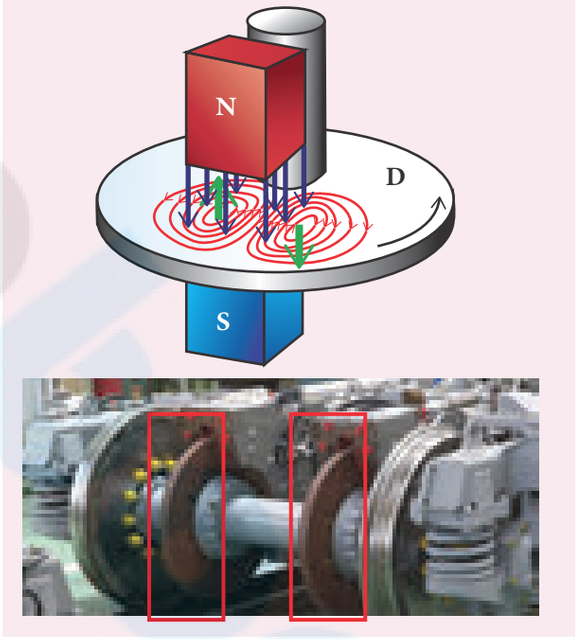


Figure 4.14(b) Circular Eddy current brake

iii. Eddy current testing

It is one of the simple non–destructive testing methods to find defects like surface cracks, air bubbles present in a specimen. A coil of insulated wire is given an alternating electric current so that it produces an alternating magnetic field. When this coil is brought near the test surface, eddy current is induced in the test surface. The presence

UNIT-4(XII-Physics_Vol-1).indd 210 17-02-2020 13:17:55

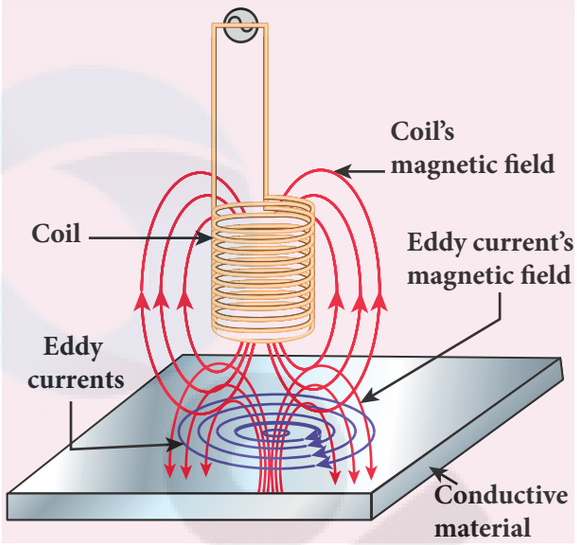


Figure 4.15 Eddy current testing

of defects causes the change in phase and amplitude of the eddy current that can be detected by some other means.In this way, the defects present in the specimen are identified (Figure 4.15).

iv. Electro magnetic damping

The armature of the galvanometer coil is wound on a soft iron cylinder. Once the armature is deflected, the relative motion between the soft iron cylinder and the radial magnetic field induces eddy current in the cylinder (Figure 4.16). The damping force due to the flow of eddy current brings

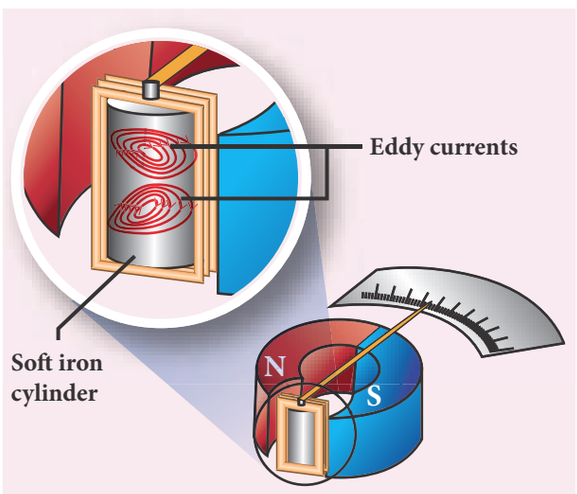


Figure 4.16 Electromagnetic damping

the armature to rest immediately and then galvanometer shows a steady deflection. This is called electromagnetic damping.

4.3 SELF–INDUCTION

4.3.1 Introduction

Inductor is a device used to store energy in a magnetic field when an electric current flows through it. The typical examples are coils, solenoids and toroids shown in Figure 4.17.

Inductance is the property of inductors to generate emf due to the change in current flowing through that circuit (self-induction) or a change in current through a neighbouring circuit with which it is magnetically linked (mutual induction). We will study about self-induction and mutual induction in the next sections.

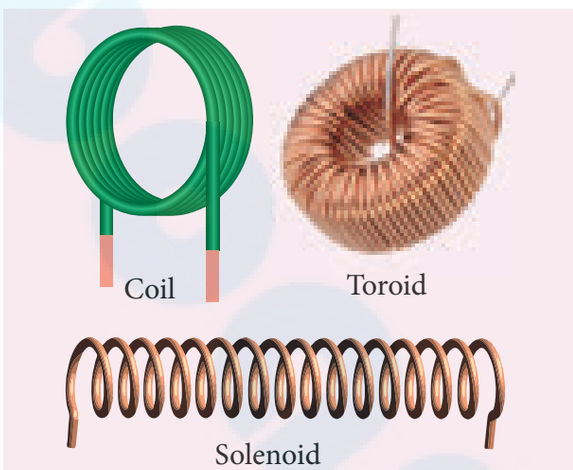


Figure 4.17 Examples for inductor

Self-induction

An electric current flowing through a coil will set up a magnetic field around it. Therefore, the magnetic flux of the magnetic field is linked with that coil itself. If this flux

UNIT-4(XII-Physics_Vol-1).indd 211 17-02-2020 13:17:56

is changed by changing the current, an emf is induced in that same coil (Figure 4.18). This phenomenon is known as **self-induction**. The emf induced is called self-induced emf.

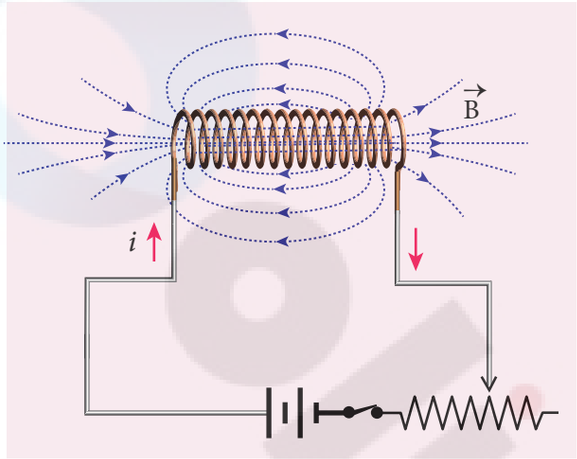


Figure 4.18 Self-Induction

Let Φ_B be the magnetic flux linked with each turn of the coil of N turns, then the total flux linked with the coil $N\Phi_B$ (flux linkage) is proportional to the current i in the coil.

$$N\Phi_B \propto i$$

$$N\Phi_B = Li \qquad (4.8)$$

$$\text{(or) } L = \frac{N\Phi_B}{i}$$

The constant of proportionality L is called self-inductance or coefficient of self-induction of the coil.

If $i = 1\text{A}$, then $L = N\Phi_B$. **Self-inductance or simply inductance of a coil is defined as the flux linkage with the coil when 1A current flows through it.**

When the current i changes with time, an emf is induced in it. From Faraday's law of electromagnetic induction, this self-induced emf in the coil is given by

$$\varepsilon = -\frac{d(N\Phi_B)}{dt}$$

$$= -\frac{d(Li)}{dt} \qquad \text{(using equation 4.8)}$$

$$\therefore \varepsilon = -L\frac{di}{dt} \qquad (4.9)$$

$$\text{(or) } L = \frac{-\varepsilon}{di/dt}$$

The negative sign in the above equation implies that the self-induced emf always opposes the change in current with respect to time. If $di/dt = 1\ \text{A s}^{-1}$, then $L = -\varepsilon$. **Inductance of a coil is also defined as the opposing emf induced in the coil when the rate of change of current through the coil is 1 A s^{-1}.**

Unit of inductance

Inductance is a scalar and its unit is Wb A^{-1} or V s A^{-1}. It is also measured in henry (H).

$$1\ \text{H} = 1\ \text{Wb A}^{-1} = 1\ \text{V s A}^{-1}$$

The dimensional formula of inductance is $M\,L^2\,T^{-2}A^{-2}$.

If i = 1 A and $N\Phi_B = 1$ Wb turns, then $L = 1\text{H}$.

Therefore, **the inductance of the coil is said to be one henry if a current of 1 *A* produces unit flux linkage in the coil.**

If $di/dt = 1\text{A s}^{-1}$ and $\varepsilon = -1\ \text{V}$, then $L = 1\text{H}$.

Therefore, **the inductance of the coil is one henry if a current changing at the rate of 1 A s^{-1} induces an opposing emf of 1 V in it.**

Physical significance of inductance

We have learnt about inertia in XI standard. In translational motion, mass is a measure of linear inertia; in the same way, for rotational motion, moment of inertia is a measure of rotational inertia (Refer sections 3.2.1 and 5.4 of XI physics text book). Generally, inertia means opposition to change its state.

UNIT-4(XII-Physics_Vol-1).indd 212 17-02-2020 13:18:00

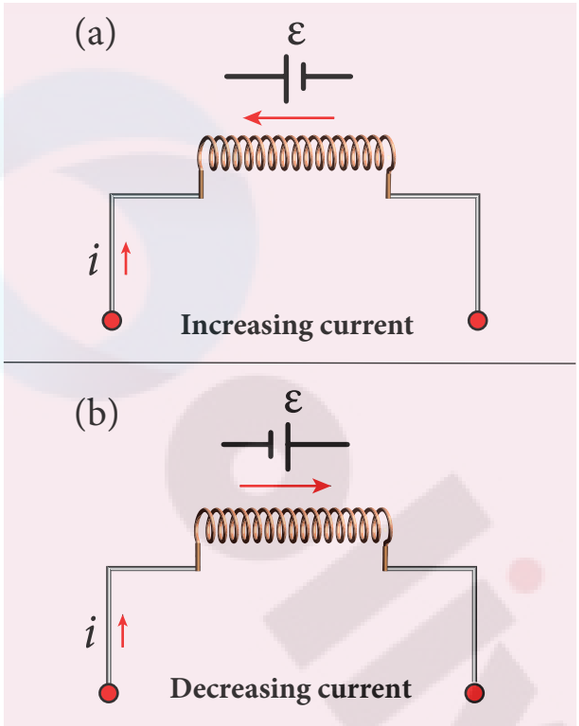


Figure 4.19 Induced emf ε opposes the changing current i

The inductance plays the same role in a circuit as mass and moment of inertia play in mechanical motion. When a circuit is switched on, the increasing current induces an emf which opposes the growth of current in a circuit (Figure 4.19(a)). Likewise, when circuit is broken, the decreasing current induces an emf in the reverse direction. This emf now opposes the decay of current (Figure 4.19(b)).

Thus, inductance of the coil opposes any change in current and tries to maintain the original state.

4.3.2 Self-inductance of a long solenoid

Consider a long solenoid of length l and cross-sectional area A. Let n be the number of turns per unit length (or turn density) of the solenoid. When an electric current i is passed through the solenoid, a magnetic

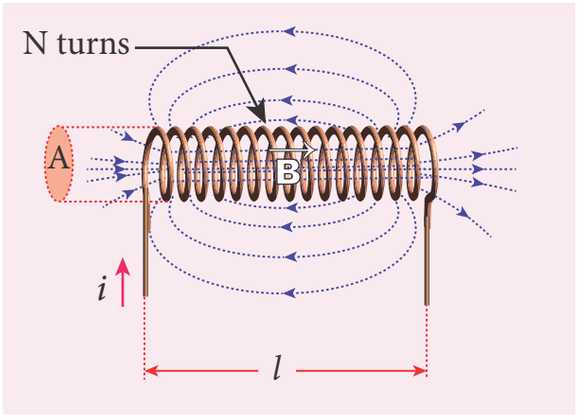


Figure 4.20 Self-inductance of a long solenoid

field produced inside is almost uniform and is directed along the axis of the solenoid as shown in Figure 4.20. The magnetic field at any point inside the solenoid is given by (Refer section 3.9.3)

$$B = \mu_{\circ} ni$$

As this magnetic field passes through the solenoid, the windings of the solenoid are linked by the field lines. The magnetic flux passing through each turn is

$$\Phi_B = \int_A \vec{B}.d\vec{A} = BA\cos\theta$$
$$= BA \qquad \text{since } \theta = 0°$$
$$= (\mu_0 ni)A$$

The total magnetic flux linked or flux linkage of the solenoid with N turns (the total number of turns N is given by $N = n\,l$) is

$$N\Phi_B = (nl)(\mu_0 ni)\,\mathrm{A}$$
$$N\Phi_B = (\mu_{\circ} n^2 Al)\,i$$

We know that

$$N\Phi_B = Li$$

UNIT-4(XII-Physics_Vol-1).indd 213 17-02-2020 13:18:02

Comparing the above equations, we get

$$L = \mu_{\circ} n^2 Al \qquad (4.10)$$

From the above equation, it is clear that inductance depends on the geometry of the solenoid (turn density n, cross-sectional area A, length l) and the medium present inside the solenoid. If the solenoid is filled with a dielectric medium of relative permeability μ_r, then

$$L = \mu n^2 Al \text{ or } \quad L = \mu_{\circ}\mu_r n^2 Al$$

Energy stored in an inductor

Whenever a current is established in the circuit, the inductance opposes the growth of the current. In order to establish a current in the circuit, work is done against this opposition by some external agency. This work done is stored as magnetic potential energy.

Let us assume that electrical resistance of the inductor is negligible and inductor effect alone is considered. The induced emf ε at any instant t is

$$\varepsilon = -L\frac{di}{dt}$$

Let dW be work done in moving a charge dq in a time dt against the opposition, then

$$dW = -\varepsilon dq$$
$$= -\varepsilon i dt \qquad \because dq = idt$$

Substituting for ε from equation (4.16),

$$= -\left(-L\frac{di}{dt}\right) i dt$$
$$dW = Lidi$$

Total work done in establishing the current i is

$$W = \int dW = \int_0^i Lidi = L\left[\frac{i^2}{2}\right]_o^i$$
$$W = \frac{1}{2}Li^2$$

This work done is stored as magnetic potential energy.

$$\therefore U_B = \frac{1}{2}Li^2 \qquad (4.11)$$

The energy density is the energy stored per unit volume of the space and is given by

$$u_B = \frac{U_B}{Al} \qquad \because \text{Volume of the solenoid} = Al$$
$$u_B = \frac{Li^2}{2Al} = \frac{\left(\mu_0 n^2 Al\right) i^2}{2Al} \qquad \because L = \mu_0 n^2 Al$$
$$= \frac{\mu_0 n^2 i^2}{2}$$
$$u_B = \frac{B^2}{2\mu_0} \qquad \because B = \mu_0 ni$$

EXAMPLE 4.10

A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.

Solution

N = 500 turns; $\mu_r = 800$;
l = 40 cm = 0.4 m; r = 3 cm = 0.03 m;
di = 3 – 0 = 3 A; dt = 0.4 s

Self inductance,

$$L = \mu n^2 Al \quad \left(\because \mu = \mu_{\circ}\mu_r; A = \pi r^2; n = \frac{N}{l}\right)$$
$$= \frac{\mu_{\circ}\mu_r N^2 \pi r^2}{l}$$
$$= \frac{4\times 3.14\times 10^{-7}\times 800\times 500^2\times 3.14\times\left(3\times 10^{-2}\right)^2}{0.4}$$
$$L = 1.77 \text{ H}$$

UNIT-4(XII-Physics_Vol-1).indd 214 17-02-2020 13:18:06

Induced emf $\varepsilon = -L\dfrac{di}{dt}$

$$= -\frac{1.77\times 3}{0.4}$$

$$\varepsilon = -13.275 \text{ V}$$

EXAMPLE 4.11

The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.

Solution

$L_{air} = 4.8\times 10^{-3}\text{H}$

$L_{iron} = 1.8\text{H}$

$L_{air} = \mu_o n^2 Al = 4.8\times 10^{-3}\text{H}$

$L_{iron} = \mu n^2 Al = \mu_o \mu_r n^2 Al = 1.8\text{H}$

$\therefore \mu_r = \dfrac{L_{iron}}{L_{air}} = \dfrac{1.8}{4.8\times 10^{-3}} = 375$

4.3.3 Mutual induction

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as **mutual induction** and the emf induced is called mutually induced emf.

Consider two coils which are placed close to each other. If an electric current i_1 is sent through coil 1, the magnetic field produced by it is also linked with coil 2 as shown in Figure 4.21(a).

If Φ_{21} is the magnetic flux linked with each turn of the coil 2 of N_2 turns due to the current in coil 1, then the total flux linked with coil 2 $(N_2\Phi_{21})$ is proportional to the current i_1 in the coil 1.

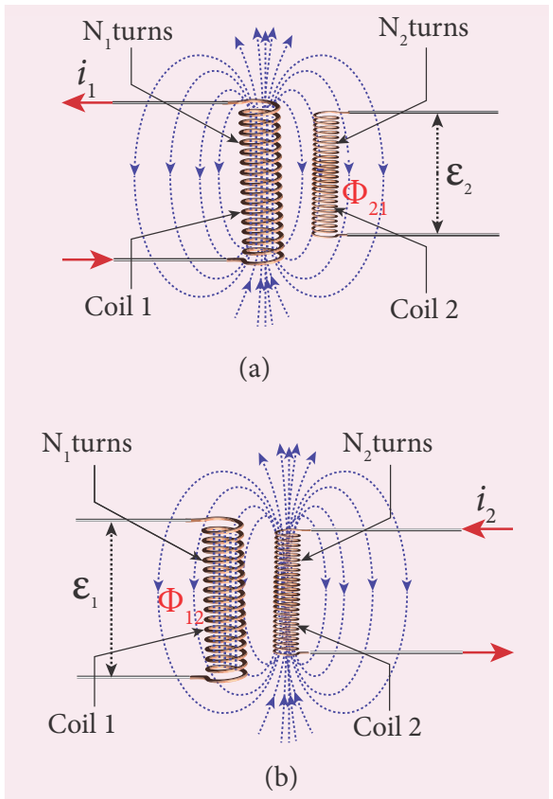


Figure 4.21 Mutual induction

$N_2\Phi_{21} \propto i_1$

$N_2\Phi_{21} = M_{21}\, i_1$ (4.12)

(or) $M_{21} = \dfrac{N_2\Phi_{21}}{i_1}$

The constant of proportionality M_{21} is the mutual inductance or coefficient of mutual induction of the coil 2 with respect to coil 1.

If $i_1 = 1A$, then $M_{21} = N_2\Phi_{21}$. Therefore, **the mutual inductance M_{21} is defined as the flux linkage with the coil 2 when 1A current flows through coil 1.**

When the current i_1 changes with time, an emf ε_2 is induced in coil 2. From Faraday's law of electromagnetic induction, this mutually induced emf ε_2 is given by

$$\varepsilon_2 = -\frac{d(N_2\Phi_{21})}{dt} = -\frac{d(M_{21}i_1)}{dt}$$

UNIT-4(XII-Physics_Vol-1).indd 215 17-02-2020 13:18:11

$$\varepsilon_2 = -M_{21}\frac{di_1}{dt}$$

$$\text{(or)}\ M_{21} = \frac{-\varepsilon_2}{di_1/dt}$$

The negative sign in the above equation shows that the mutually induced emf always opposes the change in current i_1 with respect to time. If $di_1/dt = 1\text{A s}^{-1}$, then $M_{21} = -\varepsilon_2$. **Mutual inductance M_{21} is also defined as the opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is 1As^{-1}.**

Similarly, if an electric current i_2 through coil 2 changes with time, then emf ε_1 is induced in coil 1. Therefore,

$$M_{12} = \frac{N_1\Phi_{12}}{i_2} \quad \text{and} \quad M_{12} = \frac{-\varepsilon_1}{di_2/dt}$$

where M_{12} is the mutual inductance of the coil 1 with respect to coil 2. It can be shown that for a given pair of coils, the mutual inductance is same. i.e.,

$$M_{21} = M_{12} = M$$

In general, the mutual induction between two coils depends on size, shape, the number of turns of the coils, their relative orientation and permeability of the medium.

Unit of mutual-inductance

The unit of mutual inductance is also henry (H).

If $i_1 = 1\text{A}$ and $N_2\Phi_{21} = 1$ Wb turns, then $M = 1\text{H}$.

Therefore, **the mutual inductance between two neighbouring coils is said to be one henry if a current of 1A in one coil produces unit flux linkage in neighbouring coil.**

If $di_1/dt = 1\text{As}^{-1}$ and $\varepsilon_2 = -1\text{ V}$, then $M = 1\text{H}$.

Therefore, **the mutual inductance between two neighbouring coils is one henry if a current changing at the rate of 1A s^{-1} in one coil induces an opposing emf of 1V in neighbouring coil.**

4.3.4 Mutual inductance between two long co-axial solenoids

Consider two long co-axial solenoids of same length l. The length of these solenoids is large when compared to their radii so that the magnetic field produced inside the solenoids is uniform and the fringing effect at the ends may be ignored. Let A_1 and A_2 be the area of cross section of the solenoids with A_1 being greater than A_2 as shown in Figure 4.22. The turn density of these solenoids are n_1 and n_2 respectively.

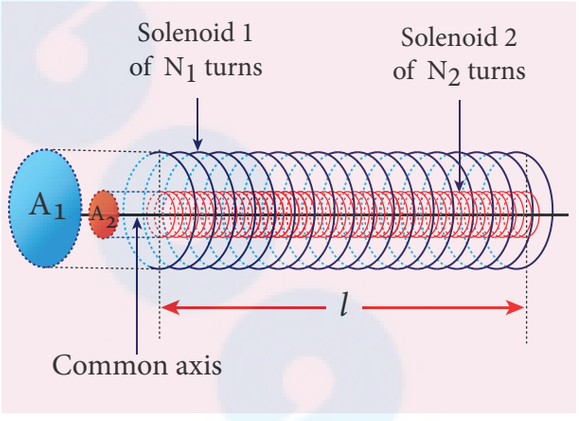


Figure 4.22 Mutual inductance of two long co-axial solenoids

Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_\circ\, n_1\, i_1$$

UNIT-4(XII-Physics_Vol-1).indd 216 17-02-2020 13:18:15

As the field lines of $\overrightarrow{B_1}$ are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to current in solenoid 1 and is given by

$$\Phi_{21} = \int_{A_2} \overrightarrow{B_1}.d\overrightarrow{A} = B_1 A_2 \quad \text{since } \theta = 0^\circ$$
$$= (\mu_\circ n_1 i_1) A_2$$

The flux linkage with solenoid 2 with total turns N_2 is

$$N_2\Phi_{21} = (n_2 l)(\mu_\circ n_1 i_1) A_2 \quad \text{since } N_2 = n_2 l$$
$$N_2\Phi_{21} = (\mu_\circ n_1 n_2 A_2 l) i_1$$

We know that $N_2\Phi_{21} = M_{21} i_1$. Comparing the above equations, we get

$$M_{21} = \mu_\circ n_1 n_2 A_2 l \qquad (4.13)$$

This gives the expression for mutual inductance M_{21} of the solenoid 2 with respect to solenoid 1. Similarly, we can find mutual inductance M_{12} of solenoid 1 with respect to solenoid 2 as given below.

The magnetic field produced by the solenoid 2 when carrying a current i_2 is

$$B_2 = \mu_\circ n_2 i_2$$

This magnetic field B_2 is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area A_2 is the effective area over which the magnetic field B_2 is present; not area A_1. Then the magnetic flux Φ_{12} linked with each turn of solenoid 1 due to current in solenoid 2 is

$$\Phi_{12} = \int_{A_2} \vec{B}_2 \cdot d\vec{A} = B_2 A_2 = (\mu_\circ n_2 i_2) A_2$$

The flux linkage of solenoid 1 with total turns N_1 is

$$N_1\Phi_{12} = (n_1 l)(\mu_\circ n_2 i_2) A_2 \quad \text{since } N_1 = n_1 l$$
$$N_1\Phi_{12} = (\mu_\circ n_1 n_2 A_2 l) i_2$$

We know that $N_1\Phi_{12} = M_{12} i_2$. Comparing the above equations, we get

$$\therefore M_{12} = \mu_\circ n_1 n_2 A_2 l \qquad (4.14)$$

From equation (4.22) and (4.23), we can write

$$M_{12} = M_{21} = M \qquad (4.15)$$

In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu_\circ n_1 n_2 A_2 l \qquad (4.16)$$

If a dielectric medium of relative permeability μ_r is present inside the solenoids, then

$$M = \mu n_1 n_2 A_2 l \text{ (or)}$$
$$M = \mu_\circ \mu_r n_1 n_2 A_2 l$$

EXAMPLE 4.12

The current flowing in the first coil changes from 2 A to 10 A in 0.4 s. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the magnitude of induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in 0.03 s. Consider only the magnitude of induced emf.

Solution

Case (i):

$di_1 = 10 - 2 = 8$ A; $dt = 0.4$ s;

$\varepsilon_2 = 60 \times 10^{-3}$ V

Case (ii):

$di_1 = 16 - 4 = 12$ A; $dt = 0.03$ s

(i) Mutual inductance between the coils.

$$M = \frac{\varepsilon_2}{di_1/dt} = \frac{60 \times 10^{-3} \times 0.4}{8}$$

$$M = 3 \times 10^{-3} \text{ H}$$

UNIT-4(XII-Physics_Vol-1).indd 217 17-02-2020 13:18:21

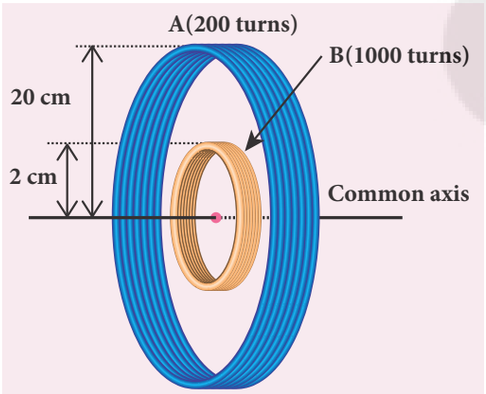
(ii) Induced emf in the second coil due to the rate of change of current in the first coil is

$$\varepsilon_2 = M\frac{di_1}{dt} = \frac{3\times10^{-3}\times12}{0.03}$$

$$\varepsilon_2 = 1.2\,\text{V}$$

EXAMPLE 4.13

Consider two coplanar, co-axial circular coils *A* and *B* as shown in figure. The radius of coil *A* is 20 cm while that of coil *B* is 2 cm. The number of turns in coils *A* and *B* are 200 and 1000 respectively. Calculate the mutual inductance between the coils. If the current in coil *A* changes from 2 A to 6 A in 0.04 s, determine the induced emf in coil *B* and the rate of change of flux through the coil *B* at that instant.



Solution

$N_A = 200$ turns; $N_B = 1000$ turns;
$r_A = 20\times10^{-2}$ m; $r_B = 2\times10^{-2}$ m;
$dt = 0.04$ s; $di_A = 6-2 = 4$ A

Let i_A be the current flowing in coil *A*, then the magnetic field B_A at the centre of the circular coil *A* is

$$B_A = \frac{\mu_\circ N_A i_A}{2r_A} = \frac{4\pi\times10^{-7}N_A i_A}{2r_A}$$
$$= \frac{10^{-7}\times2\times3.14\times200}{20\times10^{-2}}\times i_A$$
$$= 6.28\times10^{-4} i_A \ \text{Wb m}^{-2}$$

The magnetic flux linkage with coil *B* is

$$N_B\Phi_B = N_B B_A A_B$$
$$= 1000\times6.28\times10^{-4}\times i_A\times3.14\times\left(2\times10^{-2}\right)^2$$
$$= 7.89\times10^{-4} i_A \ \text{Wb turns}$$

The mutual inductance between the coils

$$M = \frac{N_B\Phi_B}{i_A} = 7.89\times10^{-4}\,\text{H}$$

Induced emf in coil *B* is

$$\varepsilon_B = -M\frac{di_A}{dt}$$
$$\varepsilon_B = \frac{7.89\times10^{-4}\times(6-2)}{0.04}\ \text{(magnitude only)}$$
$$\varepsilon_B = 78.9\,\text{mV}$$

The rate of change of magnetic flux of coil *B* is

$$\frac{d(N_B\Phi_B)}{dt} = \varepsilon_B = 78.9\ \text{m Wb s}^{-1}$$

4.4 METHODS OF PRODUCING INDUCED EMF

4.4.1 Introduction

Electromotive force is the characteristic of any energy source capable of driving electric charge around a circuit. We have already learnt that it is not actually a force. It is the work done in moving unit electric charge around the circuit. It is measured in J C^{-1} or volt.

Some examples of energy source which provide emf are electrochemical cells, thermoelectric devices, solar cells and electrical generators. Of these, electrical

UNIT-4(XII-Physics_Vol-1).indd 218 17-02-2020 13:18:25

generators are most powerful machines. They are used for large scale power generation.

According to Faraday's law of electromagnetic induction, an emf is induced in a circuit when magnetic flux linked with it changes. This emf is called induced emf. The magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} \quad or$$

$$\varepsilon = \frac{d}{dt}(BA\cos\theta) \qquad (4.17)$$

From the above equation, it is clear that induced emf can be produced by changing magnetic flux in any of the following ways.

(i) By changing the magnetic field B

(ii) By changing the area A of the coil and

(iii) By changing the relative orientation θ of the coil with magnetic field

4.4.2 Production of induced emf by changing the magnetic field

From Faraday's experiments on electromagnetic induction, it was discovered that an emf is induced in a circuit by changing the magnetic flux of the field through it. The change in flux is brought about by (i) relative motion between the circuit and the magnet (First experiment) (ii) variation in current flowing through the nearby coil (Second experiment).

4.4.3 Production of induced emf by changing the area of the coil

Consider a conducting rod of length l moving with a velocity v towards left on a rectangular fixed metallic framework as shown in Figure 4.23. The whole arrangement is placed in a uniform magnetic field $\vec{B}$ whose magnetic lines are perpendicularly directed into the plane of the paper.

As the rod moves from AB to DC in a time dt, the area enclosed by the loop and hence the magnetic flux through the loop decreases.

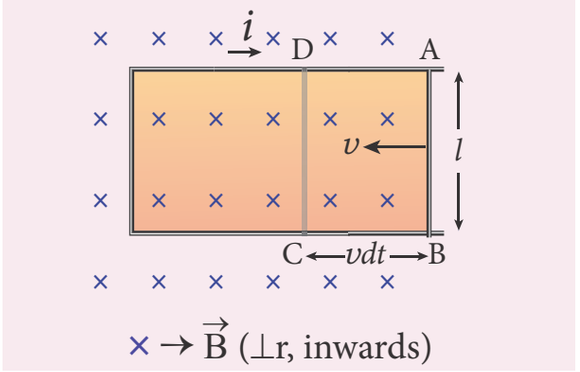


Figure 4.23 Production of induced emf by changing the area enclosed by the loop

The change in magnetic flux in time dt is

$$d\Phi_B = B \times \text{Change in area } (dA)$$

$$= B \times \text{Area } ABCD$$

$$\text{Since Area } ABCD = l(vdt)$$

$$d\Phi_B = Blvdt \quad \text{(or)}$$

$$\frac{d\Phi_B}{dt} = Blv$$

As a result of change in flux, an emf is generated in the loop. The magnitude of the induced emf is

$$\varepsilon = \frac{d\Phi_B}{dt}$$

$$\varepsilon = Blv \qquad (4.18)$$

This emf is known as **motional emf** since it is produced due to the movement of the conductor in the magnetic field. The direction of induced current is found to be clockwise from Fleming's right hand rule.

If R is the resistance of the loop, then the induced current is given by

UNIT-4(XII-Physics_Vol-1).indd 219 17-02-2020 13:18:29

$$i = \frac{\varepsilon}{R}$$

$$i = \frac{Blv}{R} \qquad (4.19)$$

Energy conservation

The current-carrying movable rod AB kept in the perpendicular magnetic field experiences a force $\vec{F}_B$ in the outward direction, opposite to its motion. This force is given by

$$\vec{F}_B = i\vec{l} \times \vec{B} = ilB\sin\theta$$
$$= ilB \qquad \text{since } \theta = 90°$$

In order to move the rod with a constant velocity $\vec{v}$, a constant force that is equal and opposite to the magnetic force, must be applied.

$$\left|\vec{F}_{app}\right| = \left|\vec{F}_B\right| = ilB$$

Therefore, mechanical work is done by the applied force to move the rod. The rate of doing work or power is

$$P = \vec{F}_{app} \cdot \vec{v} = F_{app} v\cos\theta \qquad \text{Here } \theta = 0°$$
$$= ilBv$$
$$= \left(\frac{Blv}{R}\right) lBv$$
$$P = \frac{B^2 l^2 v^2}{R} \qquad (4.20)$$

When the induced current flows in the loop, Joule heating takes place. The rate at which thermal energy is dissipated in the loop or power dissipated is

$$P = i^2 R$$
$$P = \left(\frac{Blv}{R}\right)^2 R$$
$$P = \frac{B^2 l^2 v^2}{R} \qquad (4.21)$$

This equation is exactly same as the equation (4.20). Thus the mechanical energy needed to move the rod is converted into electrical energy which then appears as thermal energy in the loop. This energy conversion is consistent with the law of conservation of energy.

EXAMPLE 4.14

A circular metal of area 0.03 m^2 rotates in a uniform magnetic field of 0.4 T. The axis of rotation passes through the centre and perpendicular to its plane and is also parallel to the field. If the disc completes 20 revolutions in one second and the resistance of the disc is 4 Ω, calculate the induced emf between the axis and the rim and induced current flowing in the disc.

Solution

$A = 0.03$ m^2; $B = 0.4$ T; $f = 20$ rps;
$R = 4\ \Omega$

Area swept out by the disc in unit time
= Area of the disc × frequency

$$\frac{dA}{dt} = 0.03 \times 20$$
$$= 0.6 \text{ m}^2$$

The magnitude of the induced emf,

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = B\frac{dA}{dt}$$
$$\varepsilon = \frac{0.4 \times 0.6}{1} = 0.24 \text{ V}$$

Induced current, $i = \frac{\varepsilon}{R} = \frac{0.24}{4} = 0.06$ A

Note

Emf can be induced by changing relative orientation between the coil and the magnetic field. This can be achieved either by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. Here rotating coil type is considered.

UNIT-4(XII-Physics_Vol-1).indd 220 17-02-2020 13:18:32

4.4.4 Production of induced emf by changing relative orientation of the coil with the magnetic field

Consider a rectangular coil of N turns kept in a uniform magnetic field $\vec{B}$ as shown in Figure 4.24. The coil rotates in anti-clockwise direction with an angular velocity ω about an axis, perpendicular to the field and to the plane of the paper.

At time $t = 0$, the plane of the coil is perpendicular to the field and the flux linked with the coil has its maximum value $\Phi_m = NBA$ (where A is the area of the coil).

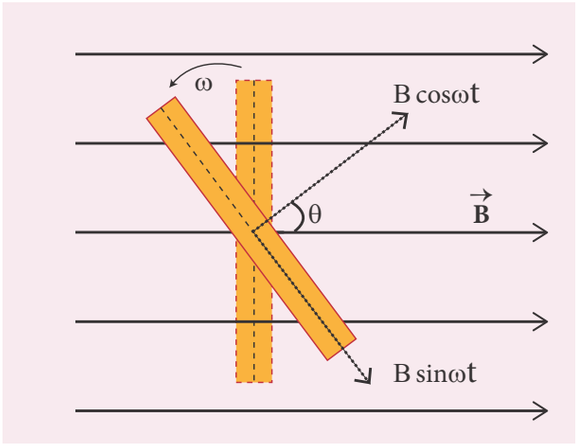


Figure 4.24 The coil has rotated through an angle $\theta = \omega t$

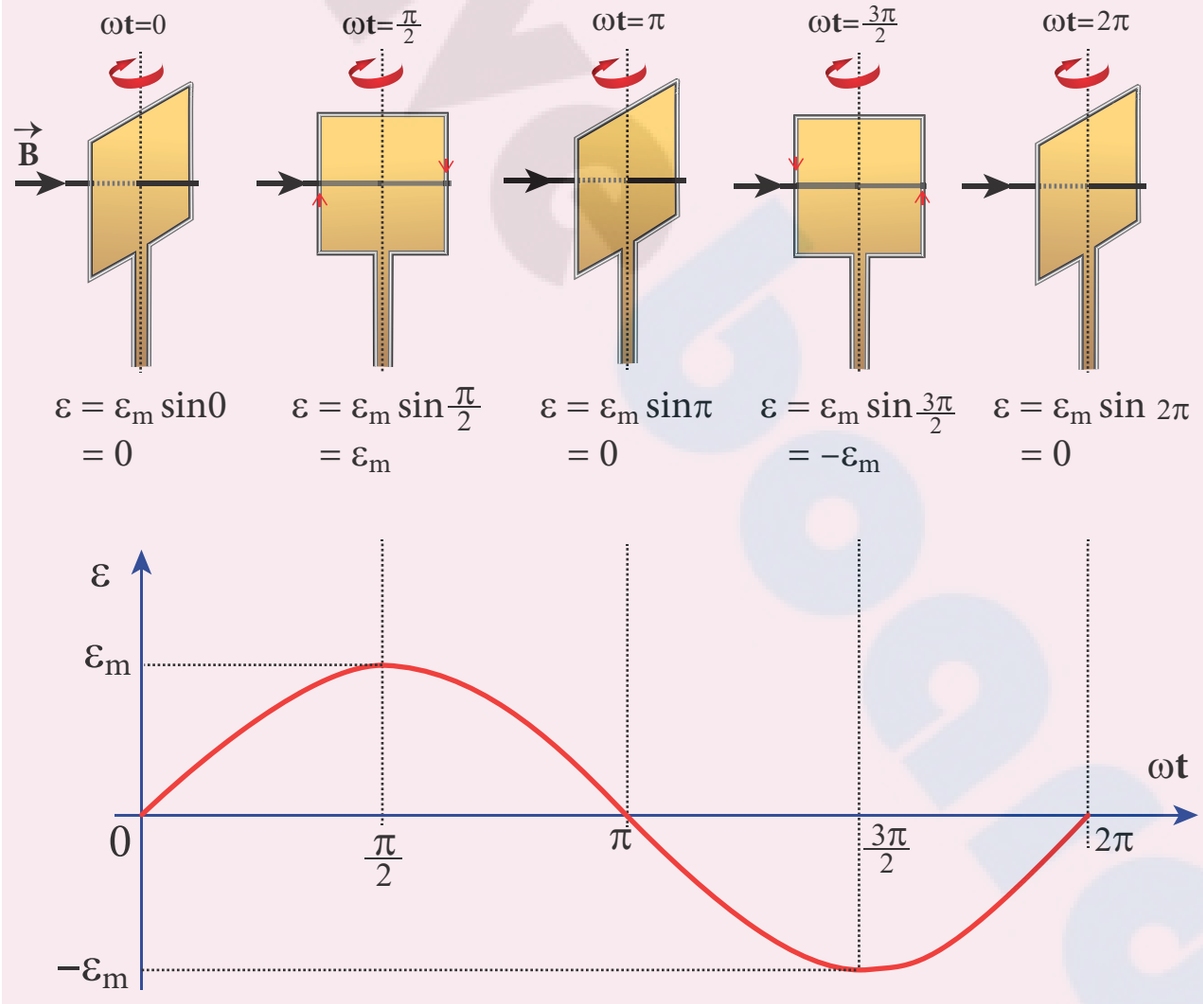


Figure 4.25 Variation of induced emf as a function of ωt

UNIT-4(XII-Physics_Vol-1).indd 221 17-02-2020 13:18:33

In a time t seconds, the coil is rotated through an angle θ (= ωt) in anti–clockwise direction. In this position, the flux linked $NBA\cos\omega t$ is due to the component of $\vec{B}$ normal to the plane of the coil. The component ($B\sin\omega t$) parallel to the plane has no role in electromagnetic induction. Therefore, the flux linkage with the coil at this deflected position is

$$N\Phi_B = NBA\cos\theta = NBA\cos\omega t$$

According to Faraday's law, the emf induced at that instant is

$$\varepsilon = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}(NBA\cos\omega t)$$
$$= -NBA(-\sin\omega t)\omega$$
$$= NBA\,\omega\sin\omega t$$

When the coil is rotated through 90° from initial position, $\sin\omega t$ = 1. Then the maximum value of induced emf is

$$\varepsilon_m = NBA\omega$$

Therefore, the value of induced emf at any instant is then given by

$$\varepsilon = \varepsilon_m \sin\omega t \qquad (4.22)$$

It is seen that the induced emf varies as sine function of the time angle ωt. The graph between induced emf and time angle for one rotation of the coil will be a sine curve (Figure 4.25) and the emf varying in this manner is called **sinusoidal emf** or **alternating emf**.

If this alternating voltage is given to a closed circuit, a sinusoidally varying current flows in it. This current is called **alternating current** and is given by

$$i = I_m \sin\omega t \qquad (4.23)$$

where I_m is the maximum value of induced current.

EXAMPLE 4.15

A rectangular coil of area 70 cm^2 having 600 turns rotates about an axis perpendicular to a magnetic field of 0.4 Wb m^{-2}. If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (i) perpendicular to the field (ii) parallel to the field and (iii) inclined at 60° with the field.

Solution

$A = 70 \times 10^{-4}$ m^2; $N = 600$ turns

$B = 0.4$ Wbm^{-2}; $f = 500$ rpm

The instantaneous emf is

$$\varepsilon = \varepsilon_m \sin\omega t$$

since $\varepsilon_m = N\Phi_m\omega = N(BA)(2\pi f)$

$$\varepsilon = NBA \times 2\pi f \times \sin\omega t$$

(i) When $\omega t = 0^\circ$,

$$\varepsilon = \varepsilon_m \sin 0 = 0$$

(ii) When $\omega t = 90^\circ$,

$$\varepsilon = \varepsilon_m \sin 90^\circ = NBA \times 2\pi f \times 1$$
$$= 600 \times 0.4 \times 70 \times 10^{-4} \times 2 \times \frac{22}{7} \times \left(\frac{500}{60}\right)$$
$$= 88\text{ V}$$

(iii) When ωt = 90° – 60° = 30°,

$$\varepsilon = \varepsilon_m \sin 30^\circ = 88 \times \frac{1}{2} = 44\text{ V}$$

UNIT-4(XII-Physics_Vol-1).indd 222 17-02-2020 13:18:38

4.5 AC GENERATOR

4.5.1 Introduction

AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy. Alternator produces a large scale electrical power for use in homes and industries. AC generator and its components are shown in Figure 4.26.

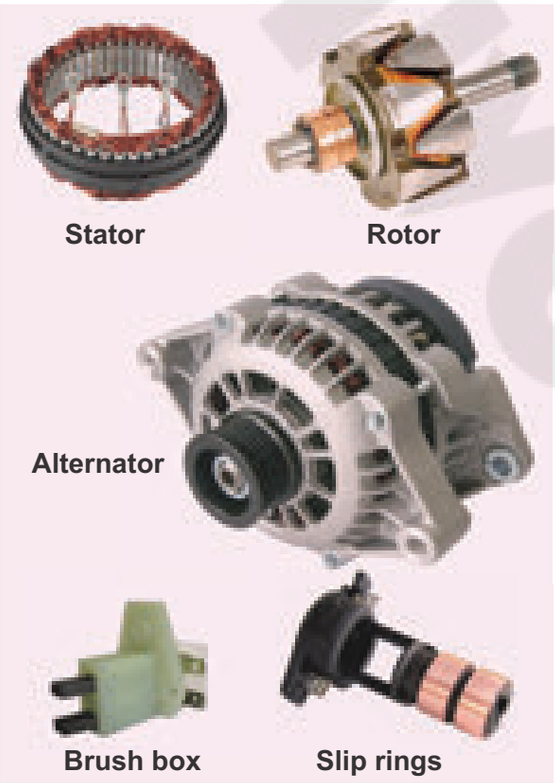


Figure 4.26 AC generator and its components

4.5.2 Principle

Alternators work on the principle of electromagnetic induction. The relative motion between a conductor and a magnetic field changes the magnetic flux linked with the conductor which in turn, induces an emf. The magnitude of the induced emf is given by Faraday's law of electromagnetic induction and its direction by Fleming's right hand rule.

Note

Alternating emf is generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. The first method is used for small AC generators while the second method is employed for large AC generators. The rotating-field method is the one which is mostly used in power stations.

4.5.3 Construction

Alternator consists of two major parts, namely stator and rotor. As their names suggest, stator is stationary while rotor rotates inside the stator. In any standard construction of commercial alternators, the armature winding is mounted on stator and the field magnet on rotor.

i) Stator

The stationary part which has armature windings mounted in it is called stator. It has two components, namely stator core and armature winding.

Stator core or armature core is made up of iron or steel alloy. It is a hollow cylinder and is laminated to minimize eddy current loss. The slots are cut on inner surface of the core to accommodate armature windings.

Armature winding is the coil, wound on slots provided in the armature core (Figure 4.27).

MN5C7X

ii) Rotor

Rotor contains magnetic field windings. The magnetic poles are magnetized by

UNIT-4(XII-Physics_Vol-1).indd 223 17-02-2020 13:18:39

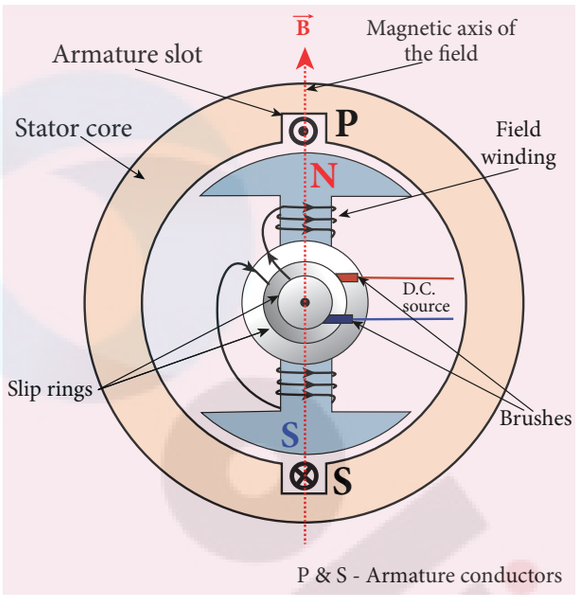


Figure 4.27 Stator core, Armature winding and 2-pole rotor

DC source. The ends of field windings are connected to a pair of slip rings, attached to a common shaft about which rotor rotates. Slip rings rotate along with rotor. To maintain connection between the DC source and field windings, two brushes are used which continuously slide over the slip rings. The 2-pole rotor is shown in Figure 4.27.

We will discuss the construction and working of two examples, namely single phase and three phase AC generators in the following sections.

4.5.4 Advantages of stationary armature-rotating field alternator

Alternators are generally high current and high voltage machines. The stationary armature-rotating field construction has many advantages. A few of them include:

1) The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
2) The insulation of stationary armature winding is easier.
3) The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
4) Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.

4.5.5 Single phase AC generator

In a single phase AC generator, the armature conductors are connected in series so as to form a single circuit which generates a single-phase alternating emf and hence it is called single-phase alternator.

In the simplified version of AC generator, a single-turn rectangular loop PQRS is mounted on the stator. The field winding is fixed inside the stator and it can be rotated about an axis, perpendicular to the plane of the paper.

The loop PQRS is stationary and is also perpendicular to the plane of the paper. When field windings are excited, magnetic field is produced around it. Let the field magnet be rotated in clockwise direction by some external means (Figure 4.28).

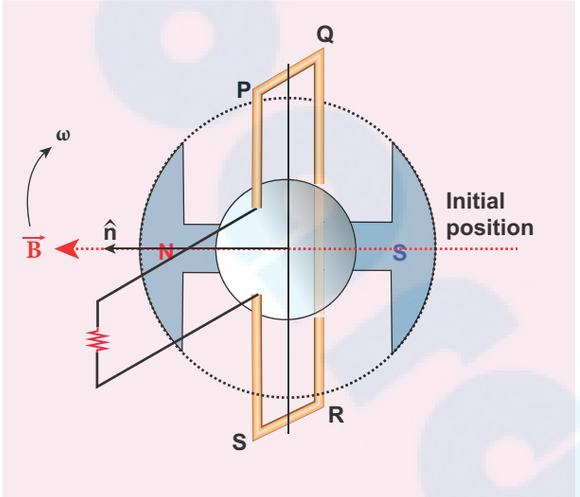


Figure 4.28 The loop PQRS and field magnet in its initial position

UNIT-4(XII-Physics_Vol-1).indd 224 17-02-2020 13:18:39

Assume that initial position of the field magnet is horizontal. At that instant, the direction of magnetic field is perpendicular to the plane of the loop PQRS. The induced emf is zero (Refer case (iii) of section 4.4). This is represented by origin O in the graph drawn between induced emf and time angle (Figure 4.29).

When field magnet rotates through 90°, magnetic field becomes parallel to PQRS. The induced emfs across PQ and RS would become maximum. Since they are connected in series, emfs are added up and the direction of total induced emf is given by Fleming's right hand rule.

Care has to be taken while applying this rule; the thumb indicates the direction of the motion of the conductor with respect to field. For clockwise rotating poles, the conductor appears to be rotating anticlockwise. Hence, thumb should point to the left. The direction of the induced emf is at right angles to the plane of the paper. For PQ, it is inwards and for RS it is outwards. Therefore, the current flows along PQRS. The point A in the graph represents this maximum emf.

For the rotation of 180° from the initial position, the field is again perpendicular to PQRS and the induced emf becomes zero. This is represented by point B.

The field magnet becomes again parallel to PQRS for 270° rotation of field magnet. The induced emf is maximum but the direction is reversed. Thus the current flows along SRQP. This is represented by point C.

On completion of 360°, the induced emf becomes zero and is represented by the point D. From the graph, it is clear that emf induced in PQRS is alternating in nature. Therefore, when field magnet completes one rotation, induced emf in PQRS finishes one cycle.

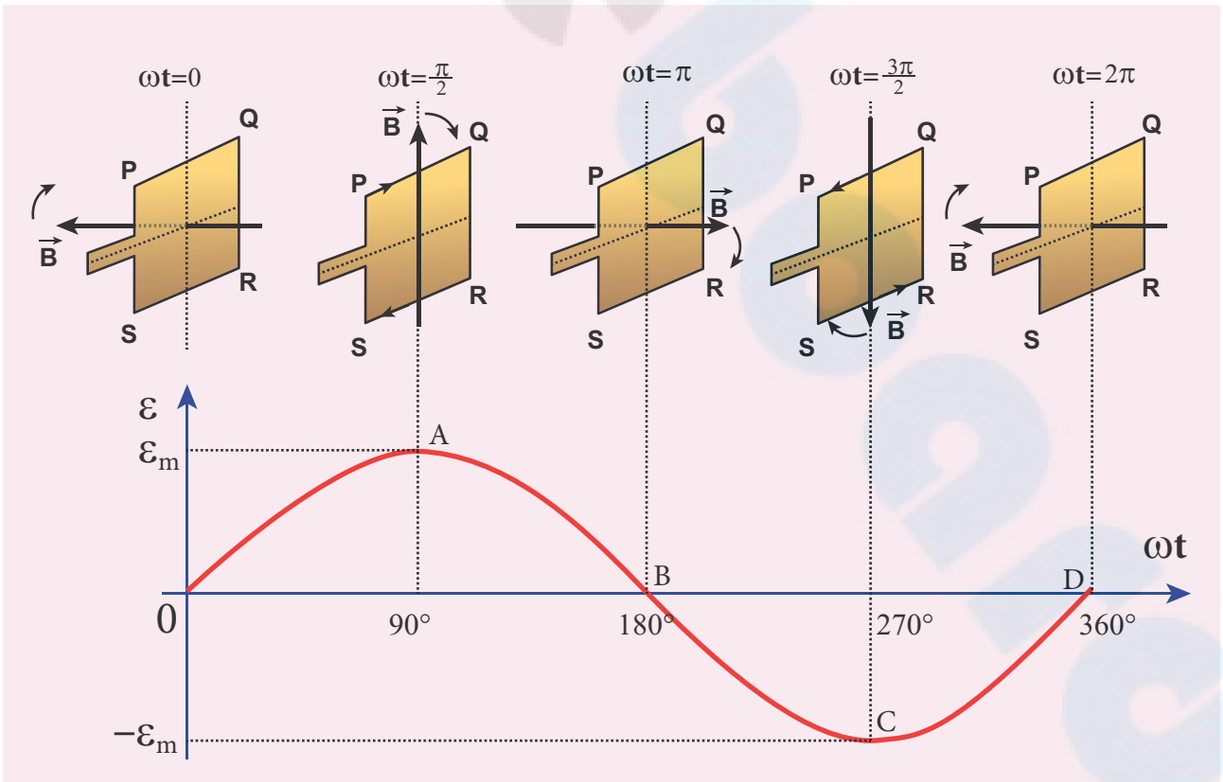


Figure 4.29 Variation of induced emf with respect to time angle

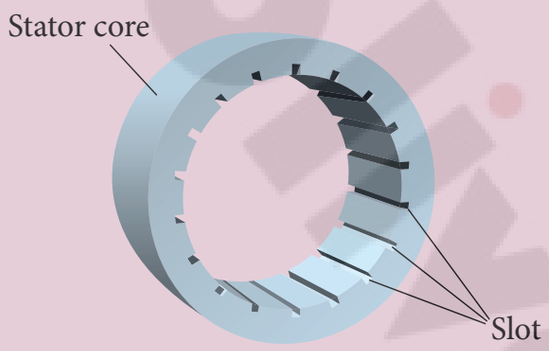
UNIT-4(XII-Physics_Vol-1).indd 225 17-02-2020 13:18:39

Construction of AC generator (Not for examination)

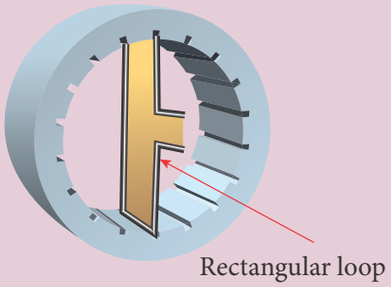
Alternator consists of two major parts, namely stator and rotor. (This box is given for better understanding of constructional details)

i) Stator

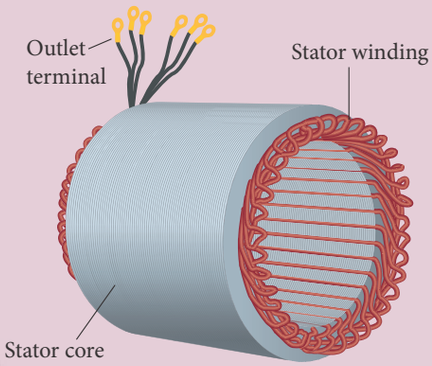
Stator has three components, namely stator frame, stator core and armature winding.



Figure(a): Stator core with empty slots



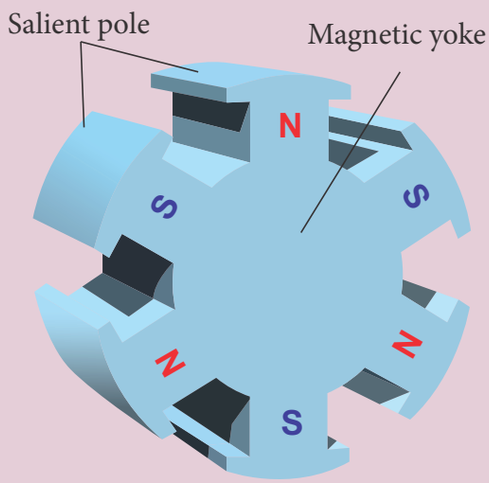
Figure(b): Stator core with rectangular loop



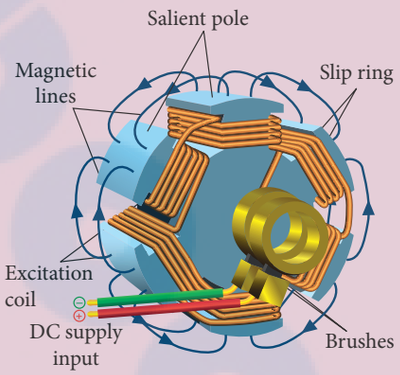
Figure(c): Stator core with armature windings

ii) Rotor

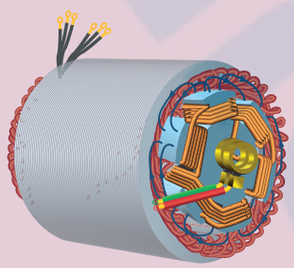
Rotor contains magnetic field windings, slip rings and brushes mounted on the same shaft.



Figure(d): 6-pole rotor



Figure(e): 6-pole rotor with field windings, slip rings and brushes



Figure(f): Stator core and rotor

UNIT-4(XII-Physics_Vol-1).indd 226 17-02-2020 13:18:44

4.5.6 Poly-phase AC generator

Some AC generators may have more than one coil in the armature core and each coil produces an alternating emf. In these generators, more than one emf is produced. Thus they are called **poly-phase generators**.

If there are two alternating emfs produced in a generator, it is called two-phase generator. In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called **three-phase AC generators**.

4.5.7 Three-phase AC generator

In the simplified construction of three-phase AC generator, the armature core has 6 slots, cut on its inner rim. Each slot is 60° away from one another. Six armature conductors are mounted in these slots. The conductors 1 and 4 are joined in series to form coil 1. The conductors 3 and 6 form coil 2 while the conductors 5 and 2 form coil 3. So, these coils are retangular in shape and are 120° apart from one another (Figure 4.30).

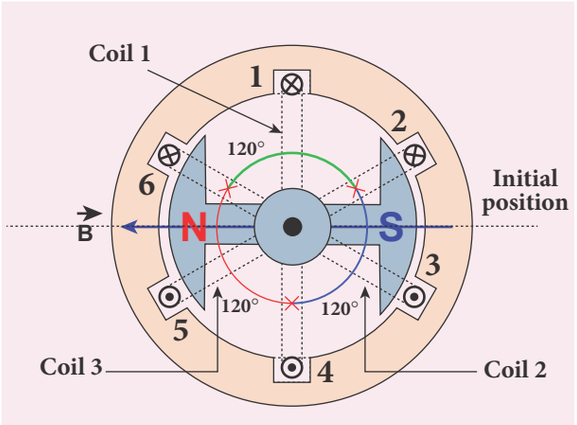


Figure 4.30 Construction of three-phase AC generator

The initial position of the field magnet is horizontal and field direction is perpendicular to the plane of the coil 1. As it is seen in single phase AC generator, when field magnet is rotated from that position in clockwise direction, alternating emf ε_1 in coil 1 begins a cycle from origin O. This is shown in Figure 4.31.

The corresponding cycle for alternating emf ε_2 in coil 2 starts at point A after field magnet has rotated through 120°. Therefore, the phase difference between ε_1 and ε_2 is 120°. Similarly, emf ε_3 in coil 3 would begin its cycle at point B after 240° rotation of field magnet from initial position. Thus these emfs produced in the three phase AC generator have 120° phase difference between one another.

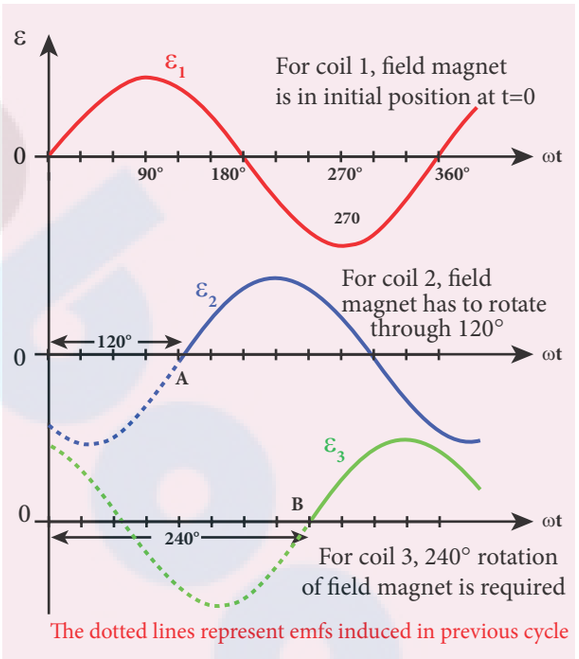


Figure 4.31 Variation of emfs ε_1, ε_2 and ε_3 with time angle.

4.5.7 Advantages of three-phase alternator

Three-phase system has many advantages over single-phase system. Let us see a few of them.

UNIT-4(XII-Physics_Vol-1).indd 227 17-02-2020 13:18:45

1) For a given dimension of the generator, three-phase machine produces higher power output than a single-phase machine.
2) For the same capacity, three-phase alternator is smaller in size when compared to single-phase alternator.
3) Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.

4.6 TRANSFORMER

Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency. The applied alternating voltage is either increased or decreased with corresponding decrease or increase of current in the circuit.

If the transformer converts an alternating current with low voltage into an alternating current with high voltage, it is called **step-up transformer**. On the contrary, if the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called **step-down transformer**.

4.6.1 Construction and working of transformer

Principle

The principle of transformer is the mutual induction between two coils. That is, when an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil.

Construction

In the simple construction of transformers, there are two coils of high mutual inductance wound over the same transformer core. The core is generally laminated and is made up of a good magnetic material like silicon steel. Coils are electrically insulated but magnetically linked via transformer core (Figure 4.32).

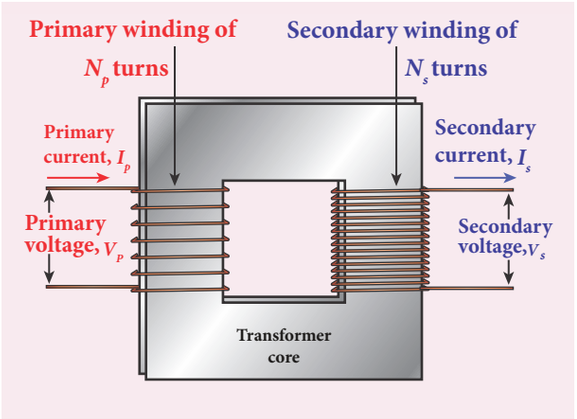


Figure 4.32(a) Construction of transformer



Figure 4.32(b) Roadside transformer

The coil across which alternating voltage is applied is called primary coil *P* and the coil from which output power is drawn out is called secondary coil *S*. The assembled core and coils are kept in a container which is filled with suitable medium for better insulation and cooling purpose.

Working

If the primary coil is connected to a source of alternating voltage, an alternating magnetic flux is set up in the laminated core. If there is no magnetic flux leakage, then whole of magnetic flux linked with primary coil is also linked with secondary

UNIT-4(XII-Physics_Vol-1).indd 228 17-02-2020 13:18:46

coil. This means that rate at which magnetic flux changes through each turn is same for both primary and secondary coils.

As a result of flux change, emf is induced in both primary and secondary coils. The emf induced in the primary coil or back emf ε_p is given by

$$\varepsilon_p = -N_p \frac{d\Phi_B}{dt}$$

But the voltage applied v_p across the primary is equal to the back emf. Then

$$v_p = -N_p \frac{d\Phi_B}{dt} \qquad (4.24)$$

The frequency of alternating magnetic flux in the core is same as the frequency of the applied voltage. Therefore, induced emf in secondary will also have same frequency as that of applied voltage. The emf induced in the secondary coil ε_s is given by

$$\varepsilon_s = -N_s \frac{d\Phi_B}{dt}$$

where N_p and N_s are the number of turns in the primary and secondary coil respectively. If the secondary circuit is open, then $\varepsilon_s = v_s$ where v_s is the voltage across secondary coil.

$$v_s = -N_s \frac{d\Phi_B}{dt} \qquad (4.25)$$

From equations (4.24) and (4.25),

$$\frac{v_s}{v_p} = \frac{N_s}{N_p} = K \qquad (4.26)$$

This constant K is known as voltage transformation ratio. For an ideal transformer,

Input power $v_p i_p$ = Output power $v_s i_s$

where i_p and i_s are the currents in the primary and secondary coil respectively. Therefore,

$$\frac{v_s}{v_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s} \qquad (4.27)$$

Equation 4.27 is written in terms of amplitude of corresponding quantities,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = K$$

i) If $N_s > N_p (K > 1)$, then $V_s > V_p$ and $I_s < I_p$. This is the case of step-up transformer in which voltage is increased and the corresponding current is decreased.

ii) If $N_s < N_p (K < 1)$, then $V_s < V_p$ and $I_s > I_p$. This is step-down transformer where voltage is decreased and the current is increased.

Efficiency of a transformer:

The efficiency η of a transformer is defined as the ratio of the useful output power to the input power. Thus

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\% \qquad (4.28)$$

Transformers are highly efficient devices having their efficiency in the range of 96 – 99%. Various energy losses in transformers will not allow them to be 100% efficient.

4.6.2 Energy losses in a transformer

Transformers do not have any moving parts so that its efficiency is much higher than that of rotating machines like generators and motors. But there are many factors which lead to energy loss in a transformer.

i) Core loss or Iron loss

This loss takes place in transformer core. Hysteresis loss (Refer section 3.6) and eddy current loss are known as core loss or Iron loss. When transformer core is magnetized and demagnetized repeatedly by the alternating voltage applied across primary coil, hysteresis takes place due to

UNIT-4(XII-Physics_Vol-1).indd 229 17-02-2020 13:18:50

which some energy is lost in the form of heat. Hysteresis loss is minimized by using steel of high silicon content in making transformer core.

Alternating magnetic flux in the core induces eddy currents in it. Therefore there is energy loss due to the flow of eddy current, called eddy current loss which is minimized by using very thin laminations of transformer core.

ii) Copper loss

Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to Joule heating. This energy loss is called copper loss which is minimized by using wires of larger diameter.

iii) Flux leakage

Flux leakage happens when the magnetic lines of primary coil are not completely linked with secondary coil. Energy loss due to this flux leakage is minimized by winding coils one over the other.

4.6.3 Advantages of AC in long distance power transmission

Electric power is produced in a large scale at electric power stations with the help of AC generators. These power stations are classified based on the type of fuel used as thermal, hydro electric and nuclear power stations. Most of these stations are located at remote places. Hence the electric power generated is transmitted over long distances through transmission lines to reach towns or cities where it is actually consumed. This process is called **power transmission**.

But there is a difficulty during power transmission. A sizable fraction of electric power is lost due to Joule heating (i^2R) in the transmission lines which are hundreds of kilometer long. This power loss can be tackled either by reducing current I or by reducing resistance R of the transmission lines. The resistance R can be reduced with thick wires of copper or aluminium. But this increases the cost of production of transmission lines and other related expenses. So this way of reducing power loss is not economically viable.

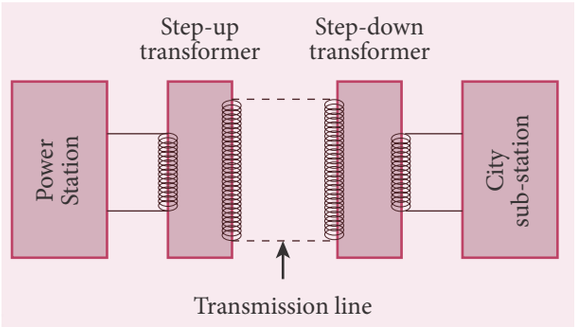


Figure 4.33 Long distance power transmissions

Since power produced is alternating in nature, there is a way out. The most important property of alternating voltage that it can be stepped up and stepped down by using transformers could be exploited in reducing current and thereby reducing power losses to a greater extent.

At the transmitting point, the voltage is increased and the corresponding current is decreased by using step-up transformer (Figure 4.33). Then it is transmitted through transmission lines. This reduced current at high voltage reaches the destination without any appreciable loss. At the receiving point, the voltage is decreased and the current is increased to appropriate values by using step-down transformer and then it is given to consumers. Thus power transmission is done efficiently and economically.

UNIT-4(XII-Physics_Vol-1).indd 230 17-02-2020 13:18:51

Illustration:

An electric power of 2 MW is transmitted to a place through transmission lines of total resistance R = 40 Ω, at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.

Case (i):

P = 2 MW; R = 40 Ω; V = 10 kV

Power, $P = VI$

$$\therefore \text{Current, } I = \frac{P}{V}$$

$$= \frac{2\times10^6}{10\times10^3} = 200\text{A}$$

Power loss = Heat produced = I^2R

$$= (200)^2 \times 40 = 1.6\times10^6\,\text{W}$$

$$\% \text{ of power loss} = \frac{1.6\times10^6}{2\times10^6}\times100\%$$

$$= 0.8\times100\% = 80\%$$

Case (ii):

P = 2 MW; R = 40 Ω; V = 100 kV

$$\therefore \text{ Current, } I = \frac{P}{V}$$

$$= \frac{2\times10^6}{100\times10^3} = 20\text{A}$$

Power loss = I^2R

$$= (20)^2 \times 40 = 0.016\times10^6\,\text{W}$$

$$\% \text{ of power loss} = \frac{0.016\times10^6}{2\times10^6}\times100\%$$

$$= 0.008\times100\% = 0.8\%$$

Thus it is clear that when an electric power is transmitted at higher voltage, the power loss is reduced to a large extent.

EXAMPLE 4.16

An ideal transformer has 460 and 40,000 turns in the primary and secondary coils respectively. Find the voltage developed per turn of the secondary if the transformer is connected to a 230 V AC mains. The secondary is given to a load of resistance 10^4 Ω. Calculate the power delivered to the load.

Solution

N_P = 460 turns; N_S = 40,000 turns

V_P = 230 V; $R_S = 10^4$ Ω

(i) Secondary voltage,

$$V_S = \frac{V_P N_S}{N_P} = \frac{230\times40{,}000}{460} = 20{,}000\,\text{V}$$

$$\text{Secondary voltage per turn, } \frac{V_S}{N_S} = \frac{20{,}000}{40{,}000}$$

$$= 0.5\ \text{V}$$

(ii) Power delivered

$$= V_S I_S = \frac{V_S^2}{R_S} = \frac{20{,}000\times20{,}000}{10^4} = 40\ \text{kW}$$

EXAMPLE 4.17

An inverter is common electrical device which we use in our homes. When there is no power in our house, inverter gives AC power to run a few electronic appliances like fan or light. An inverter has inbuilt step-up transformer which converts 12 V AC to 240 V AC. The primary coil has 100 turns and the inverter delivers 50 mA to the external circuit. Find the number of turns in the secondary and the primary current.

Solution

V_p = 12 V; V_s = 240 V

I_s = 50mA; N_p = 100 turns

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} = K$$

UNIT-4(XII-Physics_Vol-1).indd 231 17-02-2020 13:18:58

Power system at a glance (Not for Examination)

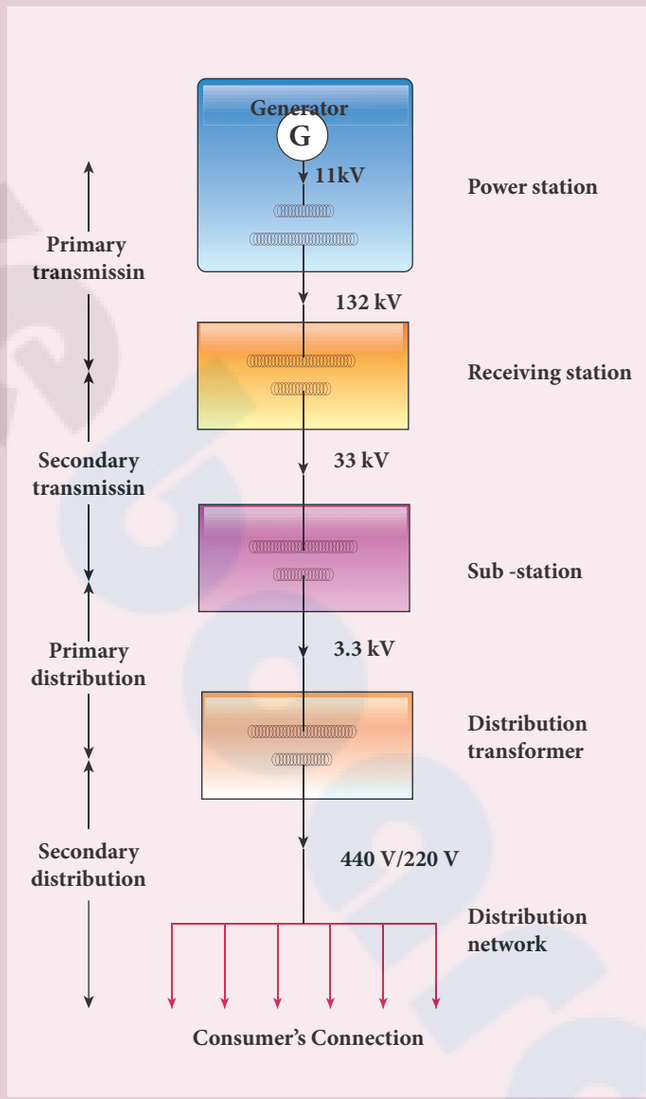
The generating stations present in a region are interconnected to form a common electrical network and are operated in parallel. This is to ensure uninterrupted power supply to a large number of consumers in the case of failure of any power station or a sudden increase of load beyond the capacity of the generating station.

The various elements such as generating stations, transmission lines, the substations and distributors etc are all tied together for continuous generation and consumption of electric energy.This is called power system. A part of power system consisting of the sub-stations and transmission lines is known as a grid.

In a power system, the transfer of electric power produced to the consumer is carried out in two stages which are further sub-divided into two as given below.

1) Transmission stage
   a) Primary transmission stage
   b) Secondary transmission stage
2) Distribution stage
   a) Primary distribution stage
   b) Secondary transmission stage

and then it is supplied to individual consumers. These two stages of power transmission is presented in a single-line diagram shown in figure. The central system usually generates power at 11 kV which is stepped up to 132 kV and is transmitted through transmission lines. This is known as primary or high-voltage transmission.



This high-voltage power reaches receiving station at the outskirts of the city where it is stepped down to 33 kV and is transmitted as secondary or low-voltage transmission to sub-stations situated within the city limits.

In the primary distribution system, the voltage is reduced from 33 kV to 3.3kV at sub-stations and is given to distribution sub-stations. The voltage is finally brought down to 440V or 230V at distribution sub-station from where secondary distribution is done to factories (440V) and homes (230V) via distribution networks.

UNIT-4(XII-Physics_Vol-1).indd 232 17-02-2020 13:18:58

Transformation ratio, $K = \frac{240}{12} = 20$

The number of turns in the secondary

$$N_S = N_P \times K = 100 \times 20 = 2000$$

Primary current,

$$I_p = K \times I_s = 20 \times 50 \text{ mA} = 1 \text{ A}$$

4.7 ALTERNATING CURRENT

4.7.1 Introduction

In section 4.5, we have seen that when the orientation of the coil with the magnetic field is changed, an alternating emf is induced and hence an alternating current flows in the closed circuit. **An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.**

In the Figure 4.34(a), an alternating voltage source is connected to a resistor R in which the upper terminal of the source is positive and lower terminal negative at an instant. Therefore, the current flows in clockwise direction. After a short time, the polarities of the source are reversed so that current now flows in anti-clockwise direction (Figure 4.34(b)). This current which flows in alternate directions in the circuit is called alternating current.

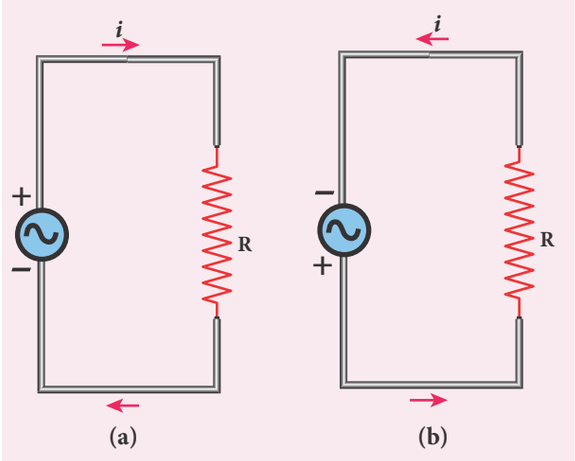


Figure 4.34 Alternating voltage and the corresponding alternating current

Sinusoidal alternating voltage

If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage which is given by the relation.

$$v = V_m \sin \omega t \qquad (4.29)$$

where v is the instantaneous value of alternating voltage; V_m is the maximum value (amplitude) and ω is the angular frequency of the alternating voltage. When sinusoidal alternating voltage is applied to a closed circuit, the resulting alternating current is also sinusoidal in nature and its relation is

$$i = I_m \sin \omega t \qquad (4.30)$$

where I_m is the maximum value (amplitude) of the alternating current. The direction of sinusoidal voltage or current is reversed after every half-cycle and its magnitude is also changing continuously as shown in Figure 4.35.

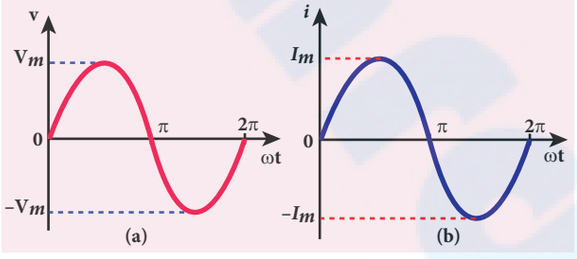


Figure 4.35 (a) Sinusoidal alternating voltage **(b)** Sinusoidal alternating current

UNIT-4(XII-Physics_Vol-1).indd 233 17-02-2020 13:19:00

Note

Interestingly, sine waves are very common in nature. The periodic motions like waves in water, swinging of pendulum are associated with sine waves. Thus sine wave seems to be nature's standard. Also refer unit 11 of XI physics text book.

4.7.1 Mean or Average value of AC

The current and voltage in a DC system remain constant over a period of time so that there is no problem in specifying their magnitudes. However, an alternating current or voltage varies from time to time. Then a question arises how to express the magnitude of an alternating current or voltage. Though there are many ways of expressing it, we limit our discussion with two ways, namely mean value and RMS (Root Mean Square) value of AC.

Mean or Average value of AC

We have learnt that the magnitude of an alternating current in a circuit changes from one instant to other instant and its direction also reverses for every half cycle. During positive half cycle, current is taken as positive and during negative cycle it is negative. Therefore mean or average value of symmetrical alternating current over one complete cycle is zero.

Therefore the average or mean value is measured over one half of a cycle. These electrical terms, average current and average voltage, can be used in both AC and DC circuit analysis and calculations.

The average value of alternating current is defined as the average of all values of current over a positive half-cycle or a negative half-cycle.

The instantaneous value of sinusoidal alternating current is given by the equation $i = I_m \sin\omega t$ or $i = I_m \sin\theta$ (where $\theta = \omega t$) whose graphical representation is given in Figure 4.36.

The sum of all currents over a half-cycle is given by area of positive half-cycle (or negative half-cycle). Therefore,

$$I_{av} = \frac{\text{Area of positive half-cycle (or negative half-cycle)}}{\text{Base length of half-cycle}} \qquad (4.31)$$

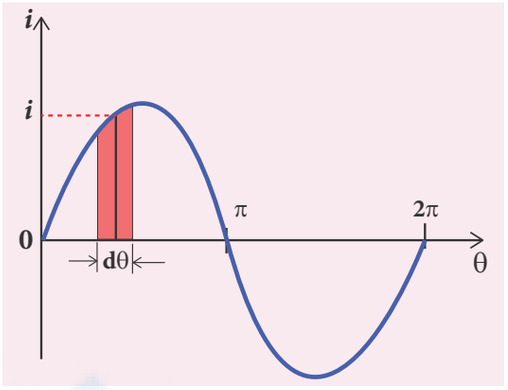


Figure 4.36 Sine wave of an alternating current

Consider an elementary strip of thickness $d\theta$ in the positive half-cycle of the current wave (Figure 4.41). Let i be the mid-ordinate of that strip.

Area of the elementary strip = $i\, d\theta$

Area of positive half-cycle

$$= \int_0^{\pi} i d\theta = \int_0^{\pi} I_m \sin\theta d\theta$$

$$= I_m\left[-\cos\theta\right]_0^{\pi} = -I_m\left[\cos\pi - \cos 0\right] = 2I_m$$

The base length of half-cycle is π. Substituting these values in equation (4.31), we get

$$\text{Average value of AC, } I_{av} = \frac{2I_m}{\pi}$$

$$I_{av} = 0.637\, I_m \qquad (4.32)$$

UNIT-4(XII-Physics_Vol-1).indd 234 17-02-2020 13:19:02

Hence the average value of AC is 0.637 times the maximum value I_m of the alternating current. For negative half-cycle, $I_{av} = -0.637\, I_m$.

Note

For example, if we consider n currents in a half-cycle of AC, namely $i_1, i_2, \ldots i_n$, then average value is given by

$$I_{av} = \frac{\text{Sum of all currents over half-cycle}}{\text{Number of currents}}$$

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n}$$

4.7.2 RMS value of AC

The term RMS refers to time-varying sinusoidal currents and voltages which is not used in DC systems.

The root mean square value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle. It is denoted by I_{RMS}. For alternating voltages, the RMS value is given by V_{RMS}.

The alternating current $i = I_m \sin\omega t$ or $i = I_m \sin\theta$, is represented graphically in Figure 4.37. The corresponding squared current wave is also shown by the dotted lines.

The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave. Therefore,

$$I_{RMS} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \qquad (4.33)$$

An elementary area of thickness $d\theta$ is considered in the first half-cycle of the squared current wave as shown in Figure 4.37. Let i^2 be the mid-ordinate of the element.

Area of the element = $i^2 d\theta$

Area of one cycle of squared wave

$$= \int_0^{2\pi} i^2 d\theta$$

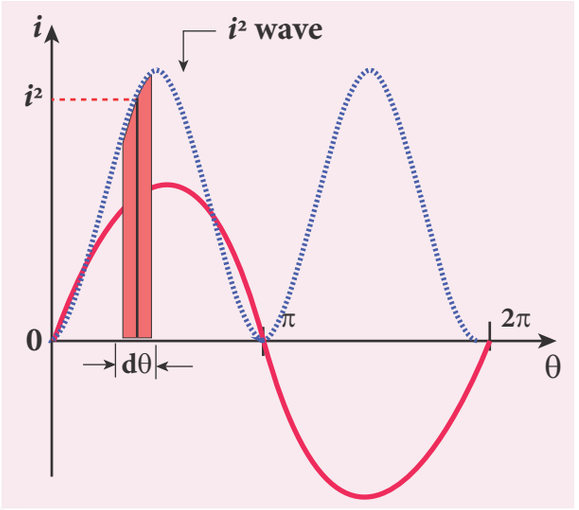


Figure 4.37 Squared wave of AC

$$= \int_0^{2\pi} I_m^2 \sin^2\theta d\theta = I_m^2 \int_0^{2\pi} \sin^2\theta d\theta \qquad (4.34)$$

$$= I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

since $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \frac{I_m^2}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2}\left[\left(2\pi - \frac{\sin 2\times 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2} \times 2\pi = I_m^2 \pi \quad [\because \sin 0 = \sin 4\pi = 0]$$

The base length of one cycle is 2π. Substituting these values in equation (4.33), we get

UNIT-4(XII-Physics_Vol-1).indd 235 17-02-2020 13:19:08

$$I_{RMS} = \sqrt{\frac{I_m^2\,\pi}{2\pi}} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = 0.707\, I_m \qquad (4.35)$$

Thus we find that for a symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

Similarly for alternating voltage, it can be shown that

$$V_{rms} = 0.707\, V_m \qquad (4.36)$$

Note

RMS value of alternating current is also called effective value and is represented as I_{eff}. It is used to compare RMS current of AC to an equivalent steady current.

RMS value is also defined as that value of the steady current which when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current when flowing through the same circuit for the same time. The effective value of an alternating voltage is represented by V_{eff}.

Note

For example, if we consider n currents in one cycle of AC, namely i_1, i_2, ... i_n, then RMS value is given by

$$I_{RMS} = \sqrt{\frac{\text{Sum of squares of all currents over one cycle}}{\text{Number of currents}}}$$

$$I_{RMS} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}}$$

DO YOU KNOW?

For common household appliances, the voltage rating and current rating are generally specified in terms of their RMS value. The domestic AC supply is 230V, 50 Hz. It is the RMS or effective value. Its peak value will be $V_m = \sqrt{2}\ V_{rms} = \sqrt{2} \times 230 = 325\,\text{V}$.

EXAMPLE 4.18

Write down the equation for a sinusoidal voltage of 50 Hz and its peak value is 20 V. Draw the corresponding voltage versus time graph.

Solution

$f = 50\,\text{Hz};\ V_m = 20\ \text{V}$

Instantaneous voltage, $v = V_m \sin\omega t$

$= V_m \sin 2\pi f t$

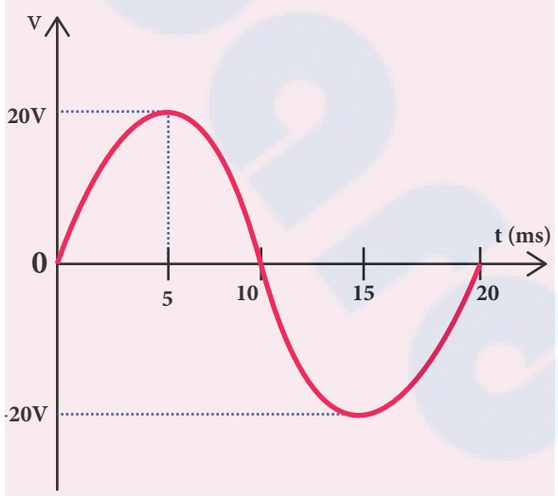
$= 20\sin(2\pi \times 50)\,\text{t} \ = 20\sin(100 \times 3.14)\,\text{t}$

$v = 20\sin 314\,\text{t}$

Time for one cycle, $T = \frac{1}{f} = \frac{1}{50} = 0.02\,\text{s}$

$= 20 \times 10^{-3}\,\text{s} = 20\,\text{m s}$

The wave form is given below.



UNIT-4(XII-Physics_Vol-1).indd 236 17-02-2020 13:19:15

EXAMPLE 4.19

The equation for an alternating current is given by $i = 77\ sin\ 314t$. Find the peak current, frequency, time period and instantaneous value of current at $t = 2$ ms.

Solution

$i = 77 \sin 314\ t$; $t = 2$ m s $= 2 \times 10^{-3}$ s

The general equation of an alternating current is $i = I_m \sin \omega t$. On comparison,

(i) Peak current, $I_m = 77$ A

(ii) Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2 \times 3.14} = 50$ Hz

(iii) Time period, $T = \dfrac{1}{f} = \dfrac{1}{50} = 0.02$ s

(iv) At $t = 2$ m s, Instantaneous current,

$$i = 77 \sin(314 \times 2 \times 10^{-3})$$

$$= 77 \sin\left(314 \times 2 \times 10^{-3} \times \frac{180^\circ}{3.14}\right)$$

$$= 77 \sin 36^\circ = 77 \times 0.5878$$

$$= 45.26 \text{A}$$

Phasor and phasor diagram

Phasor

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a phasor. A phasor is drawn in such a way that

- the length of the line segment equals the peak value V_m (or I_m) of the alternating voltage (or current)
- its angular velocity ω is equal to the angular frequency of the alternating voltage (or current)
- the projection of phasor on any vertical axis gives the instantaneous value of the alternating voltage (or current)
- the angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).

The notion of phasors is introduced to analyse phase relationship between voltage and current in different AC circuits.

Phasor diagram

The diagram which shows various phasors and their phase relations is called phasor diagram. Consider a sinusoidal alternating voltage $v = V_m \sin \omega t$ applied to a circuit. This voltage can be represented by a phasor, namely $\overrightarrow{OA}$ as shown in Figure 4.38.

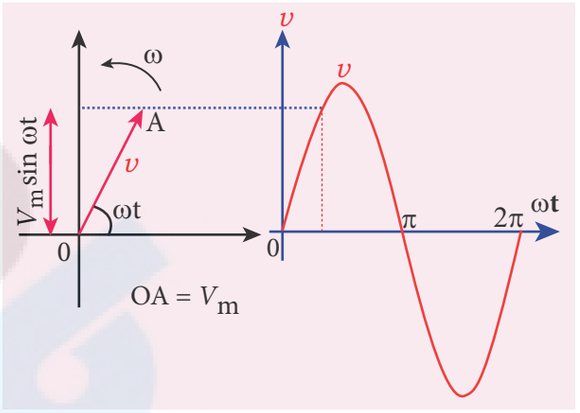


Figure 4.38 Phasor diagram for an alternating voltage $v = V_m \sin \omega t$

Here the length of $\overrightarrow{OA}$ equals the peak value (V_m), the angle it makes with x-axis gives the phase (ωt) of the applied voltage. Its projection on y-axis provides the instantaneous value ($V_m \sin \omega t$) at that instant.

When $\overrightarrow{OA}$ rotates about O with angular velocity ω in anti-clockwise direction, the waveform of the voltage is generated. For one full rotation of $\overrightarrow{OA}$, one cycle of voltage is produced.

The alternating current in the same circuit may be given by the relation $i = I_m \sin\ (\omega t + \phi)$ which is represented by

UNIT-4(XII-Physics_Vol-1).indd 237 17-02-2020 13:19:18

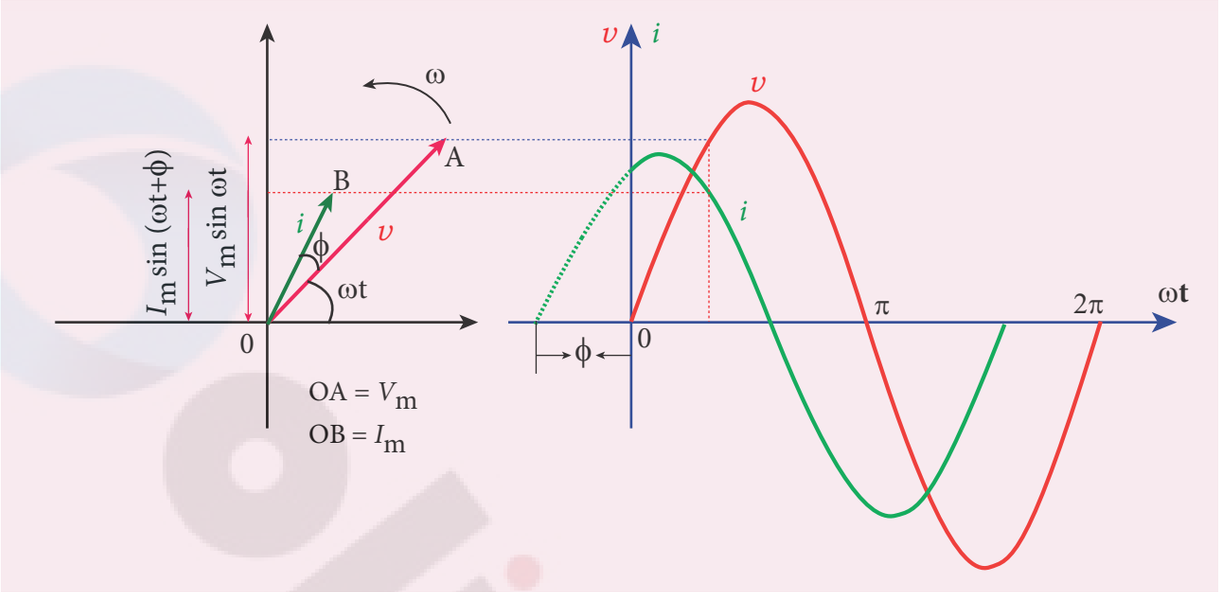


Figure 4.39 Phasor diagram and wave diagram say that i leads υ by ϕ

another phasor $\overrightarrow{OB}$. Here ϕ is the phase angle between voltage and current. In this case, the current leads the voltage by phase angle ϕ which is shown in Figure 4.39. If the current lags behind the voltage, then we write $i = I_m \sin(\omega t - \phi)$.

4.7.3 AC circuit containing pure resistor

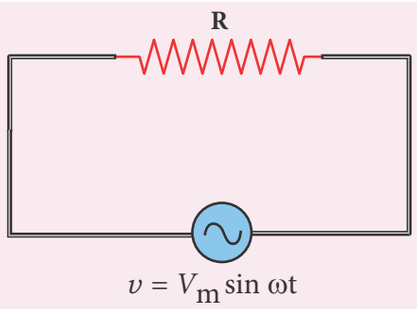


Figure 4.40 AC circuit with resistor

Consider a circuit containing a pure resistor of resistance R connected across an alternating voltage source (Figure 4.40). The instantaneous value of the alternating voltage is given by

$$\upsilon = V_m \sin \omega t \qquad (4.37)$$

An alternating current i flowing in the circuit due to this voltage, develops a potential drop across R and is given by

$$V_R = iR \qquad (4.38)$$

Kirchoff's loop rule (Refer section 2.5.2) states that the algebraic sum of potential differences in a closed circuit is zero. For this resistive circuit,

$$\upsilon - V_R = 0$$

From equation (4.37) and (4.38),

$$V_m \sin \omega t = iR$$

$$i = \frac{V_m}{R} \sin \omega t$$

$$i = I_m \sin \omega t \qquad (4.39)$$

where $\frac{V_m}{R} = I_m$, the peak value of alternating current in the circuit. From equations (4.37) and (4.38), it is clear that

UNIT-4(XII-Physics_Vol-1).indd 238 17-02-2020 13:19:21

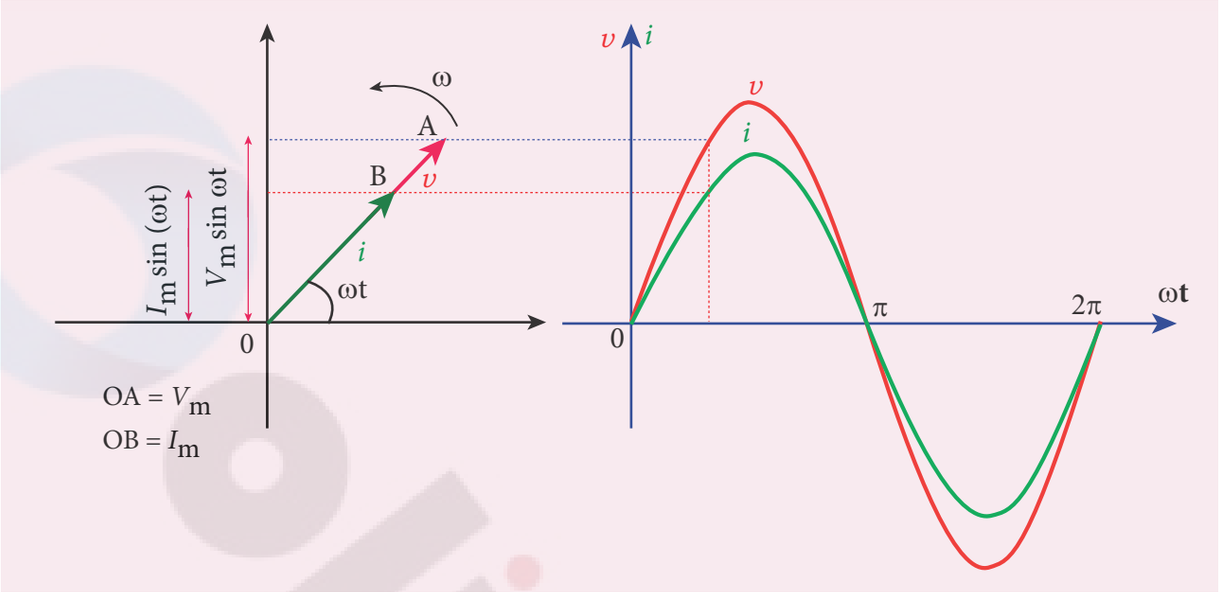


Figure 4.41 Phasor diagram and wave diagram for AC circuit with R

the applied voltage and the current are in phase with each other in a resistive circuit. It means that they reach their maxima and minima simultaneously. This is indicated in the phasor diagram (Figure 4.41). The wave diagram also depicts that current is in phase with the applied voltage (Figure 4.41).

4.7.4 AC circuit containing only an inductor

Consider a circuit containing a pure inductor of inductance L connected across an alternating voltage source (Figure 4.42). The instantaneous value of the alternating voltage is given by

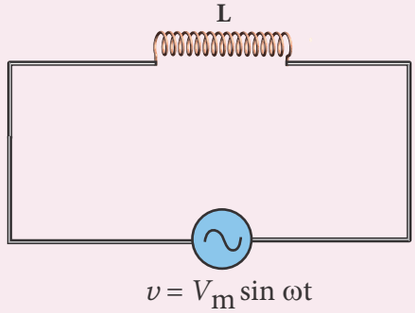


Figure 4.42 AC circuit with inductor

$$v = V_m \sin \omega t \qquad (4.40)$$

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit. The back emf is given by

$$\text{Back emf}, \varepsilon = -L\frac{di}{dt}$$

By applying Kirchoff's loop rule to the purely inductive circuit, we get

$$v + \varepsilon = 0$$
$$V_m \sin \omega t = L\frac{di}{dt}$$
$$di = \frac{V_m}{L}\sin \omega t \; dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L}\int \sin \omega t \, dt$$
$$i = \frac{V_m}{L\omega}(-\cos \omega t) + \text{constant}$$

The integration constant in the above equation is independent of time. Since the voltage in the circuit has only time dependent part, we can take the time independent part in the current (integration constant) as zero.

UNIT-4(XII-Physics_Vol-1).indd 239 17-02-2020 13:19:23

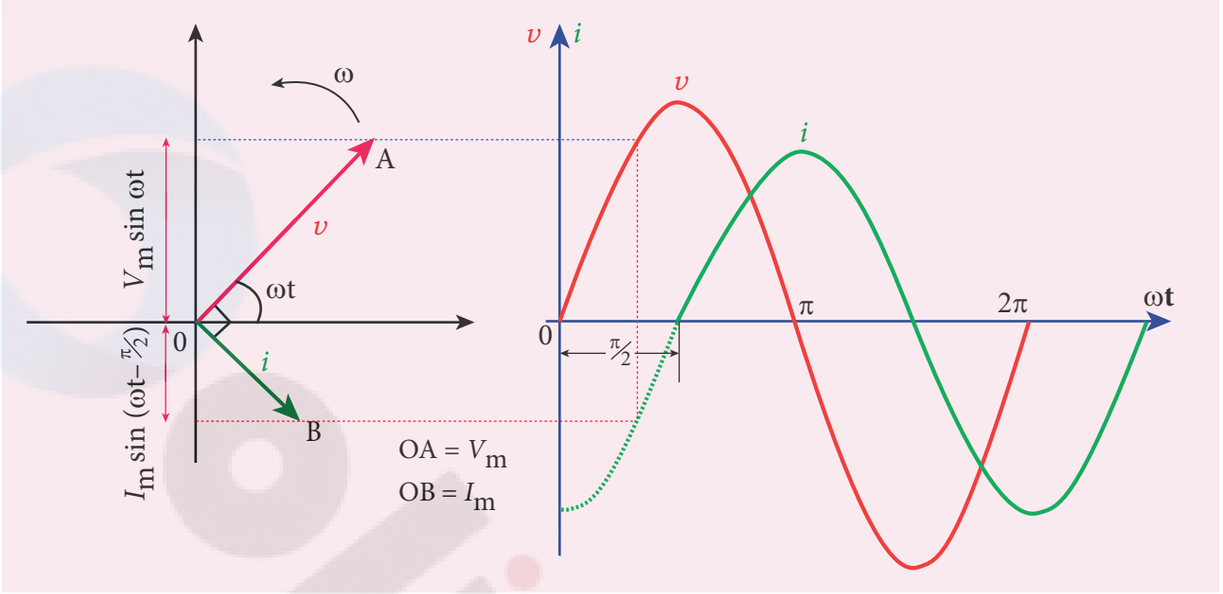


Figure 4.43 Phasor diagram and wave diagram for AC circuit with L

$$\left(\because \; -\cos\omega t = -\sin\left(\pi/2 - \omega t\right) = \sin\left(\omega t - \pi/2\right)\right)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \pi/2\right)$$

or $$i = I_m \sin\left(\omega t - \pi/2\right) \qquad (4.41)$$

where $\frac{V_m}{\omega L} = I_m$, the peak value of the alternating current in the circuit. From equation (4.40) and (4.41), it is evident that current lags behind the applied voltage by $\pi/2$ in an inductive circuit. This fact is depicted in the phasor diagram. In the wave diagram also, it is seen that current lags the voltage by 90° (Figure 4.43).

Inductive reactance X_L

The peak value of current I_m is given by $I_m = \frac{V_m}{\omega L}$. Let us compare this equation with $I_m = \frac{V_m}{R}$ from resistive circuit. The quantity ωL plays the same role as the resistance in resistive circuit. This is the resistance offered by the inductor, called inductive reactance (X_L). It is measured in ohm.

$$X_L = \omega L$$

An inductor blocks AC but it allows DC. Why? and How?

An inductor L is a closely wound helical coil. The steady DC current flowing through L produces uniform magnetic field around it and the magnetic flux linked remains constant. Therefore there is no self-induction and self-induced emf (back emf). Since inductor behaves like a resistor, DC flows through an inductor.

The AC flowing through L produces time-varying magnetic field which in turn induces self-induced emf (back emf). This back emf, according to Lenz's law, opposes any change in the current. Since AC varies both in magnitude and direction, its flow is opposed in L. For an ideal inductor of zero ohmic resistance, the back emf is equal and opposite to the applied emf. Therefore L blocks AC.

UNIT-4(XII-Physics_Vol-1).indd 240 17-02-2020 13:19:25

The inductive reactance (X_L) varies directly as the frequency.

$$X_L = 2\pi f L \qquad (4.42)$$

where f is the frequency of the alternating current. For a steady current, $f = 0$. Therefore, $X_L = 0$. Thus an ideal inductor offers no resistance to steady DC current.

4.7.5 AC circuit containing only a capacitor

Consider a circuit containing a capacitor of capacitance C connected across an alternating voltage source (Figure 4.44). The instantaneous value of the alternating voltage is given by

$$v = V_m \sin \omega t \qquad (4.43)$$

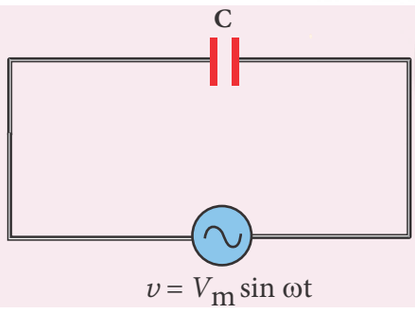


Figure 4.44 AC circuit with capacitor

Let q be the instantaneous charge on the capacitor. The emf across the capacitor at that instant is $\frac{q}{C}$. According to Kirchoff's loop rule,

$$v - \frac{q}{C} = 0$$

$$q = CV_m \sin \omega t$$

By the definition of current,

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(CV_m \sin \omega t\right)$$

$$= CV_m \frac{d}{dt}\left(\sin \omega t\right)$$

$$= CV_m \omega \cos \omega t \qquad \text{(or)}$$

$$i = \frac{V_m}{1/C\omega} \sin\left(\omega t + \pi/2\right)$$

Instantaneous value of current,

$$i = I_m \sin\left(\omega t + \pi/2\right) \qquad (4.44)$$

where $\frac{V_m}{1/C\omega} = I_m$, the peak value of the alternating current. From equations (4.43) and (4.44), it is clear that current leads the applied voltage by $\pi/2$ in a capacitive circuit. This is shown pictorially in Figure 4.45. The wave diagram for a capacitive circuit also shows that the current leads the applied voltage by 90°.

Capacitive reactance X_C

The peak value of current I_m is given by $I_m = \frac{V_m}{1/C\omega}$. Let us compare this equation with $I_m = \frac{V_m}{R}$ for a resistive circuit. The quantity $1/C\omega$ plays the same role as the resistance R in resistive circuit. This is the resistance offered by the capacitor, called capacitive reactance (X_C). It measured in ohm.

$$X_c = \frac{1}{\omega C} \qquad (4.45)$$

The capacitive reactance (X_C) varies inversely as the frequency. For a steady current, $f = 0$.

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{0} = \infty$$

Thus a capacitive circuit offers infinite resistance to the steady current. So that steady current cannot flow through the capacitor.

UNIT-4(XII-Physics_Vol-1).indd 241 17-02-2020 13:19:30

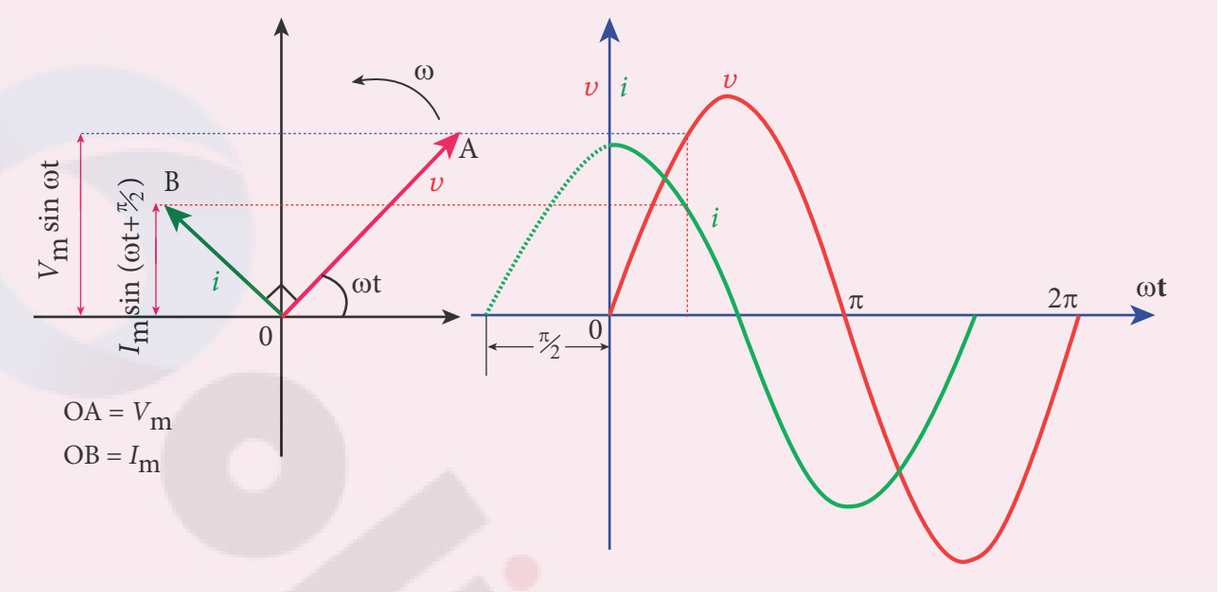
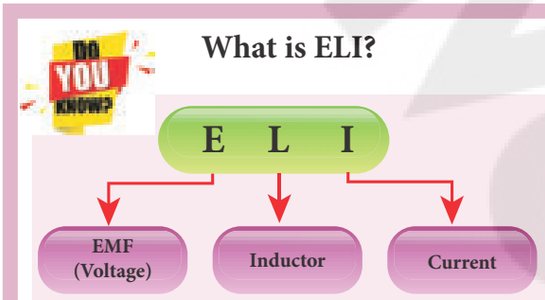


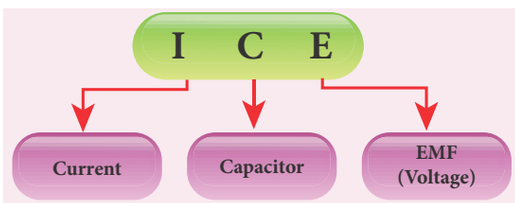
Figure 4.45 Phasor diagram and wave diagram for AC circuit with C

What is ELI?



ELI is an acronym which means that EMF (voltage) leads the current in an inductive circuit.

What is ICE?



ICE is an acronym which means that the current leads the EMF (voltage) current in a capacitive circuit.

EXAMPLE 4.20

A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.

Solution

$L = 400 \times 10^{-3}$ H; $I_{eff} = 6 \times 10^{-3}$A

$f = 1000$ Hz

Inductive reactance, $X_L = L\omega = L \times 2\pi f$

$= 2 \times 3.14 \times 1000 \times 0.4$

$= 2512\ \Omega$

Voltage across L,

$V = I X_L = 6 \times 10^{-3} \times 2512$

$V = 15.072\ \text{V}(RMS)$

EXAMPLE 4.21

A capacitor of capacitance $\frac{10^2}{\pi}\mu$F is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current.

Solution

$C = \frac{10^2}{\pi} \times 10^{-6}$ F, $V_{RMS} = 220$ V; $f = 50$ Hz

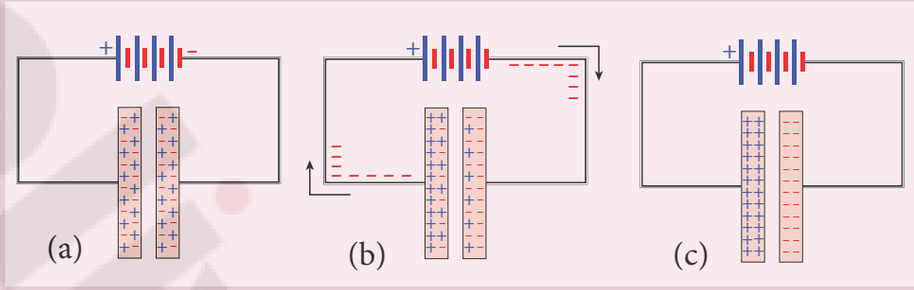
UNIT-4(XII-Physics_Vol-1).indd 242 17-02-2020 13:19:33

A capacitor blocks DC but it allows AC. Why? and How? (Not for examination)

Capacitors have two parallel metallic plates placed close to each other and there is a gap between plates. Whenever a source of voltage (either DC voltage or AC voltage) is connected across a capacitor C, the electrons from the source will reach the plate and stop. They cannot jump across the gap between plates to continue its flow in the circuit. Therefore the electrons flowing in one direction (i.e. DC) cannot pass through the capacitor. But the electrons from AC source seem to flow through C. Let us see what really happens!

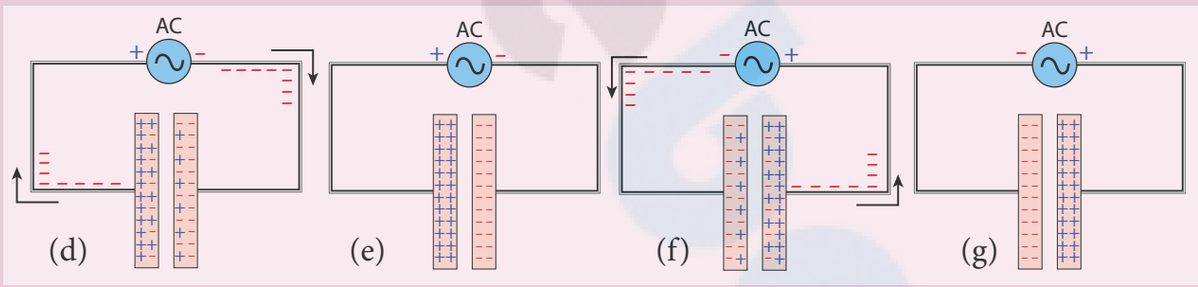
DC cannot flow through a capacitor:

Consider a parallel plate capacitor whose plates are uncharged (same amount of positive and negative charges). A DC source (battery) is connected across C as shown in Figure (a).



As soon as battery is connected, electrons start to flow from the negative terminal and are accumulated at the right plate, making it negative. Due to this negative potential, the electrons present in the nearby left plate are repelled and are moved towards positive terminal of the battery. When electrons leave the left plate, it becomes positively charged. This process is known as charging. The direction of flow of electrons is shown by arrows.

The charging of the plates continues till the level of the battery. Once C is fully charged and current will stop. At this time, we say that capacitor is blocking DC Figure (c).



AC flows (?!) through a capacitor:

Now an AC source is connected across C. At an instant, the right side of the source is at negative potential, then the electrons flow from negative terminal to the right plate and from left plate to the positive terminal as shown in Figure (d) but no electron crosses the gap between the plates. These electron-flows are represented by arrows. Thus, the charging of the plates takes place and the plates become fully charged (Figure (e)).

After a short time, the polarities of AC source are reversed and the right side of the source is now positive. The electrons which were accumulated in the right plate start to flow to the positive terminal and the electrons from negative terminal flow to the left plate to neutralize the positive charges stored in it. As a result, the net charges present in the plates begin to decrease and this is called discharging. These electron-flows are represented by arrows as shown in Figure (f). Once the charges are exhausted, C will be charged again but with reversed polarities as shown in Figure (g).

Thus the electrons flow in one direction while charging the capacitor and its direction is reversed while discharging (the conventional current is also opposite in both cases). Though electrons flow in the circuit, no electron crosses the gap between the plates. In this way, AC flows through a capacitor.

UNIT-4(XII-Physics_Vol-1).indd 243 17-02-2020 13:19:34

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times \pi \times 50 \times \frac{10^{-4}}{\pi}} = 100\,\Omega$$

(ii) RMS value of current,

$$I_{RMS} = \frac{V_{RMS}}{X_C} = \frac{220}{100} = 2.2\,\text{A}$$

(iii) $V_m = 220 \times \sqrt{2} = 311\ \text{V}$

$$I_m = 2.2 \times \sqrt{2} = 3.1\text{A}$$

Therefore,

$$v = 311 \sin 314t$$

$$i = 3.1 \sin\left(314t + \pi/2\right)$$

4.7.6 AC circuit containing a resistor, an inductor and a capacitor in series – Series RLC circuit

Consider a circuit containing a resistor of resistance R, an inductor of inductance L and a capacitor of capacitance C connected across an alternating voltage source (Figure 4.46). The instantaneous value of the alternating voltage is given by

$$v = V_m \sin \omega t$$

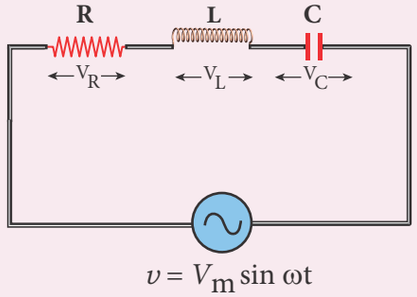


Figure 4.46 AC circuit containing R, L and C

Let i be the resulting current in the circuit at that instant. As a result, the voltage is developed across R, L and C.

We know that voltage across R (V_R) is in phase with i, voltage across L (V_L) leads i by $\pi/2$ and voltage across C (V_C) lags behind i by $\pi/2$.

The phasor diagram is drawn with current as the reference phasor. The current is represented by the phasor $\overrightarrow{OI}$, V_R by $\overrightarrow{OA}$; V_L by $\overrightarrow{OB}$ and V_C by $\overrightarrow{OC}$ as shown in Figure 4.47.

The length of these phasors are

$$OI = I_m,\ OA = I_m R,\ OB = I_m X_L;\ OC = I_m X_C$$

The circuit is either effectively inductive or capacitive or resistive depending on the value of V_L or V_C. Let us assume that $V_L > V_C$. Therefore, net voltage drop across L-C combination is $V_L - V_C$ which is represented by a phasor $\overrightarrow{OD}$.

By parallelogram law, the diagonal $\overrightarrow{OE}$ gives the resultant voltage v of V_R and $(V_L - V_C)$ and its length OE is equal to V_m. Therefore,

$$V_m^2 = V_R^2 + \left(V_L - V_C\right)^2$$

$$V_m = \sqrt{\left(I_m R\right)^2 + \left(I_m X_L - I_m X_C\right)^2}$$

$$= I_m \sqrt{R^2 + \left(X_L - X_C\right)^2} \quad \text{or}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} \quad \text{or} \qquad (4.46)$$

$$I_m = \frac{V_m}{Z}$$

where $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$ (4.47)

Z is called impedance of the circuit which refers to the effective opposition to the current by the series RLC circuit.

UNIT-4(XII-Physics_Vol-1).indd 244 17-02-2020 13:19:41

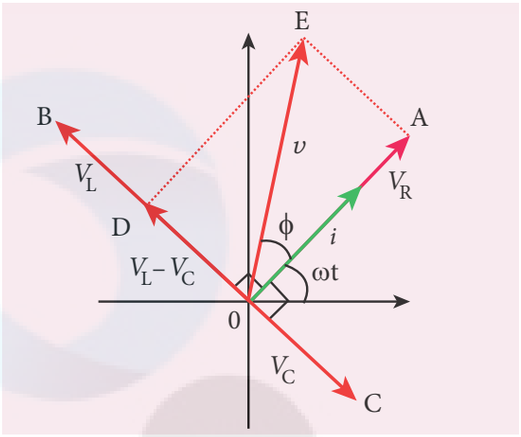


Figure 4.47 Phasor diagram for a series *RLC* – circuit when $V_L > V_C$

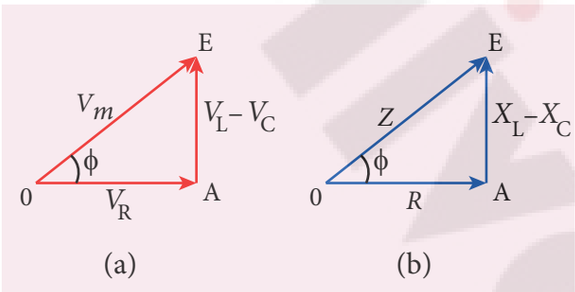


Figure 4.48 Voltage and impedance triangle when $X_L > X_C$

The voltage triangle and impedance triangle are given in the Figure 4.48.

From phasor diagram, the phase angle between v and i is found out from the following relation

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \qquad (4.48)$$

Special cases

(i) If $X_L > X_C$, $(X_L - X_C)$ is positive and phase angle ϕ is also positive. It means that the applied voltage leads the current by ϕ (or current lags behind voltage by ϕ). The circuit is inductive.

$$\therefore v = V_m \sin\omega t;\ i = I_m \sin(\omega t - \phi)$$

(ii) If $X_L < X_C$, $(X_L - X_C)$ is negative and ϕ is also negative. Therefore current leads voltage by ϕ and the circuit is capacitive.

$$\therefore v = V_m \sin\omega t;\ i = I_m \sin(\omega t + \phi)$$

(iii) If $X_L = X_C$, ϕ is zero. Therefore current and voltage are in the same phase and the circuit is resistive.

$$\therefore v = V_m \sin\omega t;\ i = I_m \sin\omega t$$

4.7.7 Resonance in series RLC Circuit

When the frequency of the applied alternating source (ω_r) is equal to the natural frequency $\left[1/\sqrt{LC}\right]$ of the *RLC* circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in **electrical resonance**. The frequency at which resonance takes place is called **resonant frequency**.

Table 4.1 Summary of results of AC circuits

Type of Impedance	Value of Impedance	Phase angle of current with voltage	Power factor
Resistance	R	0°	1
Inductance	$X_L = \omega L$	90° lag	0
Capacitance	$X_C = 1/\omega C$	90° lead	0
R- L - C	$\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}$	Between 0° and 90° lag or lead	Between 0 and 1

UNIT-4(XII-Physics_Vol-1).indd 245 17-02-2020 13:19:44

Resonant angular frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\text{or } f_r = \frac{1}{2\pi\sqrt{LC}} \qquad (4.49)$$

At series resonance,

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ or } \omega_r^2 = \frac{1}{LC}$$

$$\omega_r L = \frac{1}{\omega_r C} \text{ or}$$

$$X_L = X_C \qquad (4.50)$$

This is the condition for resonance in RLC circuit.

Since X_L and X_C are frequency dependent, the resonance condition $(X_L = X_C)$ can be achieved by varying the frequency of the applied voltage.

Effects of series resonance

When series resonance occurs, the impedance of the circuit is minimum and is equal to the resistance of the circuit. As a result of this, the current in the circuit becomes maximum. This is shown in the resonance curve drawn between current and frequency (Figure 4.49).

At resonance, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R \quad \text{since} X_L = X_C$$

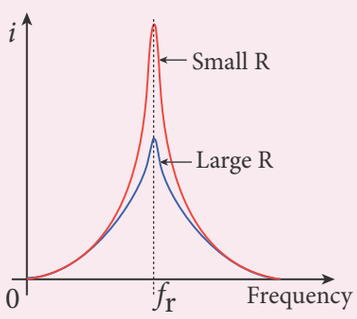


Figure 4.49 Resonance curve

Therefore, the current in the circuit is

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_m = \frac{V_m}{R} \qquad (4.51)$$

The maximum current at series resonance is limited by the resistance of the circuit. For smaller resistance, larger current with sharper curve is obtained and vice versa.

Applications of series RLC resonant circuit

RLC circuits have many applications like filter circuits, oscillators, voltage multipliers etc. An important use of series RLC resonant circuits is in the tuning circuits of radio and TV systems. The signals from many broadcasting stations at different frequencies are available in the air. To receive the signal of a particular station, tuning is done.

The tuning is commonly achieved by varying capacitance of a parallel plate variable capacitor, thereby changing the resonant frequency of the circuit. When resonant frequency is nearly equal to the frequency of the signal of the particular station, the amplitude of the current in the circuit is maximum. Thus the signal of that station alone is received.

Note

The phenomenon of electrical resonance is possible when the circuit contains both *L* and *C*. Only then the voltage across *L* and *C* cancel one another when V_L and V_C are 180° out of phase and the circuit becomes purely resistive. This implies that resonance will not occur in *RL* and *RC* circuits.

UNIT-4(XII-Physics_Vol-1).indd 246 17-02-2020 13:19:47

4.7.8 Quality factor or Q–factor

The current in the series *RLC* circuit becomes maximum at resonance. Due to the increase in current, the voltage across *L* and *C* are also increased. This magnification of voltages at series resonance is termed as Q–factor.

It is defined as the ratio of voltage across *L* or *C* at resonance to the applied voltage.

$$\text{Q-factor} = \frac{\text{Voltage across L or C at resonance}}{\text{Applied voltage}}$$

At resonance, the circuit is purely resistive. Therefore, the applied voltage is equal to the voltage across *R*.

$$\text{Q-factor} = \frac{I_m X_L}{I_m R} = \frac{X_L}{R} \qquad (4.52)$$

$$= \frac{\omega_r L}{R}$$

$$= \frac{L}{R\sqrt{LC}} \qquad \text{since } \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (4.53)$$

The physical meaning is that Q–factor indicates the number of times the voltage across *L* or *C* is greater than the applied voltage at resonance.

EXAMPLE 4.22

Find the impedance of a series *RLC* circuit if the inductive reactance, capacitive reactance and resistance are 184 Ω, 144 Ω and 30 Ω respectively. Also calculate the phase angle between voltage and current.

Solution

$X_L = 184\ \Omega; X_C = 144\ \Omega$

$R = 30\ \Omega$

(i) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{30^2 + (184 - 144)^2}$$

$$= \sqrt{900 + 1600}$$

$$Z = 50\ \Omega$$

(ii) Phase angle ϕ between voltage and current is

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$= \frac{184 - 144}{30} = 1.33$$

$$\phi = 53.1^\circ$$

Since the phase angle is positive, voltage leads current by 53.1° for this inductive circuit.

EXAMPLE 4.23

A 500 μH inductor, $\frac{80}{\pi^2} pF$ capacitor and a 628 Ω resistor are connected to form a series *RLC* circuit. Calculate the resonant frequency and Q-factor of this circuit at resonance.

Solution

$L = 500 \times 10^{-6}\text{H}; C = \frac{80}{\pi^2} \times 10^{-12}\text{F};\ R = 628\,\Omega$

(i) Resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500 \times 10^{-6} \times \frac{80}{\pi^2} \times 10^{-12}}}$$

$$= \frac{1}{2\sqrt{40{,}000 \times 10^{-18}}}$$

$$= \frac{10{,}000 \times 10^3}{4} = 2500\ \text{KHz}$$

(ii) Q–factor

$$= \frac{\omega_r L}{R} = \frac{2 \times 3.14 \times 2500 \times 10^3 \times 500 \times 10^{-6}}{628}$$

$$Q = 12.5$$

UNIT-4(XII-Physics_Vol-1).indd 247 17-02-2020 13:19:53

EXAMPLE 4.24

Find the instantaneous value of alternating voltage $v = 10\sin(3\pi \times 10^4\, t)$ volt at i) 0 s ii) 50 μs iii) 75 μs.

Solution

The given equation is $v = 10\sin\left(3\pi \times 10^4 t\right)$

(i) At t = 0 s,

$$\upsilon = 10\sin 0^\circ = 0 \text{ V}$$

(ii) At t = 50 μs,

$$\begin{aligned} v &= 10\sin\left(3\pi \times 10^4 \times 50 \times 10^{-6}\right) \\ &= 10\sin\left(150\pi \times 10^{-2} \times \frac{180^\circ}{\pi}\right) \\ &= 10\sin(270^\circ) = 10 \times -1 \\ &= -10 \text{ V} \end{aligned}$$

(iii) At t = 75 μs,

$$\begin{aligned} v &= 10\sin\left(3\pi \times 10^4 \times 75 \times 10^{-6}\right) \\ &= 10\sin\left(225\pi \times 10^{-2} \times \frac{180^\circ}{\pi}\right) \\ &= 10\sin(405^\circ) = 10\sin 45^\circ \\ &= 10 \times {}^{1}\!/\!_{\sqrt{2}} = 7.07 \text{ V} \end{aligned}$$

EXAMPLE 4.25

The current in an inductive circuit is given by 0.3 sin (200t – 40°) A. Write the equation for the voltage across it if the inductance is 40 mH.

Solution

$L = 40 \times 10^{-3}$ H; $i = 0.3$ sin (200t – 40°)

$$X_L = \omega L = 200 \times 40 \times 10^{-3} = 8\ \Omega$$
$$V_m = I_m X_L = 0.3 \times 8 = 2.4 \text{ V}$$

In an inductive circuit, the voltage leads the current by 90°. Therefore,

$$v = V_m \sin\left(\omega t + 90^\circ\right)$$
$$v = 2.4\sin\left(200t - 40^\circ + 90^\circ\right)$$
$$v = 2.4\sin\left(200t + 50^\circ\right)\text{V}$$

4.8 POWER IN AC CIRCUITS

4.8.1 Introduction of power in AC circuits

Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It is given by the product of the voltage and current. In an AC circuit, the voltage and current vary continuously with time. Let us first calculate the power at an instant and then it is averaged over a complete cycle.

The alternating voltage and alternating current in the series inductive *RLC* circuit at an instant are given by

$$v = V_m \sin\omega t \quad \text{and} \quad i = I_m \sin\left(\omega t + \phi\right)$$

where ϕ is the phase angle between v and i. The instantaneous power is then written as

$$\begin{aligned} P &= v\, i \\ &= V_m I_m \sin\omega t \sin\left(\omega t + \phi\right) \\ &= V_m I_m \sin\omega t\left[\sin\omega t\cos\phi + \cos\omega t\sin\phi\right] \\ P &= V_m I_m\left[\cos\phi\sin^2\omega t + \sin\omega t\cos\omega t\sin\phi\right] \end{aligned} \qquad (4.54)$$

Here the average of $\sin^2\omega t$ over a cycle is ${}^{1}\!/\!_{2}$ and that of $\sin\omega t\cos\omega t$ is zero. Substituting these values, we obtain average power over a cycle.

$$\begin{aligned} P_{av} &= V_m I_m \cos\phi \times {}^{1}\!/\!_{2} \\ &= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi \\ P_{av} &= V_{RMS}\, I_{RMS}\cos\phi \end{aligned} \qquad (4.55)$$

where $V_{RMS} I_{RMS}$ is called apparent power and cos ϕ is power factor. The average power of an AC circuit is also known as the true power of the circuit.

UNIT-4(XII-Physics_Vol-1).indd 248 17-02-2020 13:19:59

Special Cases

(i) For a purely resistive circuit, the phase angle between voltage and current is zero and cos $\phi = 1$.

$$\therefore P_{av} = V_{RMS}\ I_{RMS}$$

(ii) For a purely inductive or capacitive circuit, the phase angle is $\pm \pi/2$ and $\cos\left(\pm \pi/2\right) = 0$.

$$\therefore P_{av} = 0$$

(iii) For series RLC circuit, the phase angle $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$$\therefore P_{av} = V_{RMS}\ I_{RMS}\ \cos\phi$$

(iv) For series RLC circuit at resonance, the phase angle is zero and $\cos\phi = 1$.

$$\therefore P_{av} = V_{RMS}\ I_{RMS}$$

4.8.2 Wattless current

Consider an AC circuit in which there is a phase angle of ϕ between V_{RMS} and I_{RMS} and voltage is assumed to be leading the current by ϕ as shown in the phasor diagram (Figure 4.50).

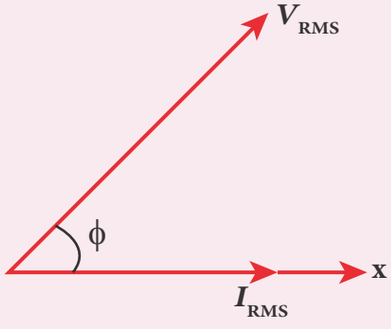


Figure 4.50 V_{RMS} leads I_{RMS} by ϕ

Now, I_{RMS} is resolved into two perpendicular components, namely $I_{RMS}\cos\phi$ along V_{RMS} and $I_{RMS}\sin\phi$ perpendicular to V_{RMS} as shown in Figure 4.51.

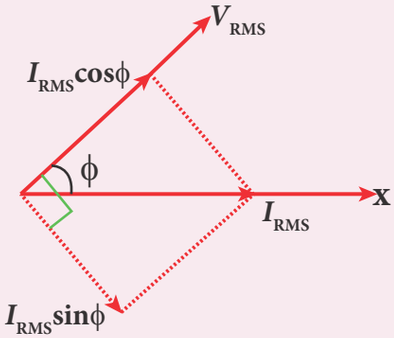


Figure 4.51 The components of I_{RMS}

(i) The component of current $\left(I_{RMS}\cos\phi\right)$ which is in phase with the voltage is called active component. The power consumed by this current $= V_{RMS} I_{RMS}\cos\phi$. So that it is also known as 'Wattful' current.

(ii) The other component $\left(I_{RMS}\sin\phi\right)$ which has a phase angle of $\pi/2$ with the voltage is called reactive component. The power consumed is zero. Hence it is also known as 'Wattless' current.

The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This wattless current occurs in a purely inductive or capacitive circuit.

4.8.3 Power factor

The power factor of a circuit is defined in one of the following ways:

(i) **Power factor = cos ϕ = cosine of the angle of lead or lag**

(ii) **Power factor** $= \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$

(iii) **Power factor** $= \frac{P_{av}}{V_{RMS} I_{RMS}}$

$= \frac{\text{True power}}{\text{Apparent power}}$

UNIT-4(XII-Physics_Vol-1).indd 249 17-02-2020 13:20:03

Some examples for power factors:

(i) Power factor = cos 0° = 1 for a pure resistive circuit because the phase angle ϕ between voltage and current is zero.

(ii) Power factor $= \cos\left(\pm \pi/2\right) = 0$ for a purely inductive or capacitive circuit because the phase angle ϕ between voltage and current is $\pm \pi/2$.

(iii) Power factor lies between 0 and 1 for a circuit having R, L and C in varying proportions.

4.8.4 Advantages and disadvantages of AC over DC

There are many advantages and disadvantages of AC system over DC system.

Advantages:

(i) The generation of AC is cheaper than that of DC.

(ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.

(iii) AC can easily be converted into DC with the help of rectifiers.

Disadvantages:

(i) Alternating voltages cannot be used for certain applications such as charging of batteries, electroplating, electric traction etc.

(ii) At high voltages, it is more dangerous to work with AC than DC.

EXAMPLE 4.26

A series RLC circuit which resonates at 400 kHz has 80 μH inductor, 2000 pF capacitor and 50 Ω resistor. Calculate (i) Q-factor of the circuit (ii) the new value of capacitance when the value of inductance is doubled and (iii) the new Q-factor.

Solution

$L = 80 \times 10^{-6}$ H; $C = 2000 \times 10^{-12}$ F

$R = 50\ \Omega$; $f_r = 400 \times 10^3$ Hz

(i) Q-factor, $Q_1 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{50}\sqrt{\frac{80\times10^{-6}}{2000\times10^{-12}}} = 4$$

(ii) When $L_2 = 2\ L$

$= 2 \times 80 \times 10^{-6}$ H

$= 160 \times 10^{-6}$ H,

$$C_2 = \frac{1}{4\pi^2 f_r^2 L_2}$$

$$= \frac{1}{4\times3.14^2\times\left(400\times10^3\right)^2\times160\times10^{-6}}$$

$$\simeq 1000\times10^{-12}\ \text{F}$$

$$C_2 \simeq 1000\ \text{pF}$$

(iii) $Q_2 = \frac{1}{R}\sqrt{\frac{L_2}{C_2}} = \frac{1}{50}\sqrt{\frac{160\times10^{-6}}{1000\times10^{-12}}}$

$$= \frac{1}{50}\sqrt{\frac{16\times10^{-5}}{10^{-9}}} = \frac{4\times10^2}{50} = 8$$

EXAMPLE 4.27

A capacitor of capacitance $\frac{10^{-4}}{\pi}$ F, an inductor of inductance $\frac{2}{\pi}$ H and a resistor of resistance 100 Ω are connected to form a series *RLC* circuit. When an AC supply of 220 V, 50 Hz is applied to the circuit, determine (i) the impedance of the circuit (ii) the peak value of current flowing in the circuit (iii) the power factor of the circuit and (iv) the power factor of the circuit at resonance.

Solution

$L = \frac{2}{\pi}$ H; $C = \frac{10^{-4}}{\pi}$ F; $R = 100\ \Omega$

$V_{RMS} = 220$ V; $f = 50$ Hz

UNIT-4(XII-Physics_Vol-1).indd 250 17-02-2020 13:20:06

$$X_L = 2\pi fL = 2\pi \times 50 \times 2/\pi = 200\,\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times \frac{10^{-4}}{\pi}} = 100\,\Omega$$

(i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{100^2 + (200-100)^2} = 141.4\ \Omega$$

(ii) Peak value of current,

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2}\,V_{RMS}}{Z}$$

$$= \frac{\sqrt{2} \times 220}{141.4} = 2.2\ \text{A}$$

(iii) Power factor of the circuit,

$$\cos\phi = \frac{R}{Z} = \frac{100}{141.4} = 0.707$$

(iv) Power factor at resonance,

$$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

4.9 OSCILLATION IN LC CIRCUITS

4.9.1 Energy conversion during LC oscillations

We have learnt that energy can be stored in both inductors and capacitors (Refer sections 1.8.2 and 4.3.2). In inductors, the energy is stored in the form of magnetic field while in capacitors, it is stored as the electric field.

Whenever energy is given to a circuit containing a pure inductor of inductance L and a capacitor of capacitance C, the energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called ***LC* oscillations**.

Generation of LC oscillations

Let us assume that the capacitor is fully charged with maximum charge Q_m at the initial stage. So that the energy stored in the capacitor is maximum and is given by $U_E = Q_m^2/2C$. As there is no current in the inductor, the energy stored in it is zero i.e., $U_B = 0$. Therefore, the total energy is wholly electrical. This is shown in Figure 4.52(a).

The capacitor now begins to discharge through the inductor that establishes current i in clockwise direction. This current produces a magnetic field around the inductor and the energy stored in the inductor is given by $U_B = Li^2/2$. As the charge in the capacitor decreases, the energy stored in it also decreases and is given by $U_E = q^2/2C$. Thus there is a transfer of some part of energy from the capacitor to the inductor. At that instant, the total energy is the sum of electrical and magnetic energies (Figure 4.52(b)).

When the charges in the capacitor are exhausted, its energy becomes zero i.e., $U_E = 0$. The energy is fully transferred to the magnetic field of the inductor and its energy is maximum. This maximum energy is given by $U_B = LI_m^2/2$ where I_m is the

UNIT-4(XII-Physics_Vol-1).indd 251 17-02-2020 13:20:10

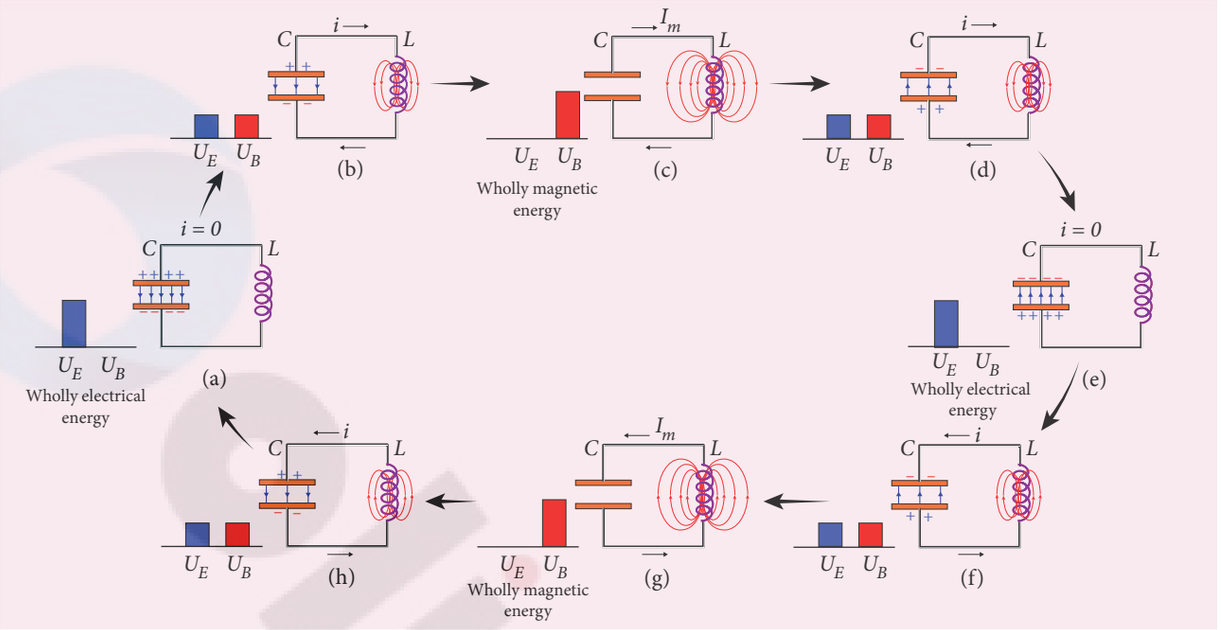


Figure 4.52 LC oscillations

maximum current flowing in the circuit. The total energy is wholly magnetic (Figure 4.52(c)).

Even though the charge in the capacitor is zero, the current will continue to flow in the same direction because the inductor will not allow it to stop immediately. The current is made to flow with decreasing magnitude by the collapsing magnetic field of the inductor. As a result of this, the capacitor begins to charge in the opposite direction. A part of the energy is transferred from the inductor back to the capacitor. The total energy is the sum of the electrical and magnetic energies (Figure 4.52(d)).

When the current in the circuit reduces to zero, the capacitor becomes fully charged in the opposite direction. The energy stored in the capacitor becomes maximum. Since the current is zero, the energy stored in the inductor is zero. The total energy is wholly electrical (Figure 4.52(e)).

The state of the circuit is similar to the initial state but the difference is that the capacitor is charged in opposite direction. The capacitor then starts to discharge through the inductor with anti-clockwise current. The total energy is the sum of the electrical and magnetic energies (Figure 4.52(f)).

As already explained, the processes are repeated in opposite direction (Figure 4.52(g) and (h)). Finally, the circuit returns to the initial state (Figure 4.52(a)). Thus, when the circuit goes through these stages, an alternating current flows in the circuit. As this process is repeated again and again, the electrical oscillations of definite frequency are generated. These are known as *LC* oscillations.

In the ideal *LC* circuit, there is no loss of energy. Therefore, the oscillations will continue indefinitely. Such oscillations are called undamped oscillations.

UNIT-4(XII-Physics_Vol-1).indd 252 17-02-2020 13:20:11

Note

But in practice, the Joule heating and radiation of electromagnetic waves from the circuit decrease the energy of the system. Therefore, the oscillations become damped oscillations.

4.9.2 Conservation of energy in LC oscillations

During LC oscillations in LC circuits, the energy of the system oscillates between the electric field of the capacitor and the magnetic field of the inductor. Although, these two forms of energy vary with time, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

$$\text{Total energy, } U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Let us consider 3 different stages of LC oscillations and calculate the total energy of the system.

Case (i) When the charge in the capacitor, $q = Q_m$ and the current through the inductor, $i = 0$, the total energy is given by

$$U = \frac{Q_m^2}{2C} + 0 = \frac{Q_m^2}{2C} \qquad (4.56)$$

The total energy is wholly electrical.

Case (ii) When charge = 0 ; current = I_m, the total energy is

$$U = 0 + \frac{1}{2}LI_m^2 = \frac{1}{2}LI_m^2$$

$$= \frac{L}{2} \times \left(\frac{Q_m^2}{LC}\right) \quad \text{since } I_m = Q_m\omega = \frac{Q_m}{\sqrt{LC}}$$

$$= \frac{Q_m^2}{2C} \qquad (4.57)$$

The total energy is wholly magnetic.

Case (iii) When charge = q; current = i, the total energy is

$$U = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Since $q = Q_m \cos\omega t$, $i = -\frac{dq}{dt} = Q_m\omega \sin\omega t$. The negative sign in current indicates that the charge in the capacitor decreases with time.

$$U = \frac{Q_m^2 \cos^2\omega t}{2C} + \frac{L\omega^2 Q_m^2 \sin^2\omega t}{2}$$

$$= \frac{Q_m^2 \cos^2\omega t}{2C} + \frac{LQ_m^2 \sin^2\omega t}{2LC}$$

$$\text{since } \omega^2 = \frac{1}{LC}$$

$$= \frac{Q_m^2}{2C}\left(\cos^2\omega t + \sin^2\omega t\right)$$

$$U = \frac{Q_m^2}{2C} \qquad (4.58)$$

From above three cases, it is clear that the total energy of the system remains constant.

4.9.3 Analogies between LC oscillations and simple harmonic oscillations

Qualitative treatment

The electromagnetic oscillations of LC system can be compared with the mechanical oscillations of a spring-mass system.

There are two forms of energy involved in LC oscillations. One is electrical energy of the charged capacitor; the other magnetic energy of the inductor carrying current.

UNIT-4(XII-Physics_Vol-1).indd 253 17-02-2020 13:20:16

Table 4.2 Energy in two oscillatory systems

LC oscillator		Spring-mass system	
Element	**Energy**	**Element**	**Energy**
Capacitor	Electrical Energy = $\frac{1}{2}\left(\frac{1}{C}\right)q^2$	Spring	Potential energy = $\frac{1}{2}kx^2$
Inductor	Magnetic energy = $\frac{1}{2}Li^2$ $i=\frac{dq}{dt}$	Mass	Kinetic energy = $\frac{1}{2}mv^2$ $v=\frac{dx}{dt}$

Likewise, the mechanical energy of the spring-mass system exists in two forms; the potential energy of the compressed or extended spring and the kinetic energy of the mass. The Table 4.2 lists these two pairs of energy.

By examining the Table 4.2, the analogies between the various quantities can be understood and these correspondences are given in Table 4.3.

The angular frequency of oscillations of a spring-mass is given by (Refer equation 10.22 of section 10.4.1 of XI physics text book).

$$\omega=\sqrt{\frac{k}{m}}$$

From Table 4.3, $k \to 1/C$ and $m \to L$. Therefore, the angular frequency of LC oscillations is given by

$$\omega=\frac{1}{\sqrt{LC}} \qquad (4.58)$$

Table 4.3 Analogies between electrical and mechanical quantities

Electrical system	Mechanical system
Charge q	Displacement x
Current $i=\frac{dq}{dt}$	Velocity $v=\frac{dx}{dt}$
Inductance L	Mass m
Reciprocal of capacitance $\frac{1}{C}$	Force constant k
Electrical energy $=\frac{1}{2}\left(\frac{1}{C}\right)q^2$	Potential energy $=\frac{1}{2}kx^2$
Magnetic energy $=\frac{1}{2}Li^2$	Kinetic energy $=\frac{1}{2}mv^2$
Electromagnetic energy $U=\frac{1}{2}\left(\frac{1}{C}\right)q^2+\frac{1}{2}Li^2$	Mechanical energy $E=\frac{1}{2}kx^2+\frac{1}{2}mv^2$

UNIT-4(XII-Physics_Vol-1).indd 254 17-02-2020 13:20:25

SUMMARY

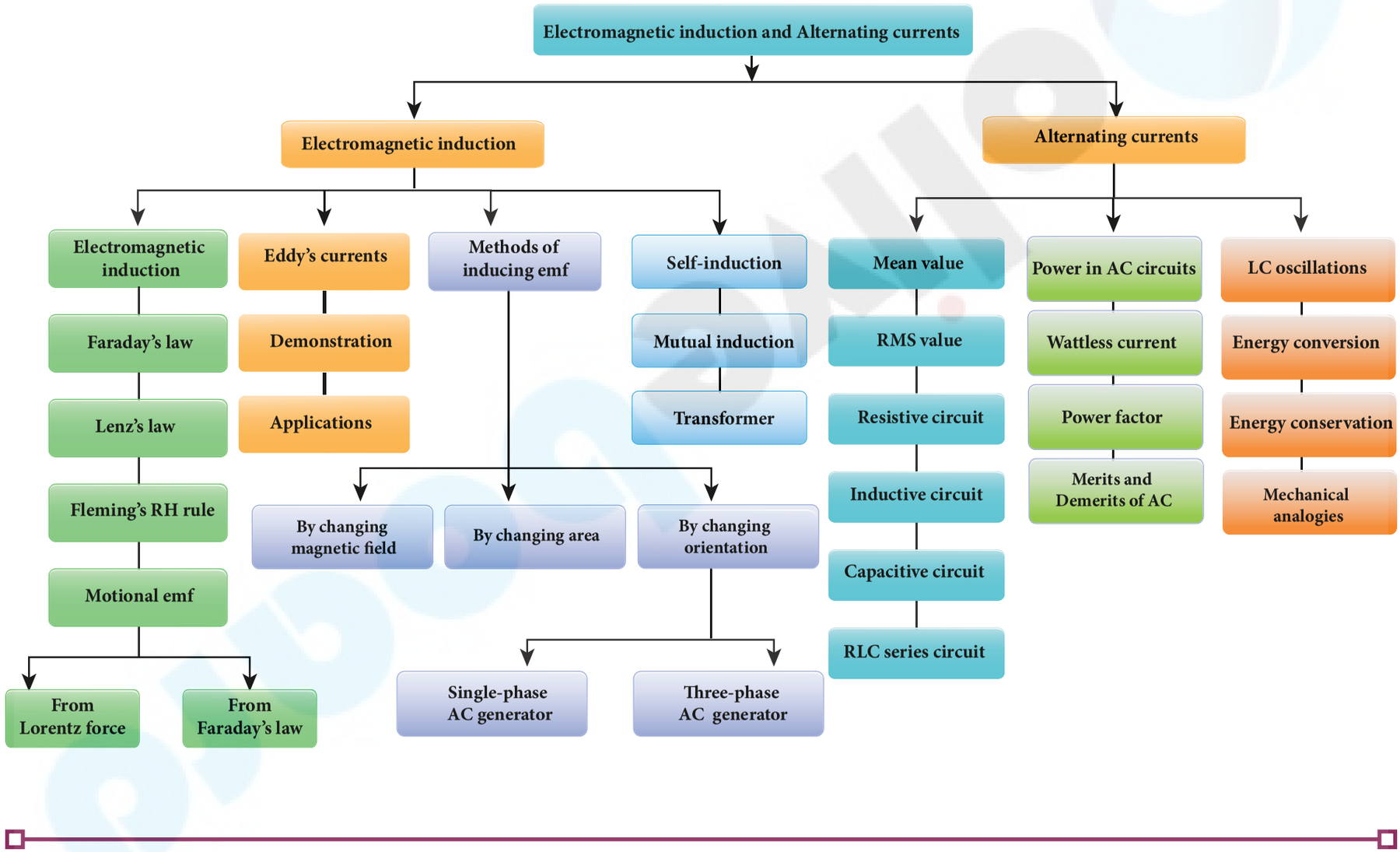
- Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This phenomenon is known as electromagnetic induction.
- Faraday's first law states that whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.
- Faraday's second law states that the magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.
- Lenz's law states that the direction of the induced current is such that it always opposes the cause responsible for its production.
- Lenz's law is established on the basis of the law of conservation of energy.
- Fleming's right hand rule states that if the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.
- Even for a conductor in the form of a sheet or a plate, an emf is induced when magnetic flux linked with it changes. The induced currents flow in concentric circular paths called Eddy currents or Foucault currents.
- Inductor is a device used to store energy in a magnetic field when an electric current flows through it.
- If the flux linked with the coil is changed, an emf is induced in that same coil. This phenomenon is known as self-induction. The emf induced is called self-induced emf.
- When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction and the emf is called mutually induced emf.
- AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.
- In some AC generators, there are three separate coils, which would give three separate emfs. Hence they are called three-phase AC generators.
- Transformer is a stationary device used to transform AC electric power from one circuit to another without changing its frequency.
- The efficiency of a transformer is defined as the ratio of the useful output power to the input power.
- An alternating voltage is a voltage which changes polarity at regular intervals of time and the resulting alternating current changes direction accordingly.
- The average value of alternating current is defined as the average of all values of current over a positive half-circle or negative half-circle.
- The root mean square value or effective value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.

UNIT-4(XII-Physics_Vol-1).indd 255 17-02-2020 13:20:26

- A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity. Such a rotating vector is called a phasor.
- When the frequency of the applied alternating source is equal to the natural frequency of the RLC circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in electrical resonance.
- The magnification of voltages at series resonance is termed as Q–factor.
- Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It depends on the components of the circuit.
- Whenever energy is given to a LC circuit, the electrical oscillations of definite frequency are generated. These oscillations are called LC oscillations.
- During LC oscillations, the total energy remains constant. It means that LC oscillations take place in accordance with the law of conservation of energy.

UNIT-4(XII-Physics_Vol-1).indd 256 17-02-2020 13:20:26

CONCEPT MAP

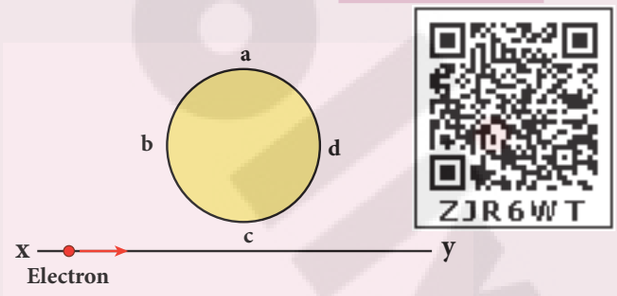


UNIT-4(XII-Physics_Vol-1).indd 257 17-02-2020 13:20:26

I Multiple Choice Questions

1. An electron moves on a straight line path XY as shown in the figure. The coil *abcd* is adjacent to the path of the electron. What will be the direction of current, if any, induced in the coil?

(NEET 2015)



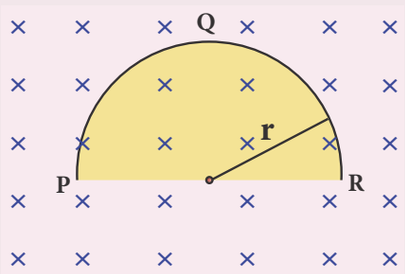
(a) The current will reverse its direction as the electron goes past the coil

(b) No current will be induced

(c) abcd

(d) adcb

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.



The potential difference developed across the ring when its speed υ, is

(NEET 2014)

(a) Zero

(b) $\frac{B\upsilon\pi r^2}{2}$ and P is at higher potential

(c) $\pi rB\upsilon$ and R is at higher potential

(d) $2rB\upsilon$ and R is at higher potential

3. The flux linked with a coil at any instant t is given by $\Phi_B = 10t^2 - 50t + 250$. The induced emf at t = 3 s is

(a) −190 V (b) −10 V

(c) 10 V (d) 190 V

4. When the current changes from +2A to −2A in 0.05 s, an emf of 8 V is induced in a coil. The co-efficient of self-induction of the coil is

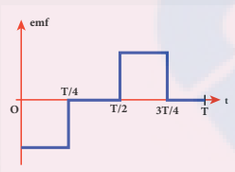
(a) 0.2 H (b) 0.4 H

(c) 0.8 H (d) 0.1 H

5. The current *i* flowing in a coil varies with time as shown in the figure. The variation of induced emf with time would be (NEET 2011)



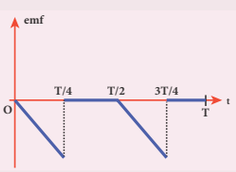
(a)



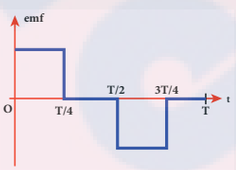
(b)



(c)



(d)



UNIT-4(XII-Physics_Vol-1).indd 258 17-02-2020 13:20:30

6. A circular coil with a cross-sectional area of 4 cm^2 has 10 turns. It is placed at the centre of a long solenoid that has 15 turns/cm and a cross-sectional area of 10 cm^2. The axis of the coil coincides with the axis of the solenoid. What is their mutual inductance?

(a) 7.54 μH (b) 8.54 μH

(c) 9.54 μH (d) 10.54 μH

7. In a transformer, the number of turns in the primary and the secondary are 410 and 1230 respectively. If the current in primary is 6A, then that in the secondary coil is

(a) 2 A (b) 18 A

(c) 12 A (d) 1 A

8. A step-down transformer reduces the supply voltage from 220 V to 11 V and increase the current from 6 A to 100 A. Then its efficiency is

(a) 1.2 (b) 0.83

(c) 0.12 (d) 0.9

9. In an electrical circuit, *R, L, C* and AC voltage source are all connected in series. When *L* is removed from the circuit, the phase difference between the voltage and current in the circuit is $\pi/3$. Instead, if C is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is

(NEET 2012)

(a) $1/2$ (b) $1/\sqrt{2}$

(c) 1 (d) $\sqrt{3}/2$

10. In a series RL circuit, the resistance and inductive reactance are the same. Then the phase difference between the voltage and current in the circuit is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$ (d) zero

11. In a series resonant *RLC* circuit, the voltage across 100 Ω resistor is 40 V. The resonant frequency ω is 250 rad/s. If the value of C is 4 μF, then the voltage across L is

(a) 600 V (b) 4000 V

(c) 400V (d) 1 V

12. An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of emf V = 10 sin 340 t. The power loss in AC circuit is

(a) 0.76 W (b) 0.89 W

(c) 0.46 W (d) 0.67 W

13. The instantaneous values of alternating current and voltage in a circuit are

$i = \frac{1}{\sqrt{2}}\sin(100\pi t)$ A and

$v = \frac{1}{\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{3}\right)$ V.

The average power in watts consumed in the circuit is (IIT Main 2012)

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q

UNIT-4(XII-Physics_Vol-1).indd 259 17-02-2020 13:20:35

15. $\frac{20}{\pi^2}$H inductor is connected to a capacitor of capacitance C. The value of C in order to impart maximum power at 50 Hz is

(a) 50 μF (b) 0.5 μF

(c) 500 μF (d) 5 μF

Answers

1) a	**2)** d	**3)** b	**4)** d	**5)** a
6) a	**7)** a	**8)** b	**9)** c	**10)** a
11) c	**12)** c	**13)** d	**14)** c	**15)** d

II Short Answer Questions

1. What is meant by electromagnetic induction?
2. State Faraday's laws of electromagnetic induction.
3. State Lenz's law.
4. State Fleming's right hand rule.
5. How is Eddy current produced? How do they flow in a conductor?
6. Mention the ways of producing induced emf.
7. What for an inductor is used? Give some examples.
8. What do you mean by self-induction?
9. What is meant by mutual induction?
10. Give the principle of AC generator.
11. List out the advantages of stationary armature-rotating field system of AC generator.
12. What are step-up and step-down transformers?
13. Define average value of an alternating current.
14. How will you define RMS value of an alternating current?
15. What are phasors?
16. Define electric resonance.
17. What do you mean by resonant frequency?
18. How will you define Q-factor?
19. What is meant by wattles current?
20. Give any one definition of power factor.
21. What are LC oscillations?

III Long Answer Questions

1. Establish the fact that the relative motion between the coil and the magnet induces an emf in the coil of a closed circuit.
2. Give an illustration of determining direction of induced current by using Lenz's law.
3. Show that Lenz's law is in accordance with the law of conservation of energy.
4. Obtain an expression for motional emf from Lorentz force.
5. Give the uses of Foucault current.
6. Define self-inductance of a coil interms of (i) magnetic flux and (ii) induced emf.
7. How will you define the unit of inductance?
8. What do you understand by self-inductance of a coil? Give its physical significance.
9. Assuming that the length of the solenoid is large when compared to its diameter, find the equation for its inductance.

UNIT-4(XII-Physics_Vol-1).indd 260 17-02-2020 13:20:35

10. An inductor of inductance L carries an electric current i. How much energy is stored while establishing the current in it?
11. Show that the mutual inductance between a pair of coils is same ($M_{12} = M_{21}$).
12. How will you induce an emf by changing the area enclosed by the coil?
13. Show mathematically that the rotation of a coil in a magnetic field over one rotation induces an alternating emf of one cycle.
14. Elaborate the standard construction details of AC generator.
15. Explain the working of a single-phase AC generator with necessary diagram.
16. How are the three different emfs generated in a three-phase AC generator? Show the graphical representation of these three emfs.
17. Explain the construction and working of transformer.
18. Mention the various energy losses in a transformer.
19. Give the advantage of AC in long distance power transmission with an illustration.
20. Find out the phase relationship between voltage and current in a pure inductive circuit.
21. Derive an expression for phase angle between the applied voltage and current in a series RLC circuit.
22. Define inductive and capacitive reactance. Give their units.
23. Obtain an expression for average power of AC over a cycle. Discuss its special cases.
24. Explain the generation of LC oscillations in a circuit containing an inductor of inductance L and a capacitor of capacitance C.
25. Prove that the total energy is conserved during LC oscillations.
26. Compare the electromagnetic oscillations of LC circuit with the mechanical oscillations of block-spring system qualitatively to find the expression for angular frequency of LC oscillator.

IV. Numerical problems

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil. (Ans: 9 Wb)
2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire. (Ans: 10 mV)
3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\Phi_B = (2t^3 + 4t^2 + 8t + 8)$ Wb. If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time t = 3 second. (Ans: 17.2 A)
4. A closely wound circular coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced in it. Calculate the number of turns in the coil. (Ans: 35 turns)

UNIT-4(XII-Physics_Vol-1).indd 261 17-02-2020 13:20:36

5. A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180°. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

(Ans: 48×10^{-3} C)

6. An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.

(Ans: 250 $mWbs^{-1}$)

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of 4×10^{-3}T. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

(Ans: 9.95 revolutions/second)

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of 4×10^{-5}T. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.

(Ans: 250 revolutions/second)

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m. (Ans: 12.62 mH)

10. A coil of 200 turns carries a current of 4 *A*. If the magnetic flux through the coil is 6×10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil. (Ans: 0.024 J)

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux linked with each turn when it carries a current of 1 A.

(Ans: 0.63×10^{-4} Wb)

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with each turn of the coil, find the inductance of the coil. (Ans: 2H)

13. Two air core solenoids have the same length of 80 cm and same cross–sectional area 5 cm^2. Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns. (Ans: 0.38 mH)

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm^2 is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

(Ans: 0.20 V)

15. A 200 turn circular coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil and the solenoid.

(Ans: 2.84 mH)

16. The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. Their areas of their cross-section and their lengths are the same and are 4 cm^2 and 0.04 m respectively. If the number of turns in S_1 is 200 and

UNIT-4(XII-Physics_Vol-1).indd 262 17-02-2020 13:20:37

that in S_2 is 800, calculate the mutual inductance between the solenoids. If the current in solenoid 1 is increased form 2A to 8A in 0.04 second, calculate the induced emf in solenoid 2.

(Ans: 1.81H; –271.5 V)

17. A step-down transformer connected to main supply of 220 V is used to operate 11V,88W lamp. Calculate (i) Voltage transformation ratio and (ii) Current in the primary.

(Ans: 1/20 and 0.4 A)

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.

(Ans: 2 A)

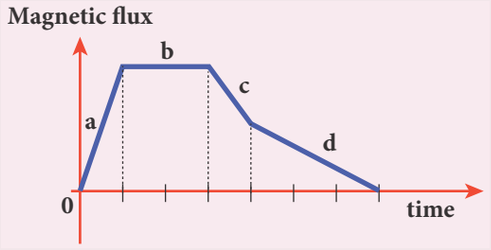
19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

(Ans: 8.2 kW and 2.48 kW)

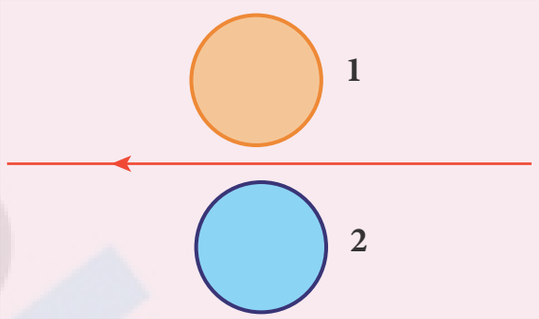
20. Calculate the instantaneous value at 60°, average value and RMS value of an alternating current whose peak value is 20 A. (Ans: 17.32A, 12.74A, 14.14A)

V. Conceptual Questions

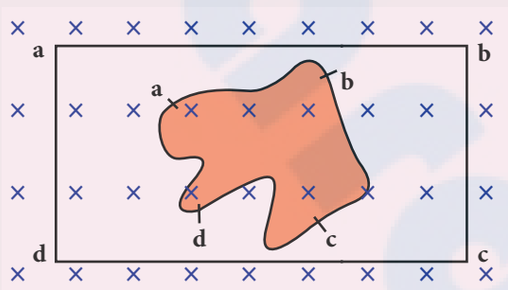
1. A graph between the magnitude of the magnetic flux linked with a closed loop and time is given in the figure. Arrange the regions of the graph in ascending order of the magnitude of induced emf in the loop.



2. Using Lenz's law, predict the direction of induced current in conducting rings 1 and 2 when current in the wire is steadily decreasing.

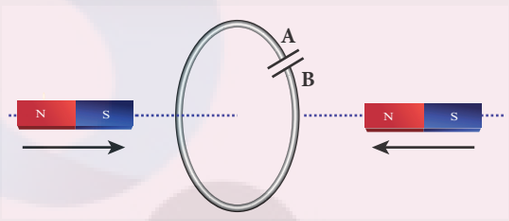


3. A flexible metallic loop abcd in the shape of a square is kept in a magnetic field with its plane perpendicular to the field. The magnetic field is directed into the paper normally. Find the direction of the induced current when the square loop is crushed into an irregular shape as shown in the figure.



UNIT-4(XII-Physics_Vol-1).indd 263 17-02-2020 13:20:37

4. Predict the polarity of the capacitor in a closed circular loop when two bar magnets are moved as shown in the figure.



25. In series LC circuit, the voltages across L and C are 180° out of phase. Is it correct? Explain.
26. When does power factor of a series RLC circuit become maximum?

BOOK FOR REFERENCES

1. H.C.Verma, Concepts of Physics, Volume 1 and 2, Bharathi Bhawan publishers.
2. Halliday, Resnick and Walker, Principles of Physics, Wiley publishers.
3. D.C.Tayal, Electricity and Magnetism, Himalaya Publishing House.
4. K.K.Tewari, Electricity and Magnetism with Electronics, S.Chand Publishers.
5. B.L.Theraja and A.K.Theraja, A text book of Electrical Technology, Volume 1 and 2, S.Chand publishers.

UNIT-4(XII-Physics_Vol-1).indd 264 17-02-2020 13:20:37

ICT CORNER

Electromagnetic induction and alternating current

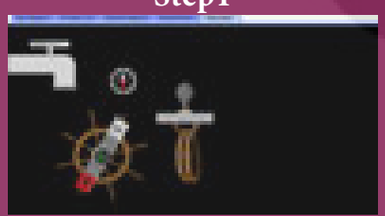
In this activity you will be able to
(1) understand electromagnetic induction.
(2) verify Faraday's laws in virtual lab.

Topic: Faraday's electromagnetic lab

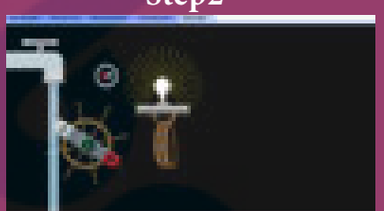
STEPS:

- Open the browser and type "phet.colorado.edu" in the address bar. Click play with simulation tab. Search Faraday's electromagnetic lab in the search box.
- Select 'pick coil' tab. Move the magnet through the coil. Note what happens when the magnetic field linked with the coil changes. Change the loop area, flux change and observe the intensity of current with the help of glowing bulb.
- Select 'Electromagnet' tab, Change the current flowing through the coil and observe the change in magnetic flux generated.
- Select 'Generator' tab. Observe induced emf in the coil if you change the angular velocity of the coil.

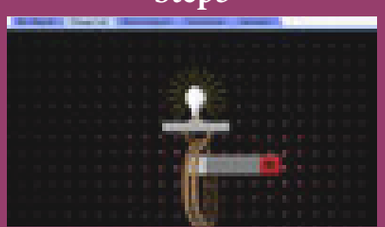
Step1



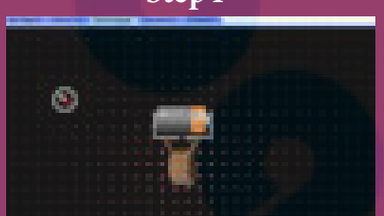
Step2



Step3



Step4



Note:

Install Java application if it is not in your system. You can download all the phet simulation and works in off line from https://phet.colorado.edu/en/offline-access.

URL:

https://phet.colorado.edu/en/simulation/legacy/faraday

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT-4(XII-Physics_Vol-1).indd 265 17-02-2020 13:20:39

UNIT 5

ELECTROMAGNETIC WAVES

"One scientific epoch ended and another began with James Clerk Maxwell"
– Albert Einstein

Learning Objectives

In this unit, the student is exposed to

- the displacement current
- Maxwell's correction to Ampere's circuital law
- Maxwell's equation in integral form
- production and properties of electromagnetic waves – Hertz's experiment
- sources of electromagnetic waves
- electromagnetic spectrum

CGD1LP

5.1 INTRODUCTION



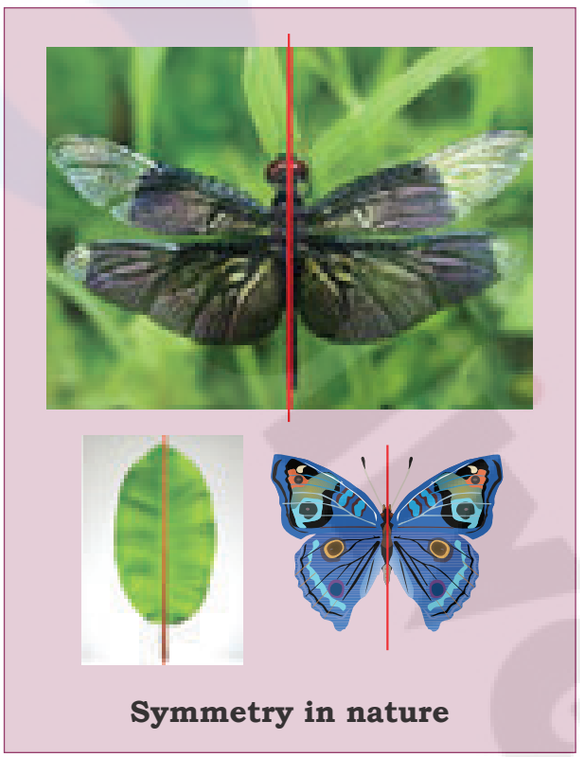
Figure 5.1 Visible spectrum – rainbow and lightning

We see the world around us through light. Light from the Sun is one of the sources of energy without which human beings cannot survive in this planet. Light plays crucial role in understanding the structure and properties of various things from atom to universe. Without light, even our eyes cannot see objects. What is light?. This puzzle made many physicists sleepless until middle of 19th century. Earlier, many scientists thought that optics and electromagnetism are two different branches of physics. But from the work of James Clerk Maxwell, who actually enlightened the concept of light from his theoretical prediction that light is an electromagnetic wave which moves with the speed equal to 3×10^8 ms^{-1} (in free space or vacuum). Later, it was confirmed that visible light is just only small portion of electromagnetic spectrum, which ranges from gamma rays to radio waves.

In unit 4, we studied that time varying magnetic field produces an electric field (Faraday's law of electromagnetic induction). Maxwell strongly believed that nature must possess symmetry and he asked

UNIT-5(XII-Physics_Vol-1).indd 266 14-02-2020 20:11:14

the following question, "when the time varying magnetic field produces an electric field, why not the time varying electric field produce a magnetic field?"



Symmetry in nature

Later he proved that it is indeed true. In 1888, H. Hertz experimentally verified Maxwell's prediction and hence, this understanding resulted in new technological invention, especially in wireless communication, LASER (Light Amplification by Stimulated Emission of Radiation) technology, RADAR (Radio Detection And Ranging) etc.

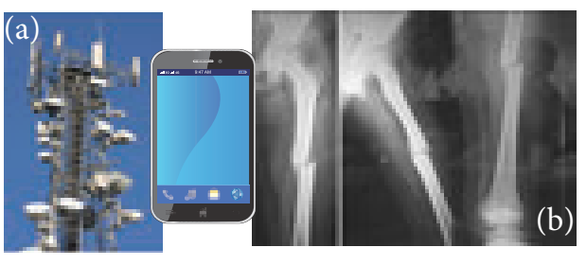


Figure 5.2 (a) Cell phone tower and cell phone (b) X-ray radiograph

In today's digital world, cell phones (Figure 5.2 (a)) have greater influence in our day to day life. It is a faster and more effective mode of transferring information from one place to another. It works on the basis that light is an electromagnetic wave. In hospitals, the location of bone fracture can be detected using X-rays as shown in Figure 5.2 (b), which is also an electromagnetic wave. For cooking microwave oven is used. The microwave is also an electromagnetic wave. There are plenty of applications of electromagnetic waves in engineering, medicine (example LASER surgery, etc), defence (example, RADAR signals) and also in fundamental scientific research. In this unit, basics of electromagnetic waves are discussed.

5.1.1 Displacement current and Maxwell's correction to Ampere's circuital law

Induced magnetic field

Faraday's law of electromagnetic induction states that the change in magnetic field produces an electric field. Mathematically, it is written as

$$\underbrace{\oint_l \vec{E}\cdot d\vec{l}}_{\text{Electric field induced along a closed loop}} = \underbrace{-\frac{d}{dt}\Phi_B}_{\text{Variation of magnetic flux with time}} = \underbrace{-\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{A}}_{\text{Changing magnetic flux } \Phi_B \text{ in the region enclosed by the loop}} \quad (5.1)$$

where Φ_B is the magnetic flux and $\frac{d}{dt}$ is the total derivative with respect to time. Equation (5.1) means that the electric field $\vec{E}$ is induced along a closed loop by the changing magnetic flux Φ_B in the region encircled by the loop.

UNIT-5(XII-Physics_Vol-1).indd 267 14-02-2020 20:11:15

From symmetry considerations, James Clerk Maxwell showed that the change in electric field also produces a magnetic field which is given by

$$\underbrace{\oint_l \vec{B}\cdot\vec{dl}}_{\text{Magnetic field induced along a closed loop}} = \underbrace{-\frac{d}{dt}\Phi_E}_{\text{Variation of electric flux with time}} = \underbrace{-\frac{d}{dt}\oint_S \vec{E}\cdot\vec{dA}}_{\text{Changing electric flux } \Phi_E \text{ in the region enclosed by the loop}} \quad (5.2)$$

where Φ_E is the electric flux. This is known as **Maxwell's law of induction** which explains that the magnetic field $\vec{B}$ is induced along a closed loop by the changing electric flux Φ_E in the region encircled by that loop. This symmetry between electric and magnetic fields explains the existence of electromagnetic waves such as radio waves, gamma rays, infrared rays etc.

Displacement current – Maxwell's correction

In order to understand how the changing electric field induces magnetic field, let us consider a situation of charging a parallel plate capacitor which contains non–conducting medium between the plates.

Let a time–dependent current i_c, called conduction current be passed through the wire to charge the capacitor.

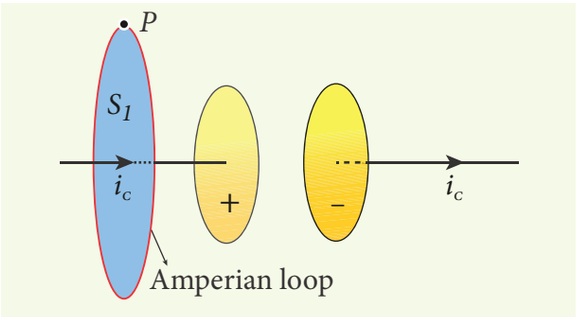


Figure 5.3 Applying Ampere's circuital law - loop enclosing surface

Ampere's circuital law can be used to find the magnetic field produced around the current carrying wire.

To calculate the magnetic field at a point P near the wire and outside the capacitor, let us draw a circular Amperian loop which encloses the circular surface S_1 (Figure 5.3). Using Ampere's circuital law for this loop, we get

$$\oint_{\text{enclosing } S_1} \vec{B}\cdot\vec{dl} = \mu_0 i_C \quad (5.3)$$

where μ_o is the permeability of free space.

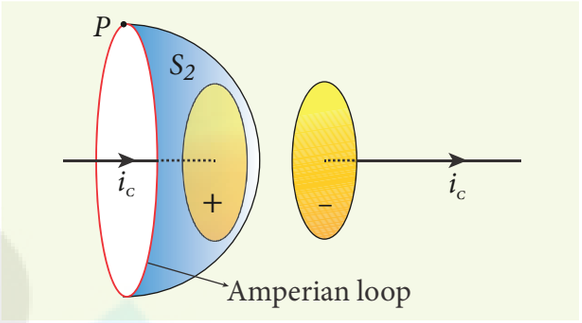


Figure 5.4 Applying Ampere's circuital law - loop enclosing surface S_2

Now, the same loop is enclosed by balloon shaped surface S_2 such that boundaries of two surfaces S_1 and S_2 are same but the shape of the surfaces is different (Figure 5.4). As Ampere's law applied for a given closed loop does not depend on the shape of the enclosing surface, the integrals should give the same answer. But by applying Ampere's circuital law for the surface S_2, we get

$$\oint_{\text{enclosing } S_2} \vec{B}\cdot\vec{dl} = 0 \quad (5.4)$$

The right hand side of equation is zero because the surface S_2 nowhere touches the wire carrying conduction current and further, there is no current flowing between

UNIT-5(XII-Physics_Vol-1).indd 268 14-02-2020 20:11:17

the plates of the capacitor (gap between the plates). So the magnetic field at a point P is zero. Hence there is an inconsistency between equations (5.3) and (5.4).

Maxwell resolved this inconsistency as follows: While the capacitor is being charged up, varying electric field is produced between capacitor plates. There must be a current associated with the changing electric field between capacitor plates. In other words, time–varying electric field (or time–varying electric flux) produces a current. This is known as displacement current flowing between the plates of the capacitor (Figure 5.5).

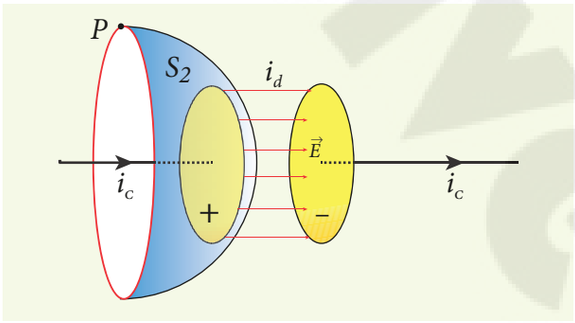


Figure 5.5 Applying Gauss's law between the plates of the capacitor

From Gauss's law of electrostatics, the electric flux between the plates of the capacitor is

$$\Phi_E = \oint_S \vec{E}\cdot d\vec{A} = EA = \frac{q}{\epsilon_\circ}$$

where A is the area of the plates of capacitor. The change in electric flux is given by

$$\frac{d\Phi_E}{dt} = \frac{1}{\epsilon_o}\frac{dq}{dt} \quad \text{(or)}$$

$$\frac{dq}{dt} = \epsilon_o \frac{d\Phi_E}{dt}$$

$$i_d = \epsilon_o \frac{d\Phi_E}{dt} \qquad (5.5)$$

where $\frac{dq}{dt} = i_d$ is known as displacement current or Maxwell's displacement current.

The **displacement current can be defined as the current which comes into play in the region in which the electric field (or the electric flux) is changing with time**. In other words, whenever the change in electric field takes place, displacement current is produced.

Maxwell modified Ampere's law as

$$\oint_l \vec{B}.d\vec{l} = \mu_0 i = \mu_0 [i_c + i_d]$$

$$\oint_l \vec{B}.d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \qquad (5.6)$$

where the total current enclosed by the surface becomes the sum of conduction current and displacement current. Therefore, $i = i_c + i_d$. The equation (5.6) is known as **Ampere–Maxwell law**. When the current in the circuit is constant, the displacement current is zero.

Between the plates, the conduction current is zero while the displacement current is non–zero. This displacement current or time–varying electric field can also produce a magnetic field between the plates of the capacitor. The magnetic field at a point inside the capacitor is perpendicular to the electric field and is shown in Figure 5.6. This magnetic field can be determined using equation (5.6).

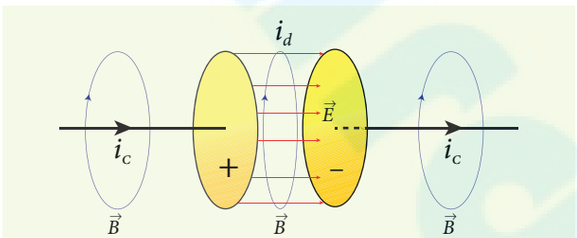


Figure 5.6 Magnetic field produced by conduction and displacement currents

UNIT-5(XII-Physics_Vol-1).indd 269 14-02-2020 20:11:18

Importance of Maxwell's correction:

Earth receives radiations from Sun and other stars. These radiations travel through empty space where there are no electric charges and hence no electric current. Ampere's law says that only electric current can produce a magnetic field. If Ampere's law alone is true, there will not be any radiation.

Maxwell's correction term $\left(\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}\right)$ in Ampere's law ensures that time–varying electric field or displacement current can also produce a magnetic field. Though conduction current is zero in an empty space, displacement current does exist. So, the equation (5.6) becomes

$$\oint_l \vec{B}.d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In stars, due to thermal excitation of atoms, time–varying electric field is produced which in turn, produces time–varying magnetic field. According to Faraday's law, this time–varying magnetic field produces again time–varying electric field and so on. The coupled time–varying electric and magnetic fields travel through empty space with the speed of light and is called electromagnetic wave.

Even though Maxwell initially started with purely symmetry argument, his correction term explains one of the important aspects of the universe, namely the existence of electromagnetic waves.



Displacement current
The name stuck because Maxwell named it. The word displacement is poorly chosen because nothing is being displaced here.

EXAMPLE 5.1

Consider a parallel plate capacitor which is connected to an 230 V RMS value and 50 Hz frequency. If the separation distance between the plates of the capacitor and area of the plates are 1 mm and 20 cm^2 respectively. Calculate the displacement current at t = 1 s.

Solution

Potential difference between the plates of the capacitor,

$$V = V_{max} \sin 2\pi ft$$
$$= 230\sqrt{2}\ \sin(2\pi \times 50t)$$
$$\therefore V = 325 \sin 100\pi t$$

d = 1 mm = 1×10^{-3} m

A = 20 cm^2 = 20×10^{-4} m^2

Displacement current, $i_d = \epsilon_\circ \frac{d\Phi_E}{dt} = \epsilon_\circ \frac{d(EA)}{dt}$

$$\therefore i_d = \frac{\epsilon_0 A}{d}\left[\frac{dV}{dt}\right] \qquad \left[\because E = \frac{V}{d}\right]$$
$$= \frac{\epsilon_0 A}{d}(325)(100\pi)\cos 100\pi t$$
$$= \left(8.85\times10^{-12} \times 20\times10^{-4} \times 325 \times 100 \times 3.14 \times \cos(100\pi \times 1)\right) \Big/ \left(1\times10^{-3}\right)$$
$$= 1.81\times10^{-6}\ \text{A} = 1.81\ \mu\text{A} \quad [\because \cos(100\pi \times 1) = 1]$$

5.1.3 Maxwell's equations in integral form

Electrodynamics can be summarized in four basic equations, known as Maxwell's equations. These equations are analogous to Newton's equations in mechanics. Maxwell's equations completely explain the behaviour of charges, currents and properties of electric and magnetic fields. These equations can be written in integral form (or integration form) or derivative form (or differential form). The

UNIT-5(XII-Physics_Vol-1).indd 270 14-02-2020 20:11:19

differential form of Maxwell's equation is beyond higher secondary level. So we focus only the integral form of Maxwell's equations.

First equation

It is nothing but the Gauss's law of electricity. It relates the net electric flux to net electric charge enclosed in a surface. Mathematically, it is expressed as

$$\oint_s \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_\circ}$$

(Gauss's law for electricity) (5.7)

where $\vec{E}$ is the electric field and Q enclosed is the net charge enclosed by the surface S. This equation is true for both discrete and continuous distribution of charges.

It also indicates that the electric field lines start from positive charge and terminate at negative charge. This implies that the electric field lines do not form a continuous closed path. In other words, it means that an isolated positive charge or negative charge can exist.

Second equation

This law is similar to Gauss's law for electricity. So this law can also be called as Gauss's law for magnetism. The surface integral of magnetic field over a closed surface is zero. Mathematically,

$$\oint_s \vec{B}\cdot d\vec{A} = 0$$

(Gauss's law for magnetism) (5.8)

where $\vec{B}$ is the magnetic field.

This equation implies that the magnetic lines of force form a continuous closed path. In other words, it means that no isolated magnetic monopole exists.

Third equation

It is Faraday's law of electromagnetic induction. This law relates electric field with the changing magnetic flux which is mathematically written as

$$\oint_l \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B \quad \text{(Faraday's law)} \qquad (5.9)$$

where $\vec{E}$ is the electric field. This equation implies that the line integral of the electric field around any closed path is equal to the rate of change of magnetic flux through the closed path bounded by the surface.

Our modern technological revolution is due to Faraday's laws of electromagnetic induction.

Fourth equation

It is modified Ampere's circuital law. This is also known as Ampere – Maxwell law. This law relates the magnetic field around any closed path to the conduction current and displacement current through that path.

$$\oint_l \vec{B}.\vec{dl} = \mu_0 i_C + \mu_0\epsilon_0 \frac{d}{dt}\oint_s \vec{E}.\vec{dA}$$

(Ampere-Maxwell law) (5.10)

where $\vec{B}$ is the magnetic field. This equation shows that both conduction current and displacement current produce magnetic field.

These four equations are known as Maxwell's equations in electrodynamics. This equation ensures the existence of electromagnetic waves. The entire communication system in the world depends on electromagnetic waves. In fact our understanding of stars, galaxy, planets etc come by analysing the electromagnetic waves emitted by these astronomical objects.

UNIT-5(XII-Physics_Vol-1).indd 271 14-02-2020 20:11:21

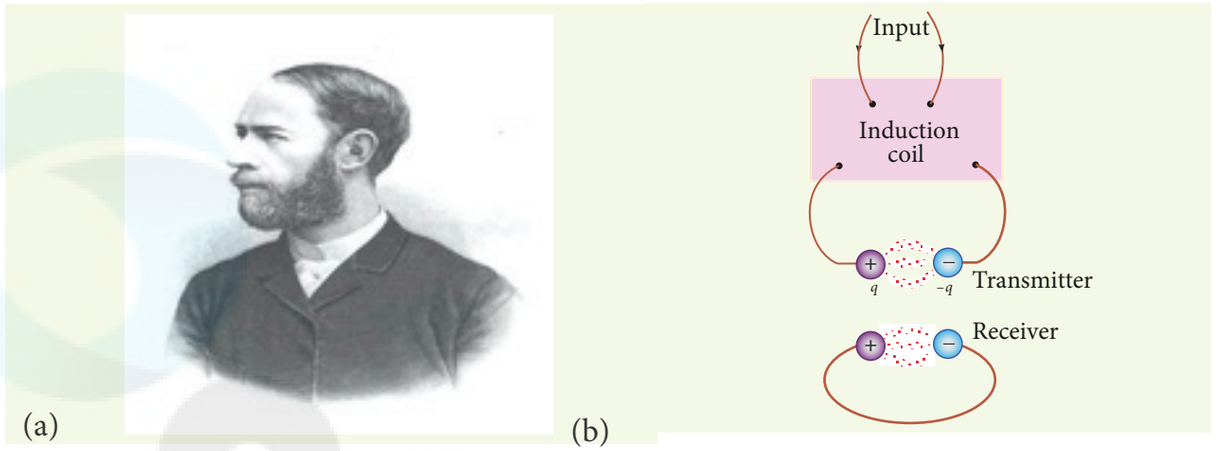


Figure 5.7 (a) Heinrich Rudolf Hertz (b) Schematic diagram of Hertz apparatus

5.2 ELECTROMAGNETIC WAVES

Electromagnetic waves are non-mechanical waves which move with speed equals to the speed of light (in vacuum). It is a transverse wave. In the following subsections, we discuss the production of electromagnetic waves and its properties, sources of electromagnetic waves and also classification of electromagnetic spectrum.

5.2.1 Production and properties of electromagnetic waves

Production of electromagnetic waves - Hertz experiment

Maxwell's prediction was experimentally confirmed by Heinrich Rudolf Hertz in 1888. The experimental set up used is shown in Figure 5.7 (b).

It consists of two metal electrodes which are made of small spherical metals. These are connected to larger spheres and the ends of them are connected to induction coil with very large number of turns. This is to produce very high electromotive force (emf).

Since the coil is maintained at very high potential, air between the electrodes gets ionized and spark (spark means discharge of electricity) is produced. This discharge of electricity affects another electrode (ring type — not completely closed) which is kept at far distance. This implies that the energy is transmitted from electrode to the receiver (ring electrode) in the form of waves, known as electromagnetic waves.

If the receiver is rotated by 90°, then no spark is observed by the receiver. This confirms that electromagnetic waves are transverse waves as predicted by Maxwell. Hertz detected radio waves and also computed the speed of radio waves which is equal to the speed of light (3×10^8 m s^{-1}).

Properties of electromagnetic waves

1. Electromagnetic waves are produced by any accelerated charge.
2. Electromagnetic waves do not require any medium for propagation. So electromagnetic wave is a non-mechanical wave.
3. Electromagnetic waves are transverse in nature. The oscillating electric field

UNIT-5(XII-Physics_Vol-1).indd 272 14-02-2020 20:11:21

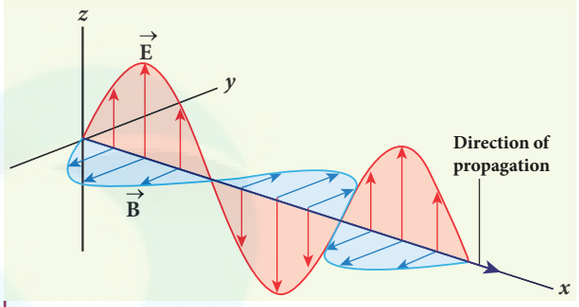


Figure 5.8 Electromagnetic waves – transverse wave

vector, oscillating magnetic field vector and propagation vector (gives direction of propagation) are mutually perpendicular to each other. For example, if the electric and magnetic fields are as shown in Figure 5.8, then the direction of propagation will be along x-direction.

4. Electromagnetic waves travel with speed which is equal to the speed of light in vacuum or free space, $c = \frac{1}{\sqrt{\epsilon_\circ \mu_\circ}} = 3\times10^8\,\mathrm{m\,s^{-1}}$, where ϵ_o is the permittivity of free space or vacuum and μ_o is the permeability of free space or vacuum (refer Unit 1 for permittivity and Unit 3 for permeability).

5. In a medium with permittivity ε and permeability μ, the speed of electromagnetic wave υ is less than that in free space or vacuum ($\upsilon < c$).

In a medium of refractive index,

$$n = \frac{c}{\upsilon} = \frac{1/\sqrt{\epsilon_\circ \mu_\circ}}{1/\sqrt{\epsilon\mu}} \quad \therefore\ n = \sqrt{\epsilon_r \mu_r}$$

where ϵ_r is the relative permittivity of the medium (also known as dielectric constant) and μ_r is the relative permeability of the medium.

6. Electromagnetic waves are not deflected by electric field or magnetic field.

7. Electromagnetic waves can exhibit interference, diffraction and polarization.

8. Like other waves, electromagnetic waves also carry energy, linear momentum and angular momentum.



- It is surprising to realize that EM waves have linear momentum and angular momentum like particles. In the year 2018, Nobel prize in physics was awarded for the invention of optical tweezers and production of high intense light pulses.
- Optical tweezer is nothing but a laser light, used to move micro sized particles or molecules from one location to another location. It has a lot of applications in the medical field. The bacteria and virus can alone be separated from regular tissue using this optical tweezer and cancerous cells can be separated from normal healthy cells. The optical tweezer utilizes momentum property of EM waves.
- In fact, the comet has tail shape because the sun light impart large amount of linear momentum which pushes the masses of the comet away from the sun.
- Angular momentum of EM waves can be understood in simple way. Consider a setup of oppositely-charged coaxial cylindrical shells and in between them a solenoid is kept. An AC current is flowing through it and when the current in the solenoid is reduced to zero, then the inner and outer cylindrical shells start to rotate in opposite directions. The rotation of these cylinders is due to the impart of angular momentum from the electromagnetic field produced by the AC current.

UNIT-5(XII-Physics_Vol-1).indd 273 14-02-2020 20:11:22

EXAMPLE 5.2

The relative magnetic permeability of the medium is 2.5 and the relative electrical permittivity of the medium is 2.25. Compute the refractive index of the medium.

Solution

Dielectric constant (relative permittivity of the medium), $\epsilon_r = 2.25$

Magnetic permeability, $\mu_r = 2.5$

Refractive index of the medium,

$$n = \sqrt{\epsilon_r \mu_r} = \sqrt{2.25 \times 2.5} = 2.37$$

5.2.2 Sources of electromagnetic waves

Any stationary charge produces only electric field (refer Unit 1). When the charge moves with uniform velocity, it produces steady current which gives rise to magnetic field (not time dependent, only space dependent) around the conductor in which charge flows. If the charged particle accelerates, it produces magnetic field in addition to electric field. Both electric and magnetic fields are time varying fields. Since the electromagnetic waves are transverse waves, the direction of propagation of electromagnetic waves is perpendicular to the planes containing electric and magnetic field vectors.

Any oscillatory motion is also an accelerated motion. So, when the charge oscillates (oscillating molecular dipole) about their mean position (Figure 5.9), it produces electromagnetic waves.

Suppose the electromagnetic field in free space propagates along z-direction and if the electric field vector points along x-axis, then the magnetic field vector will be mutually perpendicular to both electric field and the direction of wave propagation. Thus

$$E_x = E_o \sin(kz - \omega t)$$

$$B_y = B_o \sin(kz - \omega t)$$

where E_o and B_o are amplitudes of oscillating electric and magnetic field, k is a wave number, ω is the angular frequency of the

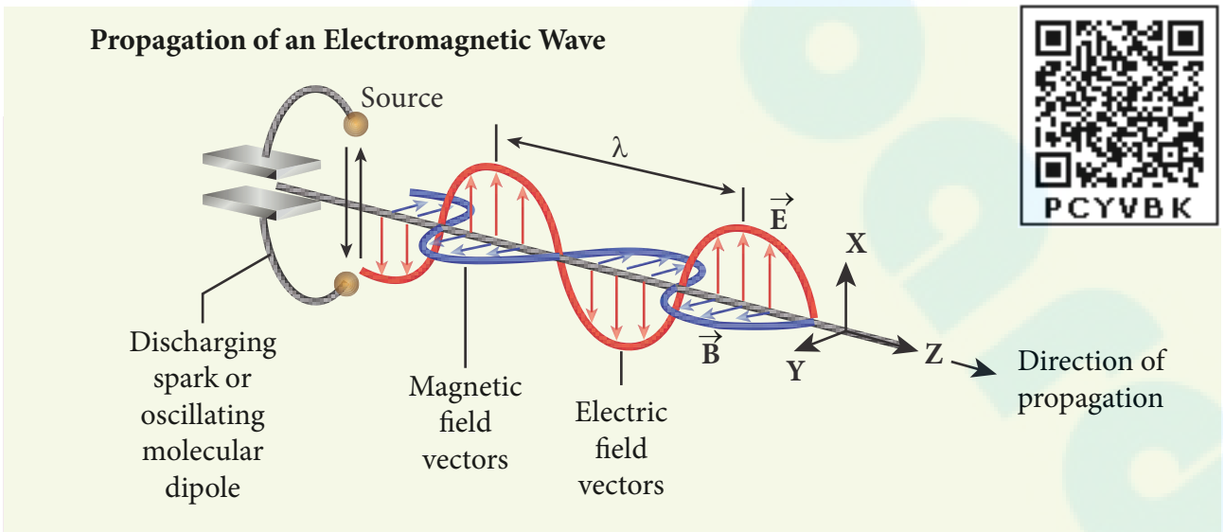


Figure 5.9 Oscillating charges - sources of electromagnetic waves

UNIT-5(XII-Physics_Vol-1).indd 274 14-02-2020 20:11:23

wave and $\hat{k}$ (unit vector, here it is called propagation vector) denotes the direction of propagation of electromagnetic wave.

Note that both electric field and magnetic field oscillate with a frequency (frequency of electromagnetic wave) which is equal to the frequency of the source (here, oscillating charge is the source for the production of electromagnetic waves). In free space or in vacuum, the ratio between E_o and B_o is equal to the speed of electromagnetic wave and is equal to speed of light c.

$$c = \frac{E_\circ}{B_\circ}$$

In any medium, the ratio of $E_\circ$ and $B_\circ$ is equal to the speed of electromagnetic wave in that medium. Thus

$$v = \frac{E_\circ}{B_\circ} < c$$

Further, the energy of electromagnetic waves comes from the energy of the oscillating charge.

EXAMPLE 5.3

Compute the speed of the electromagnetic wave in a medium if the amplitude of electric and magnetic fields are 3×10^4 N C^{-1} and 2×10^{-4} T, respectively.

Solution

The amplitude of the electric field,
$E_\circ = 3 \times 10^4$ N C^{-1}
The amplitude of the magnetic field, $B_\circ = 2 \times 10^{-4}$ T. Therefore, speed of the electromagnetic wave in that medium is

$$v = \frac{3\times10^4}{2\times10^{-4}} = 1.5\times10^8 ms^{-1}$$

5.2.3 Electromagnetic spectrum

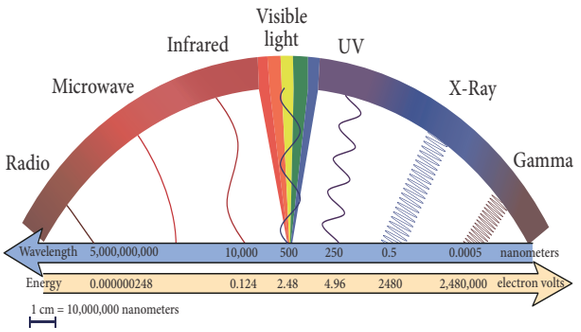


Figure 5.10 Electromagnetic spectrum -

Electromagnetic spectrum is an orderly distribution of electromagnetic waves in terms of wavelength or frequency (Figure 5.10).

Radio waves

They are produced by accelerated motion of charges in conducting wires. The frequency range is from a few Hz to 10^9 Hz. They show reflection and diffraction.

They are used in radio and television communication systems and also in cellular phones to transmit voice communication in the ultra high frequency band.

Microwaves

It is produced by special vacuum tubes such as klystron, magnetron and gunndiode. The frequency range of microwaves is 10^9 Hz to 10^{11} Hz.These waves undergo reflection and can be polarised.

It is used in radar system for aircraft navigation, speed of the vehicle, microwave oven for cooking and very long distance wireless communication through satellites.

Infrared radiation

It is produced by hot bodies (also known as heat waves) and also by when the molecules undergoing rotational and vibrational transitions. The frequency range is 10^{11} Hz to 4×10^{14} Hz.

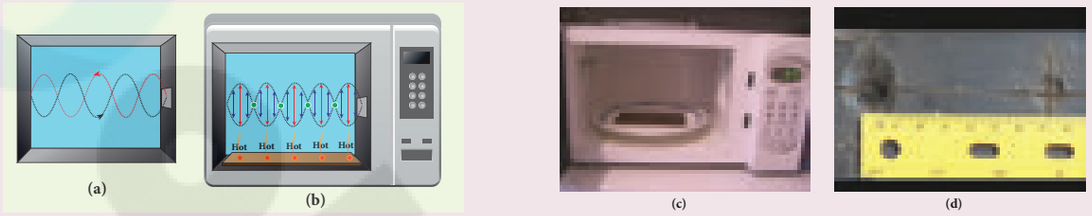
It provides electrical energy to satellites by means of solar cells. It is used to produce

UNIT-5(XII-Physics_Vol-1).indd 275 14-02-2020 20:11:24

Activity

Measuring the speed of light using the microwave oven

Nowadays the microwave oven is very commonly used to heat the food items. Micro waves of wavelengths 1 mm to 30 cm are produced in these ovens. Such waves form the standing waves between the interior walls of the oven. It is interesting to note that the speed of light can be measured using micro wave oven.



We studied about the standing waves in XI physics, Volume 2, Unit 11. The standing waves have nodes and antinodes at fixed points. At node point, the amplitude of the wave is zero and at antinodes point, the amplitude is maximum. In other words, the maximal energy of microwaves is located at antinode points. When we keep some food items like chappathi or choclate (after removing the rotating platform) inside the oven, we can notice that at antinode locations, chappathi will be burnt more than other locations. It is shown in the Figure (c) and (d). The distance between two successive burnt spots will give the half wavelength of microwave. The frequency of microwave is printed in the panel of oven. By knowing wavelength and frequency of microwaves, using the formula $v\lambda = c$, we can calculate the speed of light c.

dehydrated fruits, in green houses to keep the plants warm, heat therapy for muscular pain or sprain, TV remote as a signal carrier, to look through haze fog or mist and used in night vision or infrared photography.

Visible light

It is produced by incandescent bodies and also it is radiated by excited atoms in gases. The frequency range is from 4×10^{14} Hz to 8×10^{14} Hz.

It obeys the laws of reflection and refraction. It undergoes interference, diffraction and can be polarised. It exhibits photo-electric effect also. It can be used to study the structure of molecules, arrangement of electrons in external shells of atoms. It causes sensation of vision.

Ultraviolet radiation

It is produced by Sun, arc and ionized gases. Its frequency range is from 8×10^{14} Hz to 10^{17} Hz.

It has less penetrating power. It can be absorbed by atmospheric ozone and is harmful to human body. It is used to destroy bacteria in sterilizing the surgical instruments, burglar alarm, to detect the invisible writing, finger prints and also in the study of atomic structure.

X-rays

It is produced when there is sudden stopping of high speed electrons at high-atomic number target, and also by electronic transitions among the innermost orbits of atoms. The frequency range of X-rays is from 10^{17} Hz to 10^{19} Hz.

X-rays have more penetrating power than ultraviolet radiation. X-rays are used extensively in studying structures of inner atomic electron shells and crystal structures. It is used in detecting fractures, diseased organs, formation of bones and stones,

UNIT-5(XII-Physics_Vol-1).indd 276 14-02-2020 20:11:24

observing the progress of healing bones. Further, in a finished metal product, it is used to detect faults, cracks, flaws and holes.

Gamma rays

It is produced by transitions of radioactive nuclei and decay of certain elementary particles. They produce chemical reactions on photographic plates, fluorescence, ionisation, diffraction. The frequency range is 10^{18} Hz and above.

Gamma rays have higher penetrating power than X-rays and ultraviolet radiations; it has no charge but harmful to human body. Gamma rays provide information about the structure of atomic nuclei. It is used in radio therapy for the treatment of cancer and tumour, in food industry to kill pathogenic microorganism.

EXAMPLE 5.4

A magnetron in a microwave oven emits electromagnetic waves (em waves) with frequency f = 2450 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?.

Solution

Frequency of the electromagnetic waves given, f = 2450 MHz

The corresponding angular frequency is

$$\omega = 2\pi f = 2 \times 3.14 \times 2450 \times 10^6$$
$$= 15{,}386 \times 10^6 \text{ Hz}$$
$$= 1.54 \times 10^{10} \text{ s}^{-1}$$

The required magnetic field, $B = \dfrac{m_e \omega}{|q|}$

Mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg

Charge of the electron,

$$q = -1.60 \times 10^{-19} \text{ C}$$
$$\Rightarrow |q| = 1.60 \times 10^{-19} \text{ C}$$

$$B = \frac{\left(9.11 \times 10^{-31}\right)\left(1.54 \times 10^{10}\right)}{\left(1.60 \times 10^{-19}\right)} = 8.7683 \times 10^{-2} \text{ T}$$

$$B = 0.08768 \text{ T}$$

This magnetic field can be easily produced with a permanent magnet. So, electromagnetic waves of frequency 2450 MHz can be used for heating and cooking food because they are strongly absorbed by water molecules.

5.3 TYPES OF SPECTRUM- EMISSION AND ABSORPTION SPECTRUM- FRAUNHOFER LINES

When an object burns, it emits radiations. That is, it emits electromagnetic radiation which depends on temperature. If the object becomes hot, it glows in red colour. If the temperature of the object is further increased, then it glows in reddish-orange colour and becomes white when it is hottest. The spectrum in Figure 5.11 usually

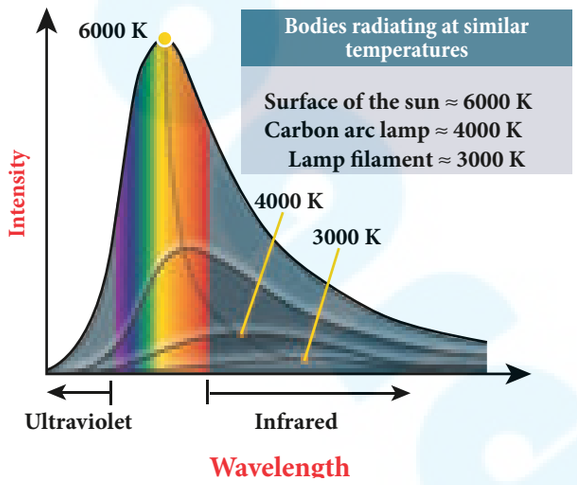


Figure 5.11 Black body radiation spectrum – variation with temperature

UNIT-5(XII-Physics_Vol-1).indd 277 14-02-2020 20:11:25

is called black body spectrum (Refer XI Physics Unit 8). It is a continuous frequency (or wavelength) curve depending on the body's temperature.

Suppose we allow a beam of white light to pass through the prism (as shown in Figure 5.12). It is split into its seven constituent colours which can be viewed on the screen as continuous spectrum. This phenomenon is known as dispersion of light and the definite pattern of colours obtained on the screen after dispersion is called as spectrum. The spectra can be broadly classified into two catagories:



Figure 5.12 White light passed through prism – dispersion

(a) Emission spectra

When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum. The emission spectrum can be divided into three types:

(i) Continuous emission spectrum (or continuous spectrum)

If the light from incandescent lamp (filament bulb) is allowed to pass through prism (simplest spectroscope), it splits up into seven colours. Thus, it consists of wavelengths containing all the visible colours ranging from violet to red (Figure 5.13). Examples: spectrum obtained from carbon arc and incandescent solids.

(ii) Line emission spectrum (or line spectrum):

Suppose light from hot gas is allowed to pass through prism, line spectrum is observed (Figure 5.14). Line spectra are also known as discontinuous spectra. The line spectra consists of sharp lines of definite wavelengths or frequencies. Such spectra arise due to excited atoms of elements. These lines are the characteristics of the element and are different for different elements. Examples: spectra of atomic hydrogen, helium, etc.

(iii) Band emission spectrum (or band spectrum)

Band spectrum consists of several number of very closely spaced spectral lines which overlap together forming specific



Figure 5.13 continuous emission spectra



Figure 5.14 line emission spectra

UNIT-5(XII-Physics_Vol-1).indd 278 14-02-2020 20:11:25

bands which are separated by dark spaces. This spectrum has a sharp edge at one end and fades out at the other end. Such spectra arise when the molecules are excited. Band spectrum is the characteristic of the molecule and hence the structure of the molecules can be studied using their band spectra. Example: spectra of ammonia gas in the discharge tube etc.

(b) Absorption spectra

When light is allowed to pass through a medium or an absorbing substance then the spectrum obtained is known as absorption spectrum. It is the characteristic of absorbing substance. Absorption spectrum is classified into three types:

(i) Continuous absorption spectrum

When we pass white light through a blue glass plate, it absorbs all the colours except blue and gives continuous absorption spectrum.

(ii) Line absorption spectrum

When light from the incandescent lamp is passed through cold gas (medium), the spectrum obtained through the dispersion due to prism is line absorption spectrum (Figure 5.15). Similarly, if the light from the carbon arc is made to pass through sodium vapour, a continuous spectrum of carbon arc with two dark lines in the yellow region are obtained.

(iii) Band absorption spectrum

When white light is passed through the iodine vapour, dark bands on continuous bright background is obtained. This type of band is also obtained when white light is passed through diluted solution of blood or chlorophyll or through certain solutions of organic and inorganic compounds.

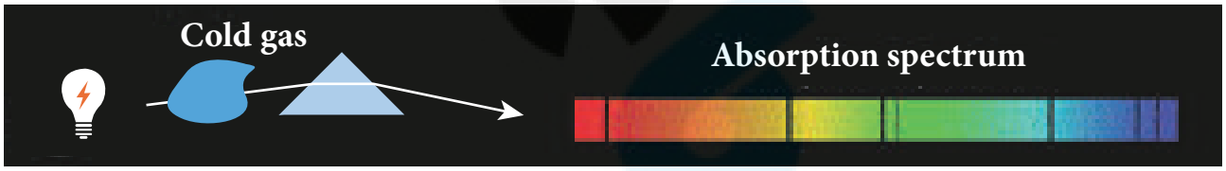


Figure 5.15 line absorption spectra

Fraunhofer lines

When the spectrum obtained from the Sun is examined, it consists of large number of dark lines (line absorption spectrum). These dark lines in the solar spectrum are known as Fraunhofer lines (Figure 5.16). The absorption spectra for various materials are compared with the Fraunhofer lines in the solar spectrum, which helps in identifying elements present in the Sun's atmosphere.

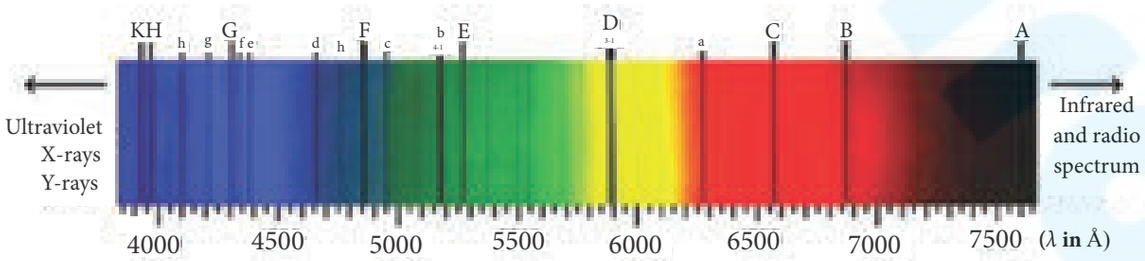


Figure 5.16 Solar spectrum - Fraunhofer lines

UNIT-5(XII-Physics_Vol-1).indd 279 14-02-2020 20:11:25

SUMMARY

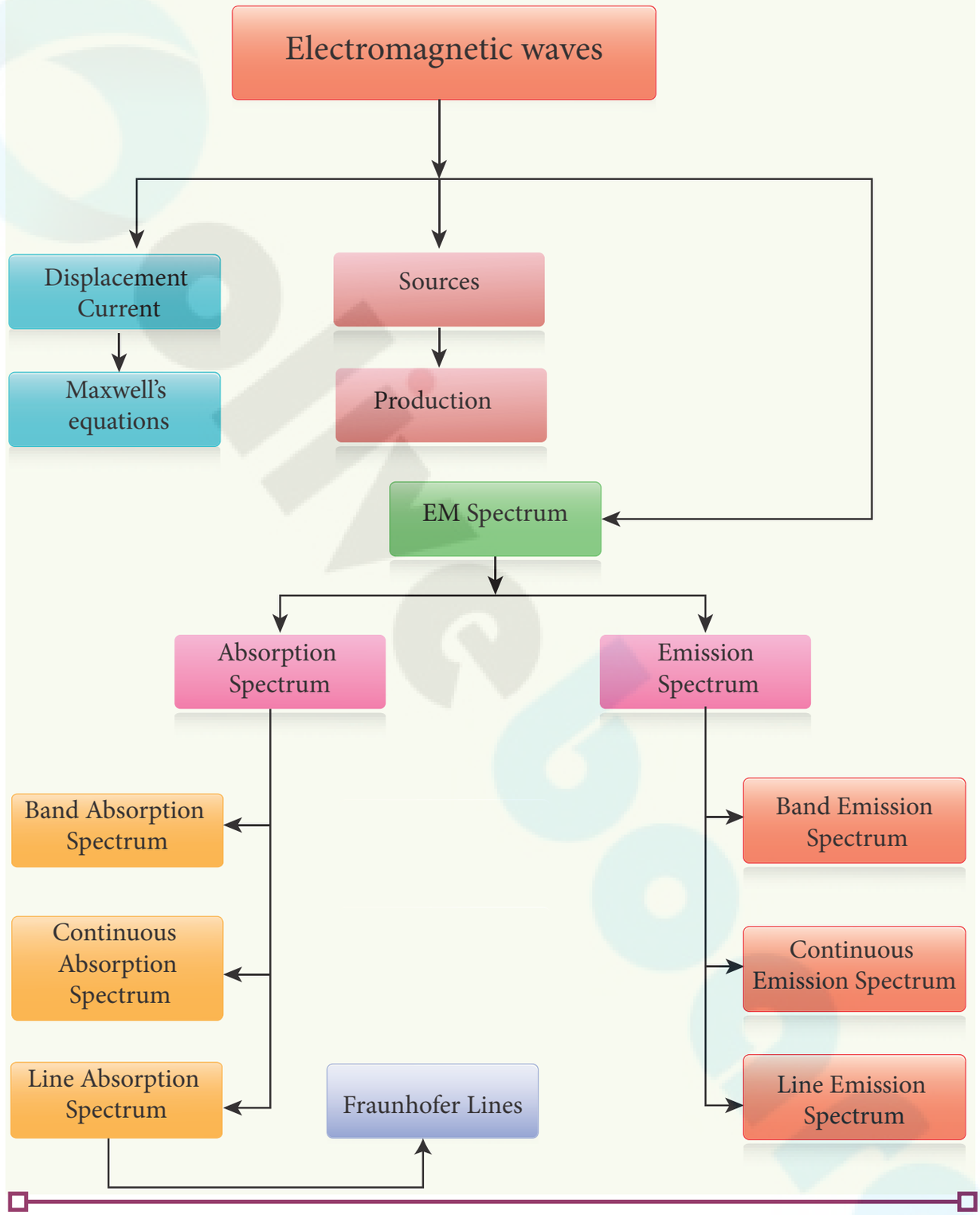
- Displacement current can be defined as 'the current which comes into play in the region in which the electric field and the electric flux are changing with time'.
- Maxwell modified Ampere's law as

$$\oint \vec{B} \cdot \vec{dl} = \mu_o i = \mu_o \left(i_c + i_d \right).$$

- An electromagnetic wave is radiated by an accelerated charge which propagates through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the wave.
- Electromagnetic waves are non-mechanical and do not require any medium for propagation.
- The instantaneous magnitude of the electric and magnetic field vectors in electromagnetic wave are related by $c = E/B$.
- Electromagnetic waves are transverse in nature. This means that the oscillating electric field vector, oscillating magnetic field vector and propagation vector are (gives direction of propagation) mutually perpendicular to each other.
- Electromagnetic waves can show interference, diffraction and also can be polarized
- Electromagnetic waves carry not only energy and momentum but also angular momentum.
- Types of spectra: emission and absorption spectra.
- When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum. The emission spectrum can be divided into three types: continuous, line and band.
- When the spectrum obtained from the Sun is examined, it consists of a large number of dark lines (line absorption spectrum). These dark lines in the solar spectrum are known as Fraunhofer lines.

UNIT-5(XII-Physics_Vol-1).indd 280 14-02-2020 20:11:26

CONCEPT MAP



UNIT-5(XII-Physics_Vol-1).indd 281 14-02-2020 20:11:26

I Multiple choice questions

1. The dimension of $\frac{1}{\mu_o \epsilon_o}$ is

(a) [L T^{-1}] (b) [L^2 T^{-2}]

(c) [L^{-1} T] (d) [L^{-2} T^2]

2. If the amplitude of the magnetic field is 3×10^{-6} T, then amplitude of the electric field for a electromagnetic waves is

(a) 100 V m^{-1} (b) 300 V m^{-1}

(c) 600 V m^{-1} (d) 900 V m^{-1}

3. Which of the following electromagnetic radiations is used for viewing objects through fog

(a) microwave (b) gamma rays

(c) X- rays (d) infrared

4. Which of the following is false for electromagnetic waves

(a) transverse

(b) non-mechanical waves

(c) longitudinal

(d) produced by accelerating charges

5. Consider an oscillator which has a charged particle oscillating about its mean position with a frequency of 300 MHz. The wavelength of electromagnetic waves produced by this oscillator is

(a) 1 m (b) 10 m

(c) 100 m (d) 1000 m

6. The electric and the magnetic fields, associated with an electromagnetic wave, propagating along negative X axis can be represented by

(a) $\vec{E} = E_\circ \hat{i}$ and $\vec{B} = B_\circ \hat{k}$

(b) $\vec{E} = E_\circ \hat{k}$ and $\vec{B} = B_\circ \hat{j}$

(c) $\vec{E} = E_\circ \hat{i}$ and $\vec{B} = B_\circ \hat{j}$

(d) $\vec{E} = E_\circ \hat{j}$ and $\vec{B} = B_\circ \hat{i}$

7. In an electromagnetic wave travelling in free space the rms value of the electric field is 3 V m^{-1}. The peak value of the magnetic field is

(a) 1.414×10^{-8} T (b) 1.0×10^{-8} T

(c) 2.828×10^{-8} T (d) 2.0×10^{-8} T

8. An e.m. wave is propagating in a medium with a velocity $\vec{v} = v\hat{i}$. The instantaneous oscillating electric field of this e.m. wave is along +y-axis, then the direction of oscillating magnetic field of the e.m. wave will be along:

E67JQC

(a) –y direction

(b) –x direction

(c) +z direction

(d) –z direction

9. If the magnetic monopole exists, then which of the Maxwell's equation to be modified?.

(a) $\oint_s \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_\circ}$

(b) $\oint_s \vec{B} \cdot d\vec{A} = 0$

(c) $\oint_l \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d}{dt} \oint_s \vec{E}.\vec{dA}$

(d) $\oint_l \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\Phi_B$

10. Fraunhofer lines are an example of _____ spectrum.

(a) line emission (b) line absorption

(c) band emission (d) band absorption

UNIT-5(XII-Physics_Vol-1).indd 282 14-02-2020 20:11:28

11. Which of the following is an electromagnetic wave?

(a) α - rays (b) β - rays

(c) γ - rays (d) all of them

12. Which one of them is used to produce a propagating electromagnetic wave?.

(a) an accelerating charge

(b) a charge moving with constant velocity (c) a stationary charge

(d) an uncharged particle

13. If $E = E_o \sin[10^6 x - \omega t]$ be the electric field of a plane electromagnetic wave, the value of ω is

(a) 0.3×10^{-14} rad s^{-1}

(b) 3×10^{-14} rad s^{-1}

(c) 0.3×10^{14} rad s^{-1}

(d) 3×10^{14} rad s^{-1}

14. Which of the following is NOT true for electromagnetic waves?.

(a) it transports energy

(b) it transports momentum

(c) it transports angular momentum

(d) in vacuum, it travels with different speeds which depend on their frequency

15. The electric and magnetic fields of an electromagnetic wave are

(a) in phase and perpendicular to each other

(b) out of phase and not perpendicular to each other

(c) in phase and not perpendicular to each other

(d) out of phase and perpendicular to each other

Answers

1) b	**2)** d	**3)** d	**4)** c	**5)** a
6) b	**7)** a	**8)** c	**9)** b	**10)** b
11) c	**12)** a	**13)** d	**14)** d	**15)** a

II Short answer questions

1. What is displacement current?
2. What are electromagnetic waves?
3. Write down the integral form of modified Ampere's circuital law.
4. Write notes on Gauss' law in magnetism.
5. Give two uses each of (i) IR radiation, (ii) Microwaves and (iii) UV radiation.
6. What are Fraunhofer lines? How are they useful in the identification of elements present in the Sun?
7. Write notes on Ampere-Maxwell law.
8. Why are e.m. waves non-mechanical?

III Long answer questions

1. Write down Maxwell equations in integral form.
2. Write short notes on (a) microwave (b) X-ray (c) radio waves (d) visible spectrum
3. Discuss the Hertz experiment.
4. Explain the Maxwell's modification of Ampere's circuital law.
5. Explain the importance of Maxwell's correction.
6. Write down the properties of electromagnetic waves.
7. Discuss the source of electromagnetic waves.
8. Explain the types of emission spectrum.
9. Explain the types of absorption spectrum.

UNIT-5(XII-Physics_Vol-1).indd 283 14-02-2020 20:11:28

IV Numerical problems

1. Consider a parallel plate capacitor whose plates are closely spaced. Let R be the radius of the plates and the current in the wire connected to the plates is 5 A, calculate the displacement current through the surface passing between the plates by directly calculating the rate of change of flux of electric field through the surface.

   Answer: $I_d = I_c = 5$ A

2. A transmitter consists of LC circuit with an inductance of 1 μH and a capacitance of 1 μF. What is the wavelength of the electromagnetic waves it emits?

   Answer: 18.84×10^2 m

3. A pulse of light of duration 10^{-6} s is absorbed completely by a small object initially at rest. If the power of the pulse is 60×10^{-3} W, calculate the final momentum of the object.

   Answer: 20×10^{-17} kg m s^{-1}

4. Let an electromagnetic wave propagate along the x - direction, the magnetic field oscillates at a frequency of 10^{10} Hz and has an amplitude of 10^{-5} T, acting along the y - direction. Then, compute the wavelength of the wave. Also write down the expression for electric field in this case.

   Answer: $\lambda = 3 \times 10^{-2}$ m and $\vec{E}(x,t) = 3\times10^3 \sin\left(2.09\times10^2 x - 6.28\times10^{10} t\right)\left(-\hat{k}\right)\text{NC}^{-1}$

5. If the relative permeability and relative permittivity of a medium are 1.0 and 2.25 respectively, find the speed of the electromagnetic wave in this medium.

   Answer: $v = 2 \times 10^8$ m s^{-1}

BOOKS FOR REFERENCE:

1. H. C. Verma, *Concepts of Physics – Volume 2*, Bharati Bhawan Publisher.
2. Halliday, Resnick and Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition.
3. Serway and Jewett, *Physics for scientist and engineers with modern physics*, Brook/Coole publishers, Eighth edition.
4. David J. Griffiths, *Introduction to electrodynamics*, Pearson publishers.
5. Paul Tipler and Gene Mosca, *Physics for scientist and engineers with modern physics*, Sixth edition, W.H.Freeman and Company.

UNIT-5(XII-Physics_Vol-1).indd 284 14-02-2020 20:11:29

ICT CORNER

Electromagnetic waves

In this activity you will be able to how do microwaves heat food?

Physics of microwaves and heating food

STEPS:

- Open the browser and type "phet.colorado.edu/en/simulation/microwaves" in the address bar. Run the simulation.
- select 'one molecule' tab. Turn on the microwave using the button in the right control panel. The arrows indicate the strength and direction of the force that would be exerted by the micro wave on the water molecules present in food. Observe the response of water molecule in response to this force?
- Observe how do microwaves heat food by rotating water molecule?
- Change amplitude and frequency of microwave and discuss how fast the water molecules are rotating?

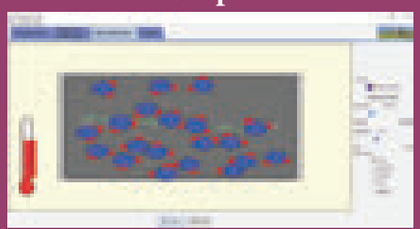
Step1



Step2



Step3



Step4



Discuss the relationship between rotating speed of the molecule with cooking time.

Note:

Install Java application if it is not in your system. You can download all the phet simulation and works in off line from https://phet.colorado.edu/en/offline-access.

URL:

https://phet.colorado.edu/en/simulation/microwaves

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B263_12_PHYSICS_EM

UNIT-5(XII-Physics_Vol-1).indd 285 14-02-2020 20:11:29

Higher Secondary Second Year

PHYSICS

PRACTICAL

Online Copy

LIST OF EXPERIMENTS

1. Determination of the specific resistance of the material of the given coil using metre bridge.
2. Determination of the value of the horizontal component of the Earth's magnetic field using tangent galvanometer.
3. Comparison of emf of two cells using potentiometer.
4. Determination of the refractive index of the material of the prism by finding angle of prism and angle of minimum deviation using spectrometer.
5. Determination of the wavelength of a composite light by normal incidence method using diffraction grating and spectrometer (The number of lines per metre length of the grating is given).
6. Investigation of the voltage-current (V-I) characteristics of PN junction diode.
7. Investigation of the voltage-current (V-I) characteristics of Zener diode.
8. Investigation of the static characteristics of a NPN Junction transistor in common emitter configuration.
9. Verification of the truth table of the basic logic gates using integrated circuits.
10. Verification of De Morgan's theorems using integrated circuits.

R2TEF8

XII-Physics_Vol-1) PRACTICAL.indd 287 14-02-2020 20:12:50

1. SPECIFIC RESISTANCE OF THE MATERIAL OF THE COIL USING METRE BRIDGE

AIM

To determine the specific resistance of the material of the given coil using metre bridge.

APPARATUS REQUIRED

Meter bridge, galvanometer, key, resistance box, connecting wires, Lechlanche cell, jockey and high resistance.

FORMULA

$$\rho = \frac{X \pi r^2}{L} \ (\Omega m)$$

where, $\rho \rightarrow$ Specific resistance of the given coil (Ωm)

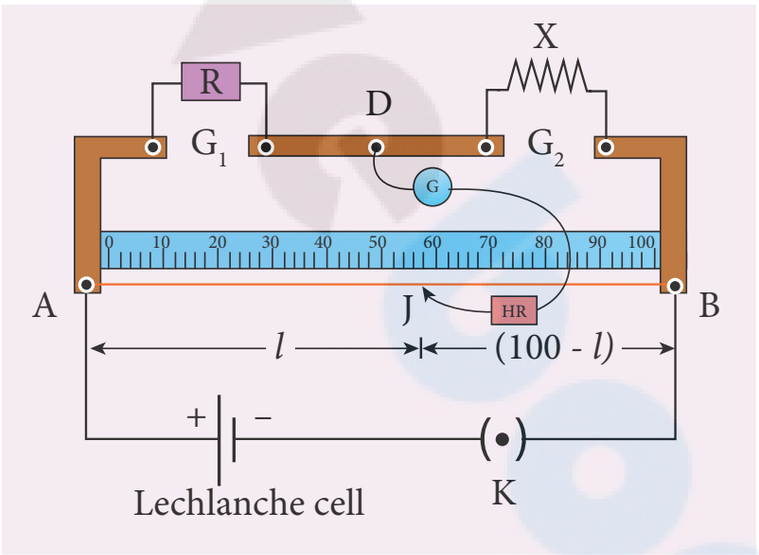
$X \rightarrow$ Resistance of the given coil (Ω)

$R \rightarrow$ Known resistance (Ω)

$L \rightarrow$ Length of the coil (m)

$r \rightarrow$ Radius of the wire (m)

CIRCUIT DIAGRAM



PROCEDURE

- A resistance box *R* is connected in the left gap and the unknown resistance X in the right gap.
- A Lechlanche cell is connected across the wire of length 1 m through a key.
- A sensitive galvanometer *G* is connected between the central strip and the jockey through a high resistance (HR).
- With a suitable resistance included in the resistance box, the circuit is switched on.
- To check the circuit connections, the jockey is pressed near one end of the wire, say A. The galvanometer will show deflection in one direction. When the jockey is pressed near the other end of the wire B, the galvanometer will show deflection in the opposite direction. This ensures that the circuit connections are correct.

XII-Physics_Vol-1) PRACTICAL.indd 288 14-02-2020 20:12:51

- By moving the jockey over the wire, the point on the wire at which the galvanometer shows null deflection i.e., balancing point J is found.
- The balancing length $AJ = l$ is noted.
- The unknown resistance X_1 is found using the formula $X_1 = \frac{R(100-l)}{l}$.
- The experiment is repeated for different values of R.
- The same procedure is repeated after interchanging R and X.
- The unknown resistance X_2 is found using the formula $X_2 = \frac{Rl}{(100-l)}$.
- The experiment is repeated for same values of R as before.
- The resistance of the given coil is found from the mean value of X_1 and X_2.
- The radius of the wire r is found using screw gauge.
- The length of the coil L is measured using meter scale.
- From the values of X, r and L, the specific resistance of the material of the wire is determined.

OBSERVATION

length of the coil, L = _____ cm.

Table 1 To find the resistance of the given coil

S.No.	Resistance R (Ω)	Before interchanging		After interchanging		Mean $X = \frac{X_1 + X_2}{2}$ (Ω)
		Balancing length l (cm)	$X_1 = \frac{R(100-l)}{l}$ (Ω)	Balancing length l (cm)	$X_2 = \frac{Rl}{(100-l)}$ (Ω)	
1						
2						
3						

Mean resistance, X = ----------Ω

XII-Physics_Vol-1) PRACTICAL.indd 289 14-02-2020 20:12:55

Table 2 To find the radius of the wire

Zero error =

Zero correction =

LC = 0.01 mm

Sl.No.	PSR (mm)	HSC (div.)	Total Reading = PSR + (HSC × LC) (mm)	Corrected Reading = TR ± ZC (mm)
1				
2				
3				
4				
5				
6				

Mean diameter 2r = ... mm

Radius of the wire r = ... mm

r = ... m

CALCULATION

(i) $\rho = \frac{X\pi r^2}{L}$ =

RESULT

The specific resistance of the material of the given coil = _____(Ωm)

Note:

i) To check the circuit connections:

The meter bridge wire is touched near one end (say, end A) with jockey, galvanometer shows a deflection in any one direction. Now the other end (say, end B) is touched. If the galvanometer shows a deflection in the opposite direction, then the circuit connections are correct.

ii) The usage of high resistance (HR):

The galvanometer is a very sensitive device. If any high current flows through the galvanometer, its coil gets damaged. Therefore in order to protect the galvanometer, a high resistance (HR) is used. When HR is connected in series with the galvanometer, the current through it is reduced so that the galvanometer is protected. But the balancing length is not accurate.

iii) To find the accurate balancing length:

The HR is first included in the circuit (that is, the plug key in HR is removed), the approximate balancing length is found. Now HR is excluded in the circuit (that is, the plug key in HR is closed), then the accurate balancing length is found.

XII-Physics_Vol-1) PRACTICAL.indd 290 14-02-2020 20:12:55

2. HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD USING TANGENT GALVANOMETER

AIM

To determine the horizontal component of the Earth's magnetic field using tangent galvanometer.

APPARATUS REQUIRED

Tangent galvanometer (TG), commutator, battery, rheostat, ammeter, key and connecting wires.

FORMULA

$$B_H = \frac{\mu_0 n k}{2r} \text{ (Tesla)}$$

$$k = \frac{I}{\tan\theta} \text{ (A)}$$

where, B_H → Horizontal component of the Earth's magnetic field (T)

μ_0 → Permeability of free space ($4\pi \times 10^{-7}$ H m^{-1})

n → Number of turns of TG in the circuit (No unit)

k → Reduction factor of TG (A)

r → Radius of the coil (m)

CIRCUIT DIAGRAM

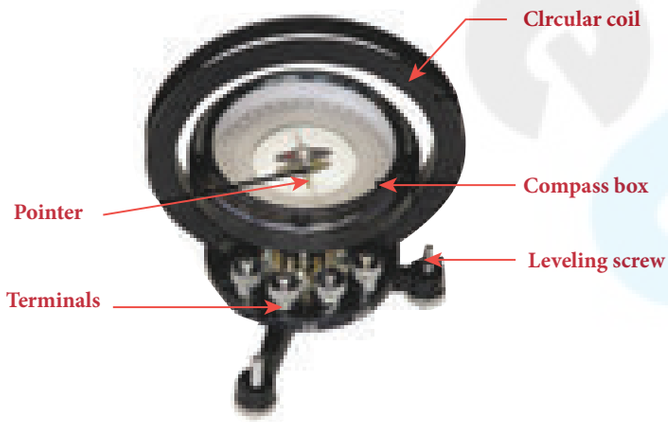
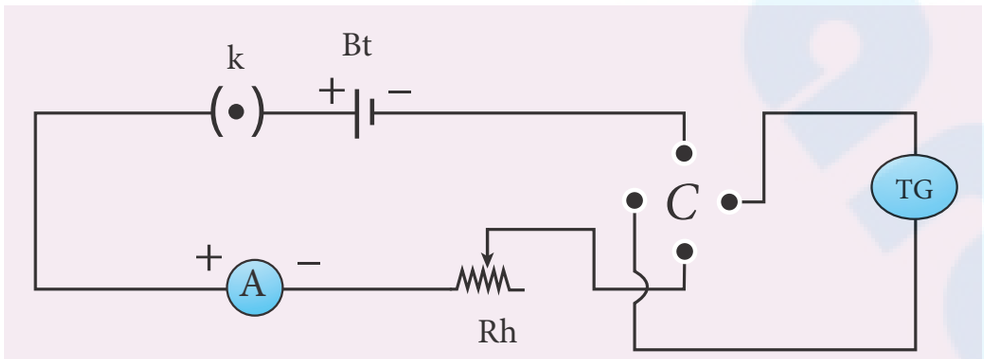


Figure (a) Tangent Galvanometer



Figure (b) Number of turns



(c) Circuit diagram

XII-Physics_Vol-1) PRACTICAL.indd 291 14-02-2020 20:12:56

PROCEDURE

- The preliminary adjustments are carried out as follows.
  a. The leveling screws at the base of TG are adjusted so that the circular turn table is horizontal and the plane of the circular coil is vertical.
  b. The circular coil is rotated so that its plane is in the magnetic meridian i.e., along the north-south direction.
  c. The compass box alone is rotated till the aluminium pointer reads $0° - 0°$.
- The connections are made as shown in Figure (c).
- The number of turns *n* is selected and the circuit is switched on.
- The range of current through TG is chosen in such a way that the deflection of the aluminium pointer lies between $30° - 60°$.
- A suitable current is allowed to pass through the circuit, the deflections θ_1 and θ_2 are noted from two ends of the aluminium pointer.
- Now the direction of current is reversed using commutator C, the deflections θ_3 and θ_4 in the opposite directions are noted.
- The mean value θ of θ_1, θ_2, θ_3 and θ_4 is calculated and tabulated.
- The reduction factor k is calculated for each case and it is found that k is a constant.
- The experiment is repeated for various values of current and the readings are noted and tabulated.
- The radius of the circular coil is found by measuring the circumference of the coil using a thread around the coil.
- From the values of *r*, *n* and *k*, the horizontal component of Earth's magnetic field is determined.

Commutator:



It is a kind of switch employed in electrical circuits, electric motors and electric generators. It is used to reverse the direction of current in the circuit.

XII-Physics_Vol-1) PRACTICAL.indd 292 14-02-2020 20:12:57

OBSERVATION

Number of turns of the coil n =

Circumference of the coil (2πr) =

Radius of the coil r =

S.No	Current I (A)	Deflection in TG (degree)				Mean θ (degree)	$k=\frac{I}{\tan\theta}$
		θ_1	θ_2	θ_3	θ_4		
1							
2							
3							
4							
						Mean	

CALCULATION

$$B_H=\frac{\mu_0 nk}{2r}=$$

RESULT

The horizontal component of Earth's magnetic field is found to be _____

Note:

i) The magnetic materials and magnets present in the vicinity of TG should be removed.

ii) The readings from the ends of the aluminium pointer should be taken without parallax error.

iii) The deflections of TG is restricted between 30° and 60°. It is because, the TG is most sensi tive for deflection around 45° and is least sensitive around 0° and 90°. We know that

$$I = k \tan\theta$$

$$\text{or } dI = k \sec^2\theta \, d\theta$$

$$\frac{d\theta}{dI}=\frac{\sin 2\theta}{2I}$$

For given current, sensitivity $\frac{d\theta}{dI}$ is maximum for sin 2θ = 1 or θ = 45°

XII-Physics_Vol-1) PRACTICAL.indd 293 14-02-2020 20:12:58

3. COMPARISON OF EMF OF TWO CELLS USING POTENTIOMETER

AIM

To compare the emf of the given two cells using a potentiometer.

APPARATUS REQUIRED

Battery eliminator, key, rheostat, DPDT switch, Lechlanche and Daniel cells, galvanometer, high resistance box, pencil jockey and connecting wires.

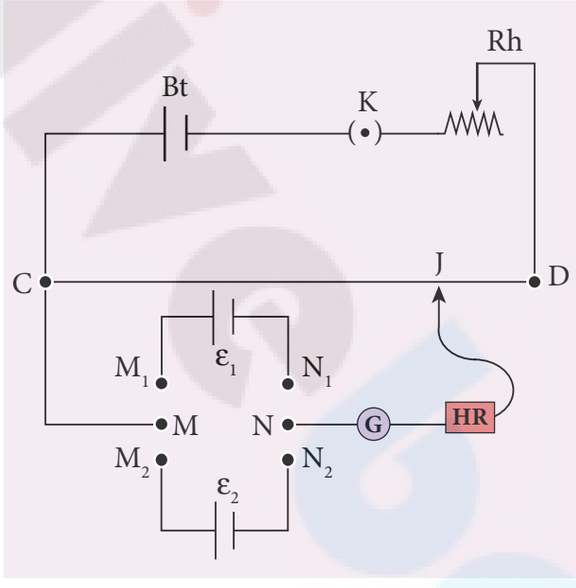
FORMULA

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \text{ (Tesla)}$$

where, ε_1 and ε_2 are the emf of Lechlanche and Daniel cells respectively (V)

l_1 and l_2 are the balancing lengths for Lechlanche and Daniel cells respectively (cm)

CIRCUIT DIAGRAM



PROCEDURE

- The apparatus is arranged as shown in the circuit diagram.
- The primary circuit consisting of battery, key and rheostat is connected to the potentiometer in series.
- The positive poles of the cells are connected to terminals M_1 & M_2 and the negative poles to terminals N_1 & N_2 of the DPDT switch. The potentiometer is connected to the common terminals M and N as shown in the circuit.
- Using the two-way key, Lechlanche cell is included in the circuit. By sliding the jockey on the potentiometer wire, the balancing point is found and the corresponding balancing length is measured.
- Similarly, the balancing length is found by including Daniel cell in the circuit.
- The experiment is repeated for different sets of balancing lengths by adjusting the rheostat.
- From different values of l_1 and l_2, the ratio of emf of the two cells is calculated.

XII-Physics_Vol-1) PRACTICAL.indd 294 14-02-2020 20:12:58

OBSERVATION

Table: To find the ratio of emf of two cells

S.No	Balancing length for Lechlanche cell, l_1 (cm)	Balancing length for Daniel cell, l_2 (cm)	$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$
1			
2			
3			
4			
5			

Mean $\frac{\varepsilon_1}{\varepsilon_2} =$

CALCULATION

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

RESULT

Ratio of emf of the given two cells = _____ (no unit)

Note:

DPDT switch



Double Pole Double Throw (DPDT) switch is a 6-terminal key with a throw switch (metal handle) attached to the common terminals M and N. The two-given cells are connected to the pairs of terminals M_1 & M_2 and
N_1 & N_2. When the handle is thrown to one side, say M_1 & M_2, the cell that is connected to that pair of terminals is included with the primary circuit.

XII-Physics_Vol-1) PRACTICAL.indd 295 14-02-2020 20:12:58

4. REFRACTIVE INDEX OF THE MATERIAL OF THE PRISM

AIM

To determine the refractive index of the material of a prism using spectrometer.

APPARATUS REQUIRED

Spectrometer, prism, prism clamp, sodium vapour lamp, spirit level.

FORMULA

$$\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \text{ (No unit)}$$

where, μ → Refractive index of the material of the prism (No unit)

A → Angle of the prism (degree)

D → Angle of minimum deviation (degree)

DIAGRAMS

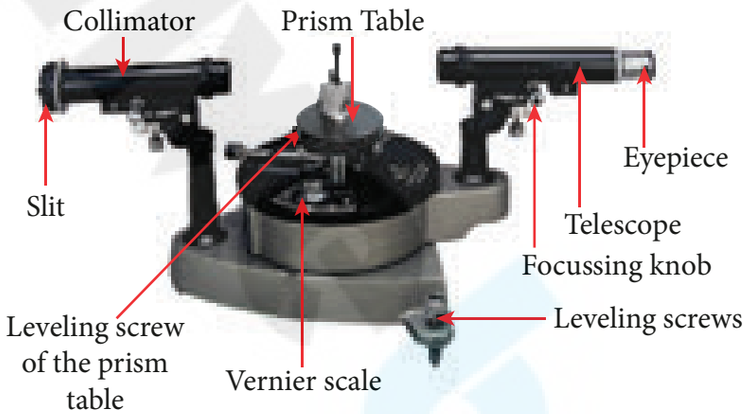


Figure (a) Angle of the prism

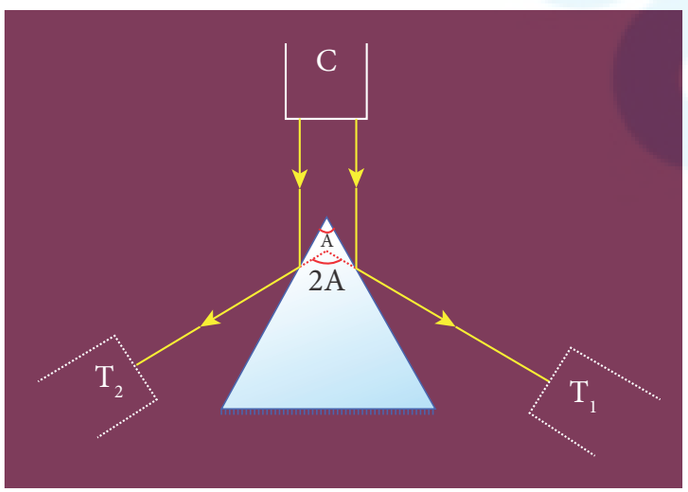


Figure (b) Angle of the prism

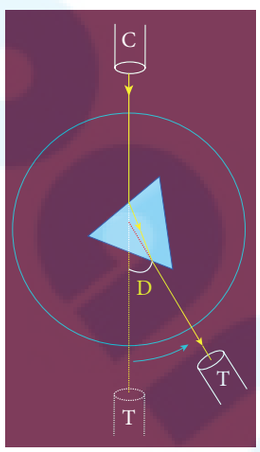


Figure (c) Angle of minimum deviation

XII-Physics_Vol-1) PRACTICAL.indd 296 14-02-2020 20:12:59

PROCEDURE

1) Initial adjustments of the spectrometer

- Eye-piece: The eye-piece of the telescope is adjusted so that the cross-wires are seen clearly.
- Slit: The slit of the collimator is adjusted such that it is very thin and vertical.
- Base of the spectrometer: The base of the spectrometer is adjusted to be horizontal using leveling screws.
- Telescope: The telescope is turned towards a distant object and is adjusted till the clear inverted image of the distant object is seen. Now the telescope is adjusted to receive parallel rays.
- Collimator: The telescope is brought in line with the collimator. Collimator is adjusted until a clear image of the slit is seen in the telescope. Now the collimator gives parallel rays.
- Prism table: Using a spirit level, the prism table is adjusted to be horizontal with the three leveling screws provided in the prism table.

2) Determination of angle of the prism (A)

- The slit is illuminated by yellow light from sodium vapour lamp.
- The given equilateral prism is placed on the prism table in such a way that refracting edge of the prism is facing the collimator.
- The light emerging from the collimator is incident on both reflecting faces of the prism and is reflected.
- The telescope is rotated towards left to obtain reflected image of the slit from face 1 of the prism and is fixed.
- Using tangential screws, the telescope is adjusted until the vertical cross-wire coincides with the reflected image of the slit.
- The main scale reading and vernier coincidence are noted from both vernier scales.
- The telescope is now rotated towards right to obtain the reflected image from face 2 of the prism. As before, the readings are taken.
- The difference between the two readings gives 2A from which the angle of the prism A is calculated.

3) Determination of angle of minimum deviation (D)

- The prism table is rotated such that the light emerging from the collimator is incident on one of the refracting faces of the prism, gets refracted and emerges out from the other refracting face.
- The telescope is turned to view the refracted image.
- Looking through the telescope, the prism table is rotated in such a direction that the image moves towards the direct ray.

XII-Physics_Vol-1) PRACTICAL.indd 297 14-02-2020 20:12:59

- At one particular position, the refracted ray begins to retrace its path. The position where the refracted image returns is the position of minimum deviation.
- The telescope is fixed in this position and is adjusted until the vertical cross-wire coincides with the refracted image of the slit.
- The readings are taken from both vernier scales.
- The prism is now removed and the telescope is rotated to obtain the direct ray image and the readings are taken.
- The readings are tabulated and the difference between these two readings gives the angle of minimum deviation D.
- From the values of A and D, the refractive index of the material of the glass prism is determined.

Least count

1 MSD = 30′

Number of vernier scale divisions = 30

For spectrometer, 30 vernier scale divisions will cover 29 main scale divisions.

$\therefore$ 30 VSD = 29 MSD

Or 1 VSD = $\left(\frac{29}{30}\right)$ MSD

Least count (LC) = 1 MSD – 1 VSD

$$= \left(\frac{1}{30}\right)\text{MSD} = \left(\frac{1}{30}\right) \times 30'$$

$$= 1'$$

OBSERVATION

Table 1 To find the angle of the prism (A)

Image	Vernier A (Degree)			Vernier B (Degree)		
	MSR	VSC	TR	MSR	VSC	TR
Reflected image from face 1						
Reflected image from face 2						
Difference 2A						

Mean 2A =

Mean A =

XII-Physics_Vol-1) PRACTICAL.indd 298 14-02-2020 20:13:00

Table 2 To find the angle of minimum deviation (D)

Image	Vernier A (Degree)			Vernier B (Degree)		
	MSR	VSC	TR	MSR	VSC	TR
Refracted image						
Direct image						
Difference D						

Mean D =

CALCULATION

$$\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

RESULT

1. Angle of the Prism (A) = ... (degree)
2. Angle of the minimum deviation of the prism (D) =..... (degree)
3. Refractive index of the material of the Prism (μ) =..... (No unit)

Note:

i) Once initial adjustments are done, spectrometer should not be disturbed.

ii) Total reading TR = MSR + (VSC × LC)

Where

MSR → Main Scale Reading

VSC → Vernier Scale Coincidence

LC → Least count (= 1′)

XII-Physics_Vol-1) PRACTICAL.indd 299 14-02-2020 20:13:00

5. WAVELENGTH OF THE CONSTITUENT COLOURS OF A COMPOSITE LIGHT USING DIFFRACTION GRATING AND SPECTROMETER

AIM

To find the wavelength of the constituent colours of a composite light using diffraction grating and spectrometer.

APPARATUS REQUIRED

Spectrometer, mercury vapour lamp, diffraction grating, grating table, and spirit level.

FORMULA

$$\lambda = \frac{\sin\theta}{nN} \text{ Å}$$

where, λ → Wavelength of the constituent colours of a composite light (Å)

N → Number of lines per metre length of the given grating (No unit) (the value of N for the grating is given)

n → Order of the diffraction (No unit)

θ → Angle of diffraction (degree)

DIAGRAMS

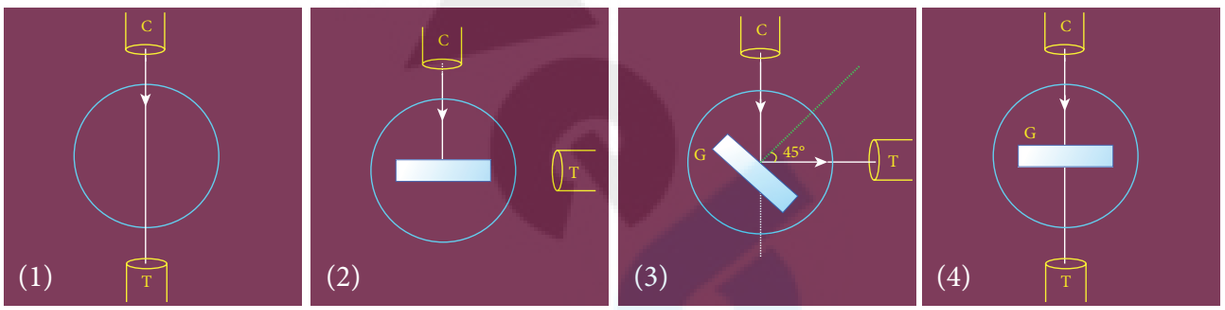


Figure (a) Normal incidence

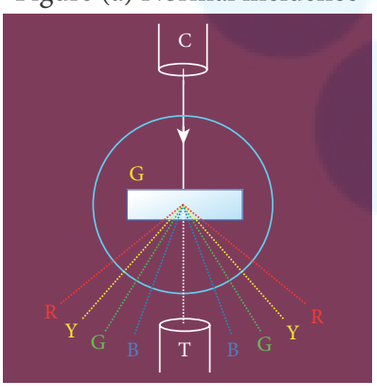


Figure (b) Angle of diffraction

PROCEDURE

1) **Initial adjustments of the spectrometer**

- Eye-piece: The eye-piece of the telescope is adjusted so that the cross-wires are seen clearly.
- Slit: The slit of the collimator is adjusted such that it is very thin and vertical.
- Base of the spectrometer: The base of the spectrometer is adjusted to be horizontal using leveling screws.

XII-Physics_Vol-1) PRACTICAL.indd 300 14-02-2020 20:13:00

- Telescope: The telescope is turned towards a distant object and is adjusted till the clear image of the distant object is seen. Now the telescope is adjusted to receive parallel rays.
- Collimator: The telescope is brought in line with the collimator. Collimator is adjusted until a clear image of the slit is seen in the telescope. Now the collimator gives parallel rays.
- Grating table: Using a spirit level, the grating table is adjusted to be horizontal with the three leveling screws provided in the grating table.

2) Adjustment of the grating for normal incidence

- The slit is illuminated with a composite light (white light) from mercury vapour lamp.
- The telescope is brought in line with the collimator. The vertical cross-wire is made to coincide with the image of the slit (Figure (a)1).
- The vernier disc alone is rotated till the vernier scale reads 0^0 - 180^0 and is fixed. This is the reading for the direct ray.
- The telescope is then rotated (anti-clockwise) through an angle of 90^0 and fixed (Figure (a)2).
- Now the plane transmission grating is mounted on the grating table.
- The grating table alone is rotated so that the light reflected from the grating coincides with vertical cross-wire of the telescope. The reflected image is white in colour (Figure (a)3).
- Now the vernier disc is released. The vernier disc along with grating table is rotated through an angle of 45^0 in the appropriate direction such that the light from the collimator is incident normally on the grating (Figure (a)4).

3) Determination of wave length of the constituent colours of the mercury spectrum

- The telescope is released and is brought in line with the collimator to receive central direct image. This undispersed image is white in colour.
- The diffracted images of the slit are observed on either side of the direct image.
- The diffracted image consists of the prominent colours of mercury spectrum in increasing order of wavelength.
- The telescope is turned to any one side (say left) of direct image to observe first order diffracted image.
- The vertical cross-wire is made to coincide with the prominent spectral lines (violet, blue, yellow and red) and the readings of both vernier scales for each case are noted.
- Now the telescope is rotated to the right side of the direct image and the first order image is observed.
- The vertical cross-wire is made to coincide with the same prominent spectral lines and the readings of both vernier scales for each case are again noted.
- The readings are tabulated.
- The difference between these two readings gives the value of 2θ for the particular spectral line.
- The number of lines per metre length of the given grating N is noted from the grating.
- From the values of N, n and θ, the wave length of the prominent colours of the mercury light is determined using the given formula.

XII-Physics_Vol-1) PRACTICAL.indd 301 14-02-2020 20:13:01

OBSERVATION

To find the wave length of prominent colours of the mercury spectrum

Colour of Light	Diffracted Ray Reading (Degree)												Difference 2θ (Degree)			θ (Degree)
	Left						Right									
	Vernier A			Vernier B			Vernier A			Vernier B						
	MSR	VSC	TR	MSR	VSC	TR	MSR	VSC	TR	MSR	VSC	TR	VER A	VER B	Mean	
Blue																
Green																
Yellow																
Red																

CALCULATION

(i) For blue, $\lambda = \frac{\sin\theta}{nN}$,

(ii) For green, $\lambda = \frac{\sin\theta}{nN}$

(iii) For yellow, $\lambda = \frac{\sin\theta}{nN}$,

(iv) For red, $\lambda = \frac{\sin\theta}{nN}$

RESULT

1. The wavelength of blue line = ----------m
2. The wavelength of green line = ----------m
3. The wavelength of yellow line = ----------m
4. The wavelength of red line = ----------m

Note:

i) Once initial adjustments are done, spectrometer should not be disturbed.

ii) Total reading TR = MSR + (VSC × LC)

Where

MSR → Main Scale Reading

VSC → Vernier Scale Coincidence

LC → Least count (= 1′)

XII-Physics_Vol-1) PRACTICAL.indd 302 14-02-2020 20:13:01

6. VOLTAGE-CURRENT CHARACTERISTICS OF A PN JUNCTION DIODE

AIM

To draw the voltage-current (V- I) characteristics of the PN junction diode and to determine its knee voltage and forward resistance.

APPARATUS REQUIRED

PN junction diode (IN4007), variable DC power supply, milli-ammeter, micro-ammeter, voltmeter, resistance and connecting wires.

FORMULA

$$R_F = \frac{\Delta V_F}{\Delta I_F} \ (\Omega)$$

where, $R_F \rightarrow$ Forward resistance of the diode (Ω)

$\Delta V_F \rightarrow$ The change in forward voltage (volt)

$\Delta I_F \rightarrow$ The change in forward current (mA)

CIRCUIT DIAGRAM

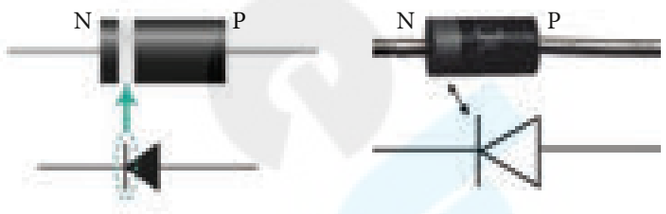


Figure (a) PN junction diode and its symbol (Silver ring denotes the negative terminal of the diode)

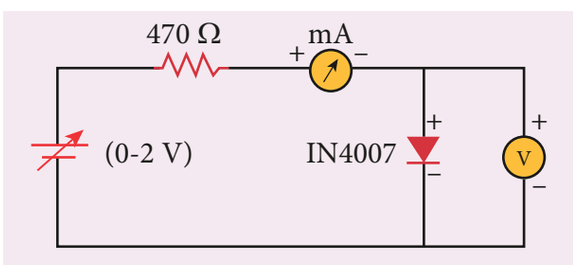


Figure (b) PN junction diode in forward bias

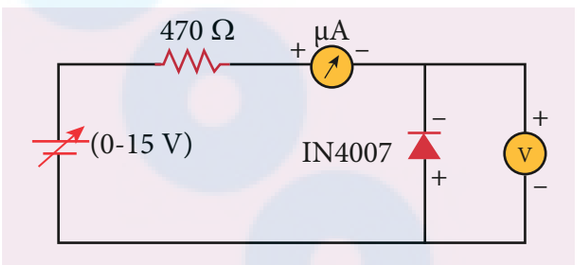


Figure (c) PN junction diode in reverse bias

Precaution

Care should be taken to connect the terminals of ammeter, voltmeter, dc power supply and the PN junction diode with right polarity.

XII-Physics_Vol-1) PRACTICAL.indd 303 14-02-2020 20:13:02

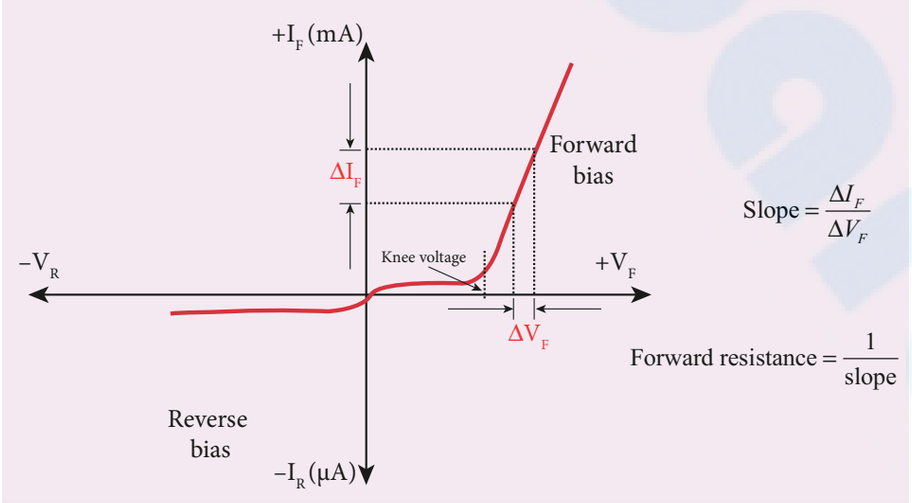
PROCEDURE

i) Forward bias characteristics

- In the forward bias, the P- region of the diode is connected to the positive terminal and N-region to the negative terminal of the DC power supply.
- The connections are given as per the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The forward voltage (V_F) across the diode is increased from 0.1 V in steps of 0.1 V up to 0.8 V and the forward current (I_F) through the diode is noted from the milli-ammeter. The readings are tabulated.
- The forward voltage V_F and the forward current I_F are taken as positive.
- A graph is drawn taking the forward voltage (V_F) along the x-axis and the forward current (I_F) along the y-axis.
- The voltage corresponding to the dotted line in the forward characteristics gives the knee voltage or threshold voltage or turn-on voltage of the diode.
- The slope in the linear portion of the forward characteristics is calculated. The reciprocal of the slope gives the forward resistance of the diode.

ii) Reverse bias characteristics

- In the reverse bias, the polarity of the DC power supply is reversed so that the P- region of the diode is connected to the negative terminal and N-region to the positive terminal of the DC power supply
- The connections are made as given in the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The reverse voltage (V_R) across the diode is increased from 1 V in steps of 1 V up to 5 V and the reverse current (I_R) through the diode is noted from the micro-ammeter. The readings are tabulated.
- The reverse voltage V_R and reverse current I_R are taken as negative.
- A graph is drawn taking the reverse bias voltage (V_R) along negative x-axis and the reverse bias current (I_R) along negative y-axis.



XII-Physics_Vol-1) PRACTICAL.indd 304 14-02-2020 20:13:02

OBSERVATION

Table 1 Forward bias characteristic curve

S.No.	Forward bias voltage V_F (volt)	Forward bias current I_F (mA)

Table 2 Reverse bias characteristic curve

S.No.	Reverse bias voltage V_R (volt)	Reverse bias current I_R (μA)

CALCULATION

(i) Forward resistance R_F=

(ii) knee voltage =

RESULT

The V-I characteristics of the PN junction diode are studied.

i) Knee voltage of the PN junction diode =……V

ii) Forward resistance of the diode =……Ω

Practical Tips

- The DC power supply voltage should be increased only up to the specified range in the forward (0 – 2V) and reverse (0 – 15V) directions. Forward bias offers very low resistance and hence an external resistance of 470Ω is connected as a safety measure.
- The voltage applied beyond this limit may damage the resistance or the diode.
- In the forward bias, the current flow will be almost zero till it crosses the junction potential or knee voltage (approximately 0.7 V). Once knee voltage is crossed, the current increases with the applied voltage.
- The diode voltage in the forward direction should be increased in steps of 0.1 V to a maximum of 0.8 V after the threshold voltage to calculate the forward resistance.
- The diode voltage in the reverse direction is increased in steps of 1 V to a maximum of 5 V. The current must be measured using micro-ammeter as the strength of current in the reverse direction is very less. This is due to the flow of the minority charge carriers called the leakage current.

XII-Physics_Vol-1) PRACTICAL.indd 305 14-02-2020 20:13:04

7. VOLTAGE-CURRENT CHARACTERISTICS OF A ZENER DIODE

AIM

To draw the voltage-current (V-I) characteristic curves of a Zener diode and to determine its knee voltage, forward resistance and reverse breakdown voltage.

APPARATUS REQUIRED

Zener diode 1Z5.6V, variable dc power supply (0 – 15V), milli ammeter, volt meter, 470 Ω resistance, and connecting wires.

FORMULA

$$R_F = \frac{\Delta V_F}{\Delta I_F} \ (\Omega)$$

where, $R_F \rightarrow$ Forward resistance of the diode (Ω)

$\Delta V_F \rightarrow$ The change in forward voltage (volt)

$\Delta I_F \rightarrow$ The change in forward current (mA)

CIRCUIT DIAGRAM

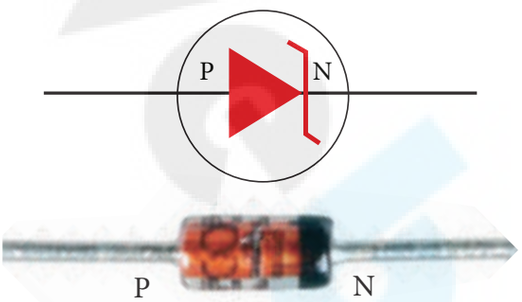


Figure (a) Zener diode and its symbol (The black colour ring denotes the negative terminal of the Zener diode)

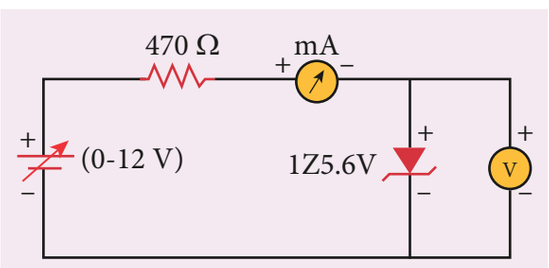


Figure (b) Zener diode in forward bias

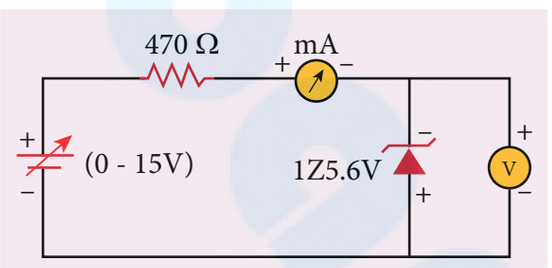


Figure (c) Zener diode in reverse bias

Precaution

Care should be taken to connect the terminals of ammeter, voltmeter, dc power supply and the Zener diode with right polarity.

XII-Physics_Vol-1) PRACTICAL.indd 306 14-02-2020 20:13:04

PROCEDURE

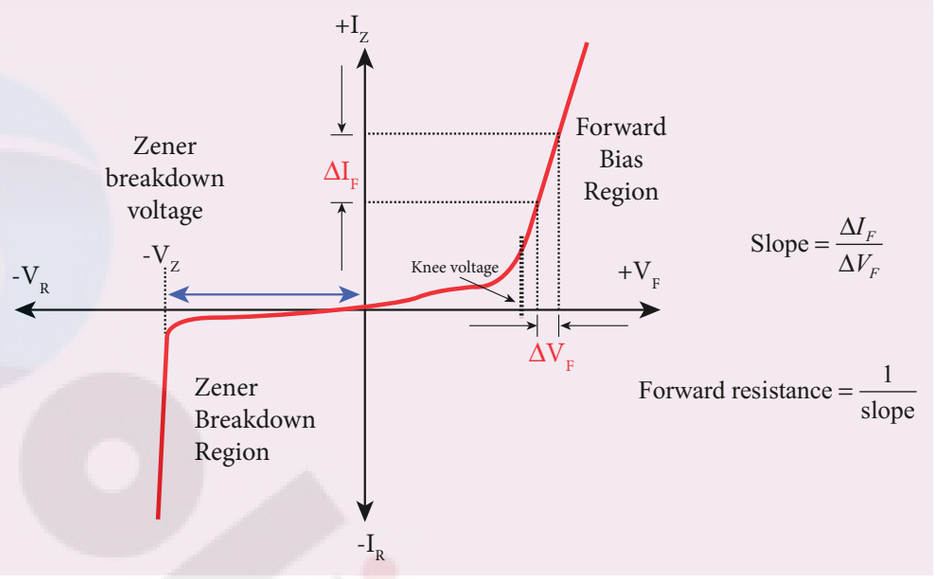
i) Forward bias characteristics

- In the forward bias, the P- region of the diode is connected to the positive terminal and N-region to the negative terminal of the DC power supply.
- The connections are given as per the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The forward voltage (V_F) across the diode is increased from 0.1V in steps of 0.1V up to 0.8V and the forward current (I_F) through the diode is noted from the milli-ammeter. The readings are tabulated.
- The forward voltage and the forward current are taken as positive.
- A graph is drawn taking the forward voltage along the x-axis and the forward current along the y-axis.
- The voltage corresponding to the dotted line in the forward characteristics gives the knee voltage or threshold voltage or turn-on voltage of the diode.
- The slope in the linear portion of the forward characteristics is calculated. The reciprocal of the slope gives the forward resistance of the diode.

ii) Reverse bias characteristics

- In the reverse bias, the polarity of the DC power supply is reversed so that the P- region of the diode is connected to the negative terminal and N-region to the positive terminal of the DC power supply
- The connections are made as given in the circuit diagram.
- The voltage across the diode can be varied with the help of the variable DC power supply.
- The reverse voltage (V_R) across the diode is increased from 0.5V in steps of 0.5V up to 6V and the reverse current (I_R) through the diode is noted from the milli-ammeter. The readings are tabulated.
- Initially, the voltage is increased in steps of 0.5V. When the breakdown region is approximately reached, then the input voltage may be raised in steps of, say 0.1V to find the breakdown voltage.
- The reverse voltage and reverse current are taken as negative.
- A graph is drawn taking the reverse bias voltage along negative x-axis and the reverse bias current along negative y-axis.
- In the reverse bias, Zener breakdown occurs at a particular voltage called Zener voltage V_Z (~5.6 to 5.8V) and a large amount of current flows through the diode which is the characteristics of a Zener diode.
- The breakdown voltage of the Zener diode is determined from the graph as shown.

XII-Physics_Vol-1) PRACTICAL.indd 307 14-02-2020 20:13:04



OBSERVATION

Table 1 Forward bias characteristic curve

S.No.	Forward bias voltage V_F (volt)	Forward bias current I_F (mA)

Table 2 Reverse bias characteristic curve

S.No.	Reverse bias voltage V_R (volt)	Reverse bias current I_R (mA)

CALCULATION

(i) Forward resistance R_F=

(ii) knee voltage =

(iii) The breakdown voltage of the Zener diode V_Z= ----V

XII-Physics_Vol-1) PRACTICAL.indd 308 14-02-2020 20:13:05

RESULT

The V-I characteristics of the Zener diode are studied.

(i) Forward resistance R_F=

(ii) knee voltage =

(iii) The breakdown voltage of the Zener diode V_Z= ----V

Practical Tips

- The DC power supply voltage should to be increased only up to the specified range in the forward (0 – 2 V) and reverse (0 – 15 V) directions.
- The voltage applied beyond this limit may damage the resistor or the diode.
- Zener diode functions like an ordinary PN junction diode in the forward direction. Hence the forward characteristic is the same for both PN junction diode and Zener diode. Therefore, knee voltage and forward resistance can be determined as explained in the previous experiment.
- Unlike ordinary PN junction diode, the reverse current in Zener diode is measured using milli-ammeter due to the large flow of current.

XII-Physics_Vol-1) PRACTICAL.indd 309 14-02-2020 20:13:05

8. CHARACTERISTICS OF A NPN-JUNCTION TRANSISTOR IN COMMON EMITTER CONFIGURATION

AIM

To study the characteristics and to determine the current gain of a NPN junction transistor in common emitter configuration.

APPARATUS REQUIRED

Transistor - BC 548/BC107, bread board, micro ammeter, milli ammeter, voltmeters, variable DC power supply and connecting wires.

FORMULA

$$r_i = \left[\frac{\Delta V_{BE}}{\Delta I_B}\right]_{V_{CE}} (\Omega), \quad r_o = \left[\frac{\Delta V_{CE}}{\Delta I_C}\right]_{I_B} (\Omega), \quad \beta = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V_{CE}} \text{(No unit)}$$

Where, r_i → Input impedance (Ω)

ΔV_{BE} → The change in base-emitter voltage (volt)

ΔI_B → The change in base current (μA)

r_o → Output impedance (Ω)

ΔV_{CE} → The change in collector-emitter voltage (volt)

ΔI_C → The change in collector current (mA)

β → Current gain of the transistor (No unit)

CIRCUIT DIAGRAM

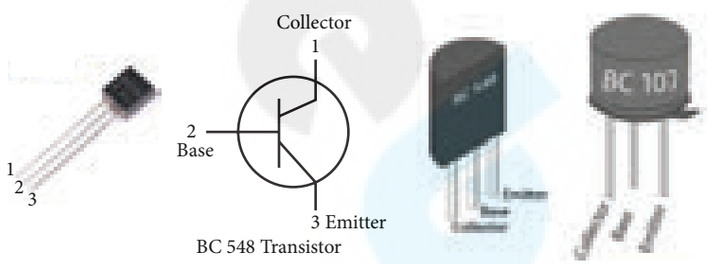


Figure (a) NPN - Junction transistor and its symbol (Transistor is held with the flat surface facing us)

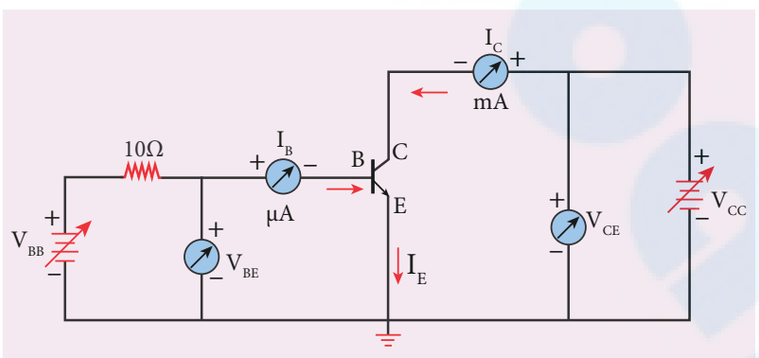


Figure (b) NPN junction transistor in CE configuration

Note

A resistor is connected in series with the base to prevent excess current flowing into the base.

XII-Physics_Vol-1) PRACTICAL.indd 310 14-02-2020 20:13:07

Precautions

- Care should be taken to connect the terminals of ammeters, voltmeters, and dc power supplies with right polarity.
- The collector and emitter terminals of the transistor must not be interchanged.

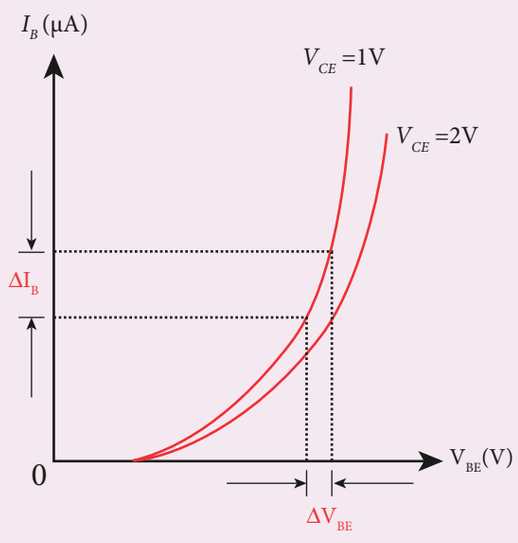
PROCEDURE

- The connections are given as shown in the diagram.
- The current and voltage at the input and output regions can be varied by adjusting the DC power supply.

(i) Input characteristic curve: V_{BE} vs I_B (V_{CE} constant)

- The collector-emitter voltage V_{CE} is kept constant.
- The base-emitter voltage V_{BE} is varied in steps of 0.1V and the corresponding base current (I_B) is noted. The readings are taken till V_{CE} reaches a constant value.
- The same procedure is repeated for different values of V_{CE}. The readings are tabulated.
- A graph is plotted by taking V_{BE} along x-axis and I_B along y-axis for both the values of V_{CE}.
- The curves thus obtained are called the input characteristics of a transistor.
- The reciprocal of the slope of these curves gives the input impedance of the transistor.

S. No	V_{CE} = 1V		V_{CE} = 2V	
	V_{BE} (V)	I_B (μA)	V_{BE} (V)	I_B (μA)



$$\text{Slope} = \frac{\Delta I_B}{\Delta V_{BE}}$$

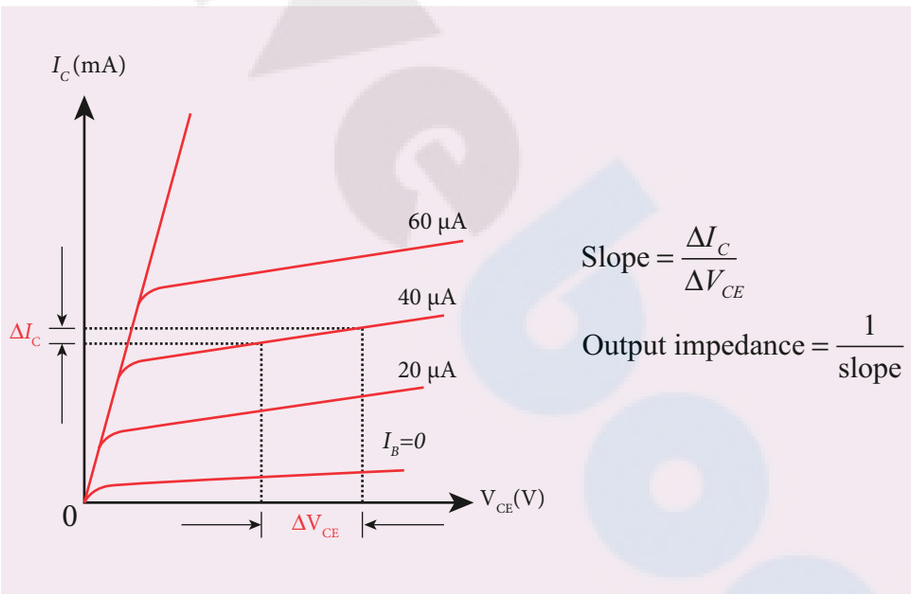
$$\text{Input impedance} = \frac{1}{\text{slope}}$$

XII-Physics_Vol-1) PRACTICAL.indd 311 14-02-2020 20:13:07

(ii) Output characteristic curve: V_{CE} vs I_C (I_B constant)

- The base current I_B is kept constant.
- V_{CE} is varied in steps of 1V and the corresponding collector current I_C is noted. The readings are taken till the collector current becomes almost constant.
- Initially I_B is kept at 0 mA and the corresponding collector current is noted. This current is the reverse saturation current I_{CEO}.
- The experiment is repeated for various values of I_B. The readings are tabulated.
- A graph is drawn by taking V_{CE} along x-axis and I_C along y-axis for various values of I_B.
- The set of curves thus obtained is called the output characteristics of a transistor.
- The reciprocal of the slope of the curve gives output impedance of the transistor.

S. No	I_B =20 μA		I_B =40 μA	
	V_{CE} (V)	I_C (mA)	V_{CE} (V)	I_C (mA)

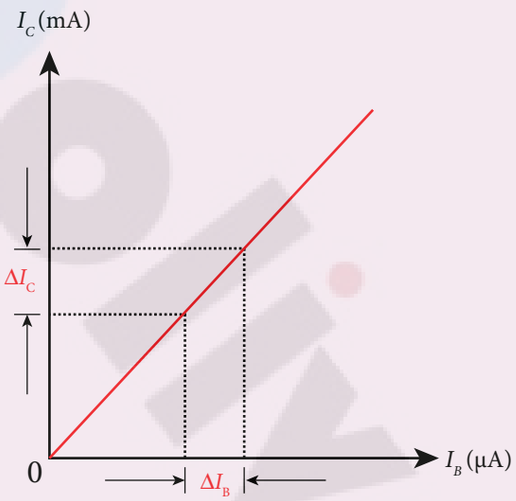


(iii) Transfer characteristic curve: I_B vs I_C (V_{CE} constant)

- The collector-emitter voltage V_{CE} is kept constant.
- The base current I_B is varied in steps of 10 μA and the corresponding collector current I_C is noted.
- This is repeated by changing the value of V_{CE}. The readings are tabulated.
- The transfer characteristics is a plot between the input current IB along x-axis and the output current I_C along y-axis keeping V_{CE} constant.
- The slope of the transfer characteristics plot gives the current gain β can be calculated.

XII-Physics_Vol-1) PRACTICAL.indd 312 14-02-2020 20:13:07

S.No	V_{CE}=1V		V_{CE}=2V	
	I_B (μA)	I_C (mA)	I_B (μA)	I_C (mA)



$$\text{Slope} = \frac{\Delta I_C}{\Delta I_B}$$

Current gain = slope

RESULT

i) The input, output and transfer characteristics of the NPN junction in common emitter mode are drawn.

ii) (a) Input impedance = _____Ω

(b) Output impedance = _____Ω

(c) Current gain β = ____(no unit)

XII-Physics_Vol-1) PRACTICAL.indd 313 14-02-2020 20:13:07

9. VERIFICATION OF TRUTH TABLES OF LOGIC GATES USING INTEGRATED CIRCUITS

AIM

To verify the truth tables of AND, OR, NOT, EX-OR, NAND and NOR gates using integrated circuits

COMPONENTS REQUIRED

AND gate (IC 7408), NOT gate (IC 7404), OR gate (IC 7432), NAND gate (IC 7400), NOR gate (IC 7402), X-OR gate (IC 7486), Power supply, Digital IC trainer kit, connecting wires.

BOOLEAN EXPRESSIONS

(i) AND gate $Y = A.B$

(ii) OR gate $Y = A+B$

(iii) NOT gate $Y = \overline{A}$

(iv) Ex OR gate $Y = \overline{A}B + A\overline{B}$

(v) NAND gate $Y = \overline{A.B}$

(vi) NOR gate $Y = \overline{A+B}$

CIRCUIT DIAGRAM

Pin Identification

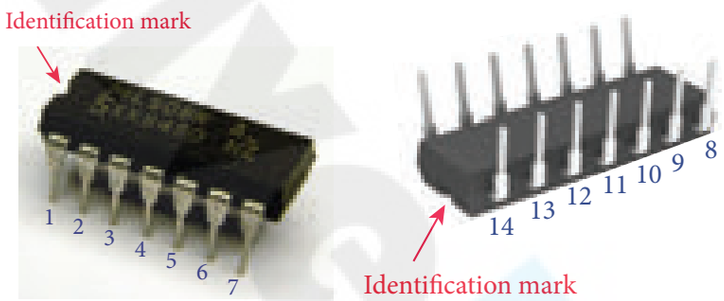


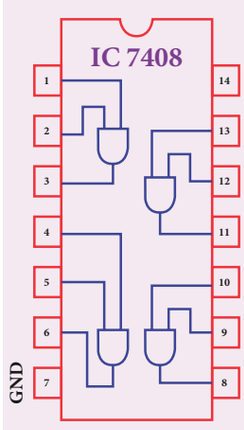
Figure (a) Integrated circuit

Note:

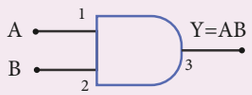
The chip must be inserted in the bread board in such a way that the identification mark should be on our left side. In this position, pin numbers are counted as marked in the picture above. Pin identification is the same for all chips that are mentioned below.

AND Gate:

PIN DIAGRAM-IC 7408



SYMBOL

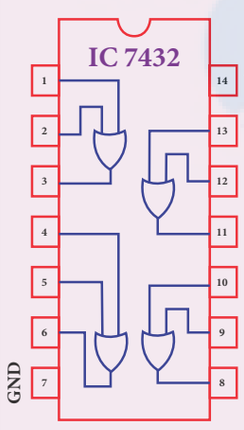


TRUTH TABLE

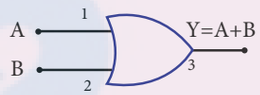
A	B	Y=AB
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate:

PIN DIAGRAM-IC 7432



SYMBOL

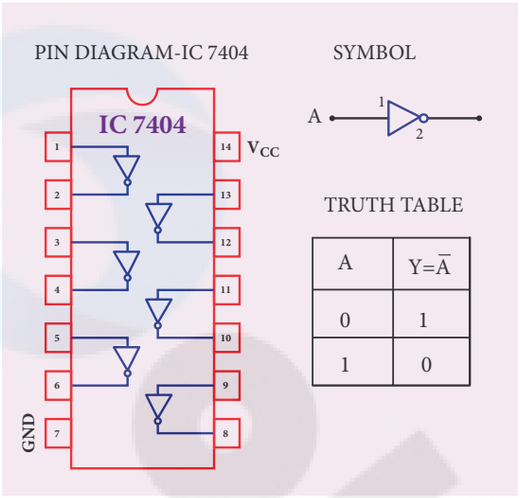


TRUTH TABLE

A	B	Y=A+B
0	0	0
0	1	1
1	0	1
1	1	1

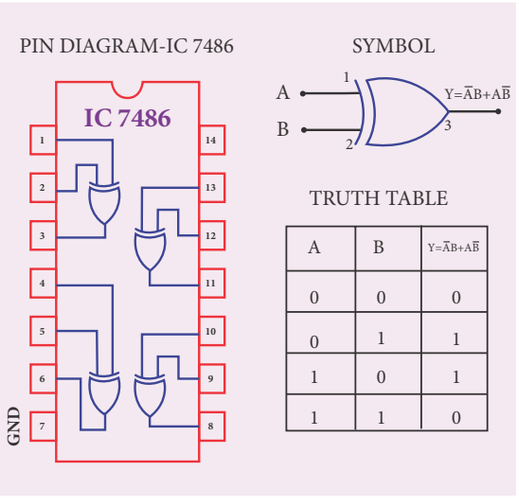
XII-Physics_Vol-1) PRACTICAL.indd 314 14-02-2020 20:13:08

NOT Gate:



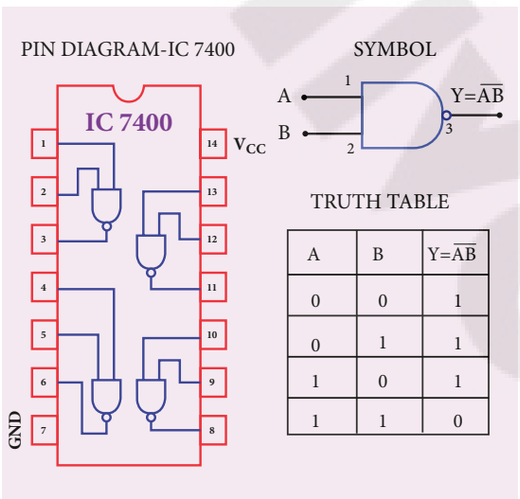
A	$Y=\overline{A}$
0	1
1	0

X-OR Gate :



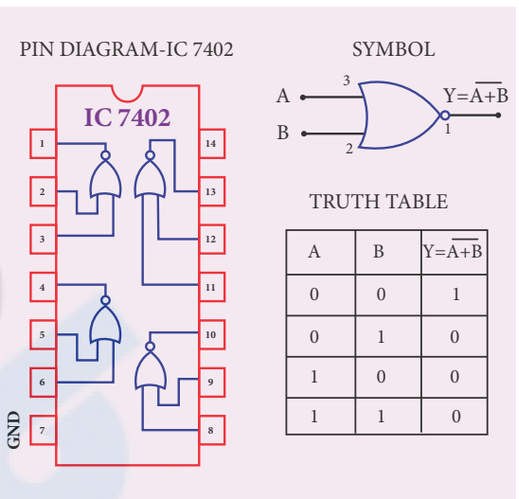
A	B	$Y=\overline{A}B+A\overline{B}$
0	0	0
0	1	1
1	0	1
1	1	0

NAND Gate:



A	B	$Y=\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate:



A	B	$Y=\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

PROCEDURE

- To verify the truth table of a logic gate, the suitable IC is taken and the connections are given using the circuit diagram.
- For all the ICs, 5V is applied to the pin 14 while the pin 7 is connected to the ground.
- The logical inputs of the truth table are applied and the corresponding output is noted.
- Similarly the output is noted for all other combinations of inputs.
- In this way, the truth table of a logic gate is verified.

RESULT

The truth table of logic gates AND, OR, NOT, Ex-OR, NAND and NOR using integrated circuits is verified.

Precautions

(i) V_{CC} and ground pins must not be interchanged while making connections. Otherwise the chip will be damaged. (ii) The pin configuration for NOR gate is different from other gates

XII-Physics_Vol-1) PRACTICAL.indd 315 14-02-2020 20:13:08

10. VERIFICATION OF DE MORGAN'S THEOREMS

AIM: To verify De Morgan's first and second theorems.

COMPONENTS REQUIRED: Power Supply (0 – 5V), IC 7400, 7408, 7432, 7404, and 7402, Digital IC trainer kit, connecting wires.

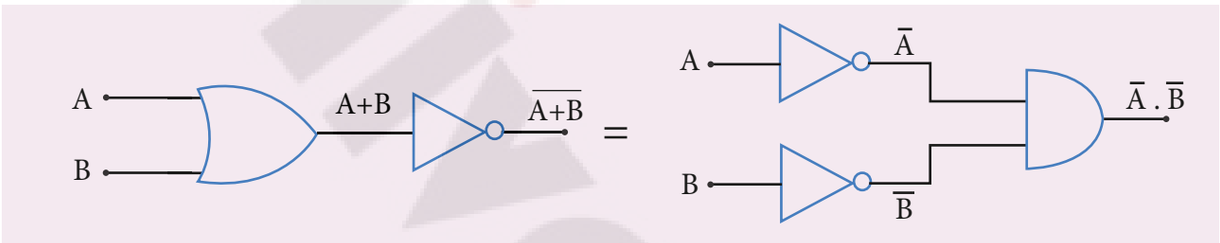
FORMULA

De Morgan's first theorem $\overline{A+B} = \overline{A}.\overline{B}$

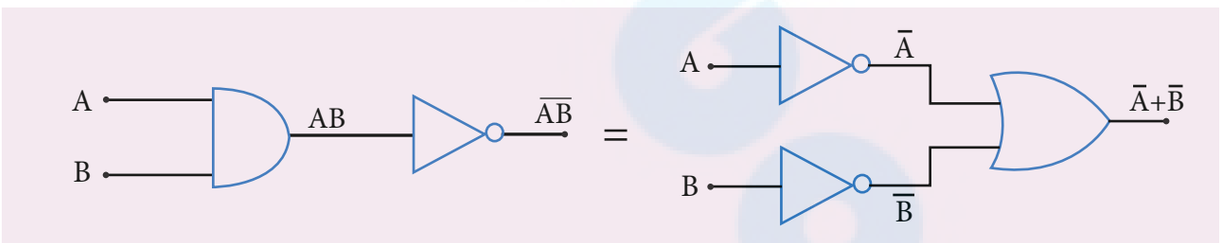
De Morgan's second theorem $\overline{A.B} = \overline{A} + \overline{B}$

CIRCUIT DIAGRAM:

De Morgan's first theorem



De Morgan's second theorem



PROCEDURE:

i) Verification of De Morgan's first theorem

- The connections are made for LHS $\left[\overline{A+B}\right]$ of the theorem as shown in the circuit diagram using appropriate ICs.
- The output is noted and tabulated for all combinations of logical inputs of the truth table.
- The same procedure is repeated for RHS $\left[\overline{A}.\overline{B}\right]$ of the theorem.
- From the truth table, it can be shown that $\overline{A+B} = \overline{A}.\overline{B}$.

XII-Physics_Vol-1) PRACTICAL.indd 316 14-02-2020 20:13:11

ii) Verification of De Morgan's second theorem

- The connections are made for LHS $\left[\overline{A.B}\right]$ of the theorem as shown in the circuit diagram using appropriate ICs.
- The output is noted and tabulated for all combinations of logical inputs of the truth table.
- The same procedure is repeated for RHS $\left[\overline{A}+\overline{B}\right]$ of the theorem.
- From the truth table, it can be shown that $\overline{A.B} = \overline{A}+\overline{B}$.

OBSERVATION

De-Morgan's first theorem

Truth Table

A	B	$\overline{A+B}$	$\overline{A}.\overline{B}$
0	0		
0	1		
1	0		
1	1		

De-Morgan's second theorem

Truth Table

A	B	$\overline{A.B}$	$\overline{A}+\overline{B}$
0	0		
0	1		
1	0		
1	1		

RESULT

De Morgan's first and second theorems are verified.

Note

The pin diagram for IC 7408, IC 7432 and IC 7404 can be taken from previous experiment

Precautions

V_{CC} and ground pins must not be interchanged while making connections. Otherwise the chip will be damaged.

For the ICs used, 5V is applied to the pin 14 while the pin 7 is connected to the ground.

XII-Physics_Vol-1) PRACTICAL.indd 317 14-02-2020 20:13:13

SUGGESTED QUESTIONS FOR THE PRACTICAL EXAMINATION

1. Determine the resistance of a given wire using metre bridge. Also find the radius of the wire using screw gauge and hence determine the specific resistance of the material of the wire. Take at least 4 readings.

2. Determine the value of the horizontal component of the Earth's magnetic field, using tangent galvanometer. Take at least 4 readings.

3. Compare the emf of two cells using potentiometer.

4. Using the spectrometer, measure the angle of the given prism and angle of minimum deviation. Hence calculate the refractive index of the material of the prism.

5. Adjust the grating for normal incidence using the spectrometer. Determine the wavelength of green, blue, yellow and red lines of mercury spectrum (The number of lines per metre length of the grating can be noted from the grating).

6. Draw the V-I characteristics of PN junction diode and determine its forward resistance and knee voltage from forward characteristics.

7. Draw the V-I characteristics of Zener diode and determine its forward resistance and knee voltage from forward characteristics. Also find break down voltage of the Zener diode from reverse characteristics.

8. Draw the input and transfer characteristic curves of the given NPN junction transistor in CE mode. Find the input impedance from input characteristics and current gain from transfer characteristics.

9. Draw the output and transfer characteristic curves of the given NPN junction transistor in CE mode. Find the output impedance from output characteristics and current gain from transfer characteristics.

10. Verify the truth table of logic gates AND, NOT, Ex-OR and NOR gates using integrated circuits.

11. Verify the truth table of logic gates OR, NOT, Ex-OR and NOR gates using integrated circuits.

12. Verify De Morgan's first and second theorems.

XII-Physics_Vol-1) PRACTICAL.indd 318 14-02-2020 20:13:13

APPENDIX

A1.1 ELECTRIC FIELD DUE TO CONTINUOUS CHARGE DISTRIBUTION

Consider the following charged object of irregular shape as shown in Figure A1.1. The entire charged object is divided into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3 ..... \Delta q_n$ and each charge element Δq is taken as a point charge.

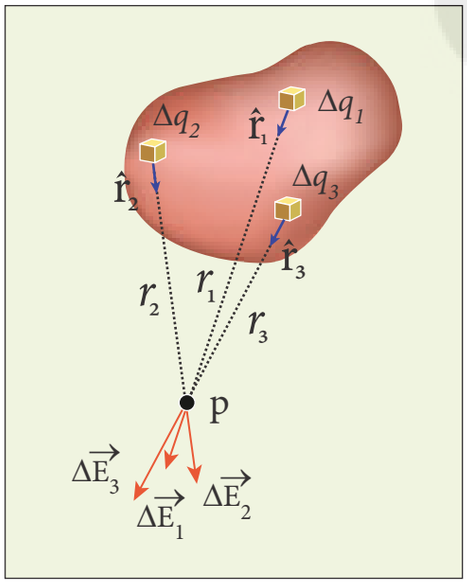


Figure A1.1 Continuous charge distributions

The electric field at a point P due to a charged object is approximately given by the sum of the fields at P due to all such charge elements.

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0}\left(\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + ... + \frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right)$$
$$\approx \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP} \qquad \text{(A1.1)}$$

Here Δq_i is the ith charge element, r_{iP} is the distance of the point P from the ith charge element and $\hat{r}_{iP}$ is the unit vector from ith charge element to the point P.

However the equation (A1.1) is only an approximation. To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (A1.1) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r^2}\hat{r} \qquad \text{(A1.2)}$$

Here r is the distance of the point P from the infinitesimal charge dq and $\hat{r}$ is the unit vector from dq to point P. Even though the electric field for a continuous charge distribution is difficult to evaluate, the force experienced by some test charge q in this electric field is still given by $\vec{F} = q\vec{E}$.

(a) If the charge Q is uniformly distributed along the wire of length L, then linear charge density (charge per unit length) is $\lambda = \frac{Q}{L}$. Its unit is coulomb per meter (Cm^{-1}).

The charge present in the infinitesimal length dl is $dq = \lambda dl$. This is shown in Figure A1.2 (a).

Appendix-1.indd 319 14-02-2020 20:14:25

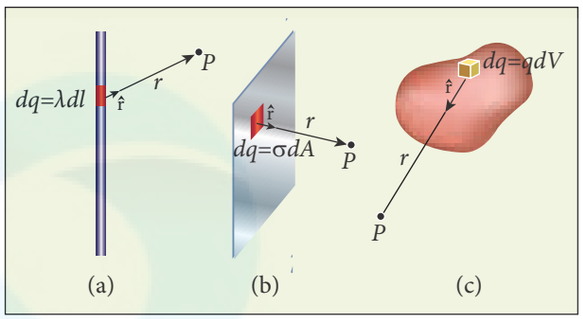


Figure A1.2 Line, surface and volume charge distribution

The electric field due to the line of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dl}{r^2}\hat{r} = \frac{\lambda}{4\pi\epsilon_0}\int \frac{dl}{r^2}\hat{r}$$

(b) If the charge Q is uniformly distributed on a surface of area A, then surface charge density (charge per unit area) is $\sigma = \frac{Q}{A}$. Its unit is coulomb per square meter (C m^{-2}).

The charge present in the infinitesimal area dA is $dq = \sigma\, dA$. This is shown in the Figure A1.2 (b).

The electric field due to a of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_\circ}\int \frac{\sigma dA}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_\circ}\sigma\int \frac{dA}{r^2}\hat{r}$$

This is shown in Figure A1.2 (b).

(c) If the charge Q is uniformly distributed in a volume V, then volume charge density (charge per unit volume) is given by $\rho = \frac{Q}{V}$. Its unit is coulomb per cubic meter (C m^{-3}).

The charge present in the infinitesimal volume element dV is $dq = \rho dV$. This is shown in Figure A1.2 (c).

The electric field due to a volume of total charge Q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_\circ}\int \frac{\rho dV}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_\circ}\rho\int \frac{dV}{r^2}\hat{r}.$$

A3.1 EXPRESSION FOR TORQUE ON A CURRENT LOOP PLACED IN A UNIFORM MAGNETIC FIELD

Consider a single rectangular loop (this means the number of turns is one) PQRS kept in a uniform magnetic field $\vec{B}$. Let a and b be the length and breadth of the rectangular loop respectively. Let $\hat{n}$ be the unit vector normal to the plane of the current loop which completely describes the orientation of the loop. The direction of magnetic field $\vec{B}$ is shown in Figure A3.1.

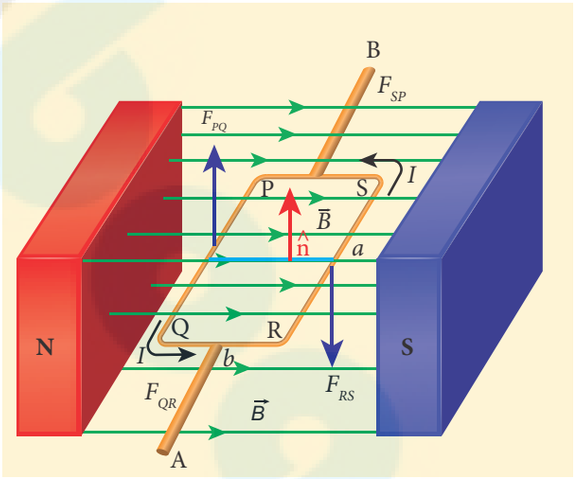


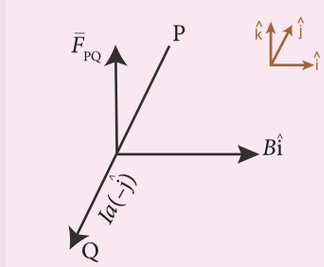
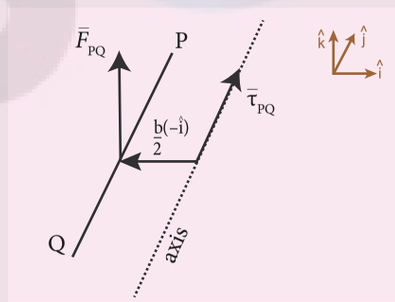
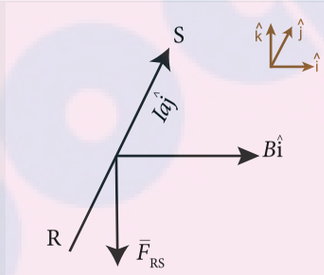
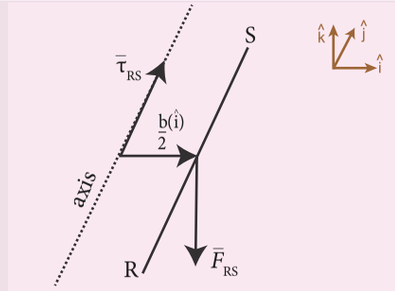
Figure A3.1 Rectangular coil placed in a magnetic field

When a steady current I passes through the loop PQRS, the net force acting on the loop is zero where as the net torque is not zero. For calculation purpose, we shall divide the rectangular loop into four sections PQ, QR, RS and SP. Now we shall consider how

Appendix-1.indd 320 14-02-2020 20:14:27

Appendix-1.indd 321

Table A3.1 Unit normal vector is perpendicular to magnetic field

Section	Force		Torque	
	$\vec{F} = I\vec{l} \times \vec{B}$	Explanatory diagram	$\vec{\tau} = \vec{r} \times \vec{F}$	Explanatory diagram
PQ	$\vec{F}_{PQ} = I(-a\hat{j}) \times B\hat{i}$ $= -IaB(\hat{j} \times \hat{i})$ $= IaB\hat{k}$		$\vec{\tau}_{PQ} = \frac{b}{2}(-\hat{i}) \times \vec{F}_{PQ}$ $= \left(-\frac{b}{2}\right)(IaB)(\hat{i} \times \hat{k})$ $= \frac{b}{2} IaB\hat{j}$	
QR	$\vec{F}_{QR} = Ib\hat{i} \times B\hat{i} = \vec{0}$		$\vec{\tau}_{QR} = \frac{a}{2}(-\hat{j}) \times \vec{F}_{QR} = \vec{0}$	
RS	$\vec{F}_{RS} = (Ia\hat{j}) \times B\hat{i}$ $= IaB(\hat{j} \times \hat{i})$ $= -IaB\hat{k}$		$\vec{\tau}_{RS} = \frac{b}{2}\hat{i} \times \vec{F}_{RS}$ $= \frac{b}{2}\hat{i} \times (-IaB\hat{k})$ $= \left(-\frac{b}{2}\right) IaB(\hat{i} \times \hat{k})$ $= \frac{b}{2} IaB\hat{j}$	
SP	$\vec{F}_{SP} = Ib(-\hat{i}) \times B\hat{i} = \vec{0}$		$\vec{\tau}_{SP} = \frac{a}{2}\hat{j} \times \vec{F}_{SP} = \vec{0}$	

14-02-2020 20:14:30

to calculate torque when the plane of the loop is parallel to the direction of magnetic field $\vec{B}$, i.e., $\hat{n} \perp \vec{B}$

$\hat{n} \perp \vec{B}$ **(unit normal vector is perpendicular to magnetic field)**

Since the current carrying wire experiences a force in a magnetic field, we shall tabulate the force experienced by each section of the loop and also the torque about an axis passing through the centre (see Table (A3.1))

$$\therefore \text{Net force } \vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP} = \vec{0}$$

And net torque

$$\vec{\tau}_{net} = \vec{\tau}_{PQ} + \vec{\tau}_{QR} + \vec{\tau}_{RS} + \vec{\tau}_{SP} = IabB\hat{j}$$

Thus, the net force on the rectangular loop is zero but net torque on the rectangular loop is not zero. Let A be the area of the rectangular loop (A = ab)

$$\text{Then } \vec{\tau}_{net} = ABI\hat{j}$$

If N be the number of turns of rectangular loop then

$$\vec{\tau}_{net} = NABI\hat{j}$$

Due to this torque, loop will start to rotate (here clockwise) and hence magnetic field $\vec{B}$ is no longer in the plane of the loop. Therefore, the above equation is a special case.

Note: When the plane of the loop is inclined to the direction of the magnetic field (i.e., $\hat{n} \not\perp \vec{B}$), then the torque is given by $\vec{\tau}_{net} = NABI\sin\theta\hat{j}$. In terms of magnetic dipole moment, $\tau_{net} = \vec{p}_m \times \vec{B}$.

Special Cases:

(i) $\theta = 90^\circ, \vec{\tau}_{net} = NABI\hat{j} = \text{max}imum$

$\vec{p}_m and \vec{B}$ are perpendicular to each other

(ii) $\theta = 0^\circ, \vec{\tau}_{net} = \vec{0}$

$\vec{p}_m \; and \; \vec{B}$ are parallel

(iii) $\theta = 180^\circ, \vec{\tau}_{net} = \vec{0}$

$\vec{p}_m \; and \; \vec{B}$ are anti-parallel

EXAMPLE A3.1

Show the time period of oscillation when a bar magnet is kept in a uniform magnetic field is $T = 2\pi\sqrt{\frac{I}{p_m B}}$ in second, where I represents moment of inertia of the bar magnet, p_m is the magnetic moment and B is the magnetic field.

Solution

The magnitude of deflecting torque (the torque which makes the object rotate) acting on the bar magnet which will tend to align the bar magnet parallel to the direction of the uniform magnetic field $\vec{B}$ is

$$|\vec{\tau}| = p_m B\sin\theta$$

The magnitude of restoring torque acting on the bar magnet can be written as

$$|\vec{\tau}| = I\frac{d^2\theta}{dt^2}$$

Under equilibrium conditions, both magnitude of deflecting torque and restoring torque will be equal but act in the opposite directions, which means

Appendix-1.indd 322 14-02-2020 20:14:34

$$I\frac{d^2\theta}{dt^2} = -p_m B\sin\theta$$

The negative sign implies that both are in opposite directions. The above equation can be written as

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\sin\theta$$

This is non-linear second order homogeneous differential equation. In order to make it linear, we use small angle approximation as we did in XI volume II (Unit 10 – oscillations, Refer section 10.4.4) i.e., $\sin\theta \approx \theta$, we get

$$\frac{d^2\theta}{dt^2} = -\frac{p_m B}{I}\theta$$

This linear second order homogeneous differential equation is a Simple Harmonic differential equation.

Comparing this equation with Simple Harmonic Motion (SHM) differential equation

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where ω is the angular frequency of the oscillation. Therefore,

$$\omega^2 = \frac{p_m B}{I} \Rightarrow \omega = \sqrt{\frac{p_m B}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B}}$$

$$T = 2\pi\sqrt{\frac{I}{p_m B_H}} \text{ in second}$$

where B_H is the horizontal component of Earth's magnetic field.

Note

1. The current in circuit can be calculated from I = K tan θ, where K is called reduction factor of tangent Galvanometer, where

$$K = \frac{2RB_H}{\mu_o N}$$

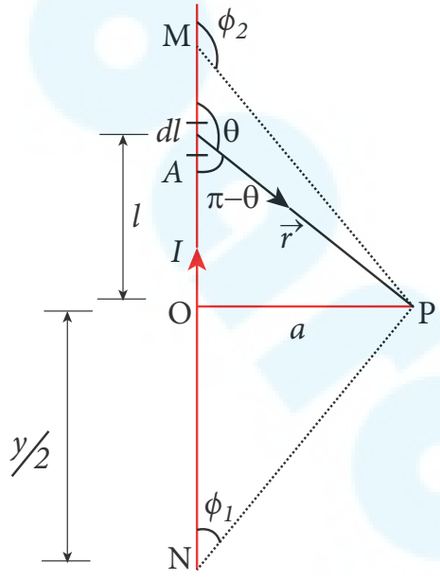
2. Sensitivity measures the change in the deflection produced by a unit current, mathematically

$$\frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

3. The tangent Galvanometer is most sensitive at a deflection of 45°. Generally the deflection is taken between 30°

EXAMPLE A3.2

Calculate the magnetic field at a point P which is perpendicular bisector to current carrying straight wire as shown in figure.



Appendix-1.indd 323 14-02-2020 20:14:36

Solution

Let the length MN = y and the point P is on its perpendicular bisector. Let O be the point on the conductor as shown in figure.

Therefore, $OM = ON = \frac{y}{2}$, then

$$\cos\varphi_1 = \frac{adjacent\,length}{hypotenuse\,length} = \frac{ON}{PN}$$

$$= \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} = \frac{y}{\sqrt{y^2+4a^2}}$$

$$\cos(\pi-\varphi_2) = \frac{adjacent\,length}{hypotenuse\,length} = \frac{OM}{PM}$$

$$\cos\varphi_2 = -\frac{OM}{PM}$$

Using the equation,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

We get $\vec{B} = \frac{\mu_\circ I}{4\pi a}\frac{2y}{\sqrt{y^2+4a^2}}\hat{n}$

For long straight wire, $y \to \infty$,

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n}$$

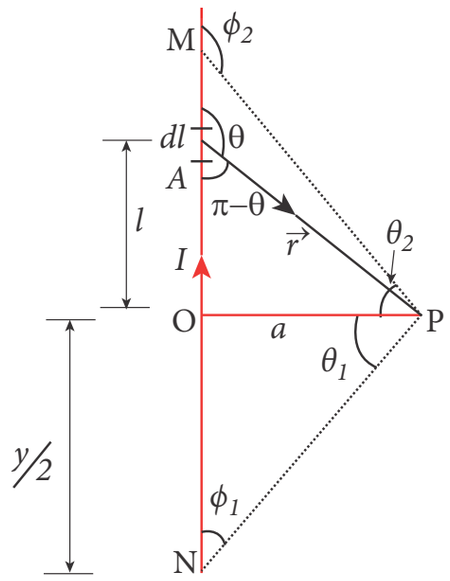
The result obtained is same as we obtained in equation (3.39).

EXAMPLE A3.3

Show that for a straight conductor, the magnetic field

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$



Solution:

In a right angle triangle OPN, let the angle $\angle OPN = \theta_1$ which implies, $\varphi_1 = \frac{\pi}{2} - \theta_1$ and also in a right angle triangle OPM, $\angle OPM = \theta_2$ which implies, $\varphi_2 = \frac{\pi}{2} + \theta_2$

Hence,

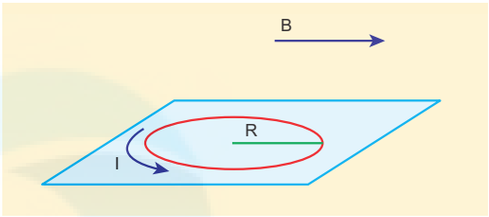
$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\left(\cos\left(\frac{\pi}{2}-\theta_1\right) - \cos\left(\frac{\pi}{2}+\theta_2\right)\right)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$

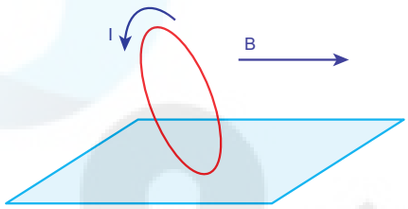
EXAMPLE A3.4

Consider a circular wire loop of radius R, mass m kept at rest on a rough surface. Let I be the current flowing through the loop and $\vec{B}$ be the magnetic field acting along horizontal as shown in Figure. Estimate the current I that should be applied so that one edge of the loop is lifted off the surface?

Appendix-1.indd 324 14-02-2020 20:14:40



Solution



When the current is passed through the loop, the torque is produced. If the torque acting on the loop is increased then the loop will start to rotate. The loop will start to lift if and only if the magnitude of magnetic torque due to current applied equals to the gravitational torque as shown in Figure

$$\tau_{magnetic} = \tau_{gravitational}$$

$$IAB = mgR$$

But $p_m = IA = I(\pi R^2)$

$$\pi IR^2 B = mgR$$

$$\Rightarrow I = \frac{mg}{\pi RB}$$

The current estimated using this equation should be applied so that one edge of loop is lifted of the surface.

A4.1 MOTIONAL EMF FROM FARADAY'S LAW

Let us consider a rectangular conducting loop of width l in a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of the loop

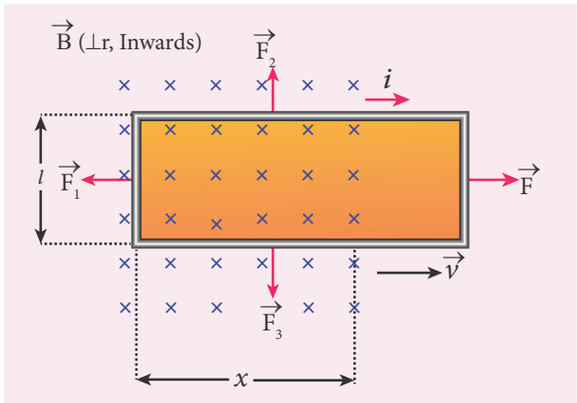


Figure A4.1 Motional emf from Faraday's law

and is directed inwards. A part of the loop is in the magnetic field while the remaining part is outside the field as shown in Figure A4.1.

When the loop is pulled with a constant velocity $\vec{v}$ to the right, the area of the portion of the loop within the magnetic field will decrease. Thus, the flux linked with the loop will also decrease. According to Faraday's law, an electric current is induced in the loop which flows in a direction so as to oppose the pull of the loop.

Let x be the length of the loop which is still within the magnetic field, then its area is lx. The magnetic flux linked with the loop is

$$\Phi_B = \int_A \vec{B}\cdot d\vec{A} = BA\cos\theta = BA$$

Here $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$$\Phi_B = Blx \qquad \text{(A4.1)}$$

As this magnetic flux decreases due to the movement of the loop, the magnitude of the induced emf is given by

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

Here, both B and l are constants. Therefore,

$$\varepsilon = Bl\frac{dx}{dt} = Blv \qquad \text{(A4.2)}$$

Appendix-1.indd 325 14-02-2020 20:14:43

where $v = \frac{dx}{dt}$ is the velocity of the loop. This emf is known as motional emf since it is produced due to the movement of the loop in the magnetic field.

From Lenz's law, it is found that the induced current flows in clockwise direction. If R is the resistance of the loop, then the induced current is given by

$$i = \frac{\varepsilon}{R} = \frac{Blv}{R} \qquad \text{(A4.3)}$$

A4.2 ANALOGIES BETWEEN LC OSCILLATIONS AND SIMPLE HARMONIC OSCILLATIONS

Quantitative treatment

The mechanical energy of the spring-mass system is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad \text{(A4.4)}$$

The energy E remains constant for varying values of x and v. Differentiating E with respect to time, we get

$$\frac{dE}{dt} = \frac{1}{2}m\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k\left(2x\frac{dx}{dt}\right) = 0$$

$$\text{or } m\frac{d^2x}{dt^2} + kx = 0 \qquad \text{(A4.5)}$$

$$\text{since } \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

This is the differential equation of the oscillations of the spring-mass system. The general solution of equation (4.68) is of the form

$$x(t) = X_m \cos(\omega t + \phi) \qquad \text{(A4.6)}$$

where X_m is the maximum value of $x(t)$, ω the angular frequency and ϕ the phase constant.

Similarly, the electromagnetic energy of the LC system is given by

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\left(\frac{1}{C}\right)q^2 = \text{constant} \qquad \text{(A4.7)}$$

Differentiating U with respect to time, we get

$$\frac{dU}{dt} = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) = 0$$

$$\text{or } L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad \text{(A4.8)}$$

$$\text{since } i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2q}{dt^2}$$

The general solution of equation (A4.8) is of the form

$$q(t) = Q_m \cos(\omega t + \phi) \qquad \text{(A4.9)}$$

where Q_m is the maximum value of $q(t)$, ω the angular frequency and ϕ the phase constant.

A5.1 PROPERTIES OF ELECTROMAGNETIC WAVES

- The energy density (energy per unit volume) associated with an electromagnetic wave propagating in vacuum or free space is

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

Appendix-1.indd 326 14-02-2020 20:14:49

where, $\frac{1}{2}\varepsilon_{\circ}E^2 = u_E$ is the energy density in an electric field and $\frac{1}{2\mu_0}B^2 = u_B$ is the energy density in a magnetic field.

Since, $E = Bc \Rightarrow u_B = u_E$.

- The energy density of the electromagnetic wave is

$$u = \varepsilon_0 E^2 = \frac{1}{\mu_0}B^2$$

- The average energy density for electromagnetic wave,

$$\langle u \rangle = \frac{1}{2}\varepsilon_{\circ}E^2 = \frac{1}{2}\frac{1}{\mu_{\circ}}B^2.$$

- The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity.

Intensity, $= \frac{\text{Energy}}{\text{speed}} = \frac{U}{c}$. or

$$I = \frac{\text{total electromagnetic energy (U)}}{\text{Surface area (A)} \times \text{time (t)}}$$

$$= \frac{\text{Power (P)}}{\text{Surface area (A)}}$$

Note

For a point source, $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

For a line source, $I \propto \frac{1}{r}$

For a plane source, I is independent of r

- Like other waves, electromagnetic waves also carry energy and momentum. For the electromagnetic wave of energy U propagating with speed c has linear momentum which is given by $p = \frac{\text{Energy}}{\text{speed}} = \frac{U}{c}$. The force exerted by an electromagnetic wave on unit area of a surface is called radiation pressure.
- If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is U and momentum imparted on the surface is $p = \frac{U}{c}$.
- If the incident electromagnetic wave of energy U is totally reflected from the surface, then the momentum delivered to the surface is

$$\Delta p = \frac{U}{c} - \left(-\frac{U}{c}\right) = 2\frac{U}{c}.$$

- The rate of flow of energy crossing a unit area is known as Poynting vector for electromagnetic waves, which is $\vec{S} = \frac{1}{\mu_{\circ}}\left(\vec{E} \times \vec{B}\right) = c^2\varepsilon_{\circ}\left(\vec{E} \times \vec{B}\right)$. The unit for Poynting vector is W m^{-2}. The Poynting vector at any point gives the direction of energy transport from that point.

Appendix-1.indd 327 14-02-2020 20:14:51

GLOSSARY
கலைச்சொற்கள்

1.	Absorption spectra	-	உட்கவர் நிறமாலை
2.	Armature	-	சுருளி
3.	Axial symmetry	-	அச்சுச் சமச்சீர்
4.	Average current	-	சராசரி மின்னோட்டம்
5.	Blackbody radiation	-	கரும்பொருள் கதிர்வீச்சு
6.	Charge	-	மின்னூட்டம்
7.	Continuous charge distribution	-	தொடர் மின்னூட்டப் பரவல்
8.	Conventional current	-	மரபு மின்னோட்டம்
9.	Conservation of charges	-	மின்னூட்டம் மாறாத் தன்மை
10.	Capacitor	-	மின்தேக்கி
11.	Corona discharge	-	ஒளிவட்ட மின்னிறக்கம் அல்லது சிதறொளி மின்னிறக்கம்
12.	Capacitance	-	மின்தேக்குத்திறன்
13.	Coercivity	-	காந்த நீக்குத்திறன்
14.	Current density	-	மின்னோட்ட அடர்த்தி
15.	Conductivity	-	மின்கடத்து எண்
16.	Configuration	-	நிலை அமைப்பு
17.	Conduction current	-	கடத்து மின்னோட்டம்
18.	Carbon Resistor	-	கார்பன் மின்தடை
19.	Current sensitivity	-	மின்னோட்ட உணர்வுநுட்பம்
20.	Dielectrics	-	மின்காப்புகள்
21.	Displacement current	-	இடப்பெயர்ச்சி மின்னோட்டம்
22.	Declination angle	-	காந்த ஒதுக்கக்கோணம்
23.	Dielectric strength	-	மின்காப்பு வலிமை
24.	Drift velocity	-	இழுப்பு திசைவேகம்
25.	Dielectric constant	-	மின்காப்பு மாறிலி
26.	Eddy current	-	சுழல் மின்னோட்டம்
27.	Electromagnetic damping	-	மின்காந்தத் தணிப்பு
28.	Electronic devices	-	மின்னணு சாதனங்கள்
29.	Electrostatics	-	நிலை மின்னியல்
30.	Electric charge	-	மின்னூட்டம்,மின்துகள்
31.	Electric field	-	மின்புலம்
32.	Electric dipole	-	மின்னிருமுனை (மின் இருமுனை)
33.	Equivalent capacitance	-	தொகுபயன் மின்தேக்குத்திறன் அல்லது இணை மின்தேக்குத்திறன்

Glossary.indd 328 14-02-2020 20:15:30

34.	Electrostatic induction	-	நிலைமின் தூண்டல்
35.	Electrostatic potential energy	-	நிலை மின்னழுத்த ஆற்றல்
36.	Electric flux	-	மின்பாயம்
37.	Equi-potential surface	-	சமமின்னழுத்தப் பரப்பு
38.	Electrostatic equilibrium	-	நிலை மின் சமநிலை
39.	Electrostatic shielding	-	நிலை மின் தடுப்புறை
40.	Energy density	-	ஆற்றல் அடர்த்தி
41.	Electrostatic potential	-	நிலை மின்னழுத்தம்
42.	Electric battery	-	மின்கலத் தொகுப்பு
43.	Emission spectra	-	வெளியிடு நிறமாலை
44.	Equivalent Resistance	-	தொகுபயன் மின்தடை
45.	Flux leakage	-	பாயக்கசிவு
46.	Figure of merit of a galvanometer	-	கால்வானா மீட்டரின் தர ஒப்பீட்டு எண்
47.	Finite value	-	வரம்பிற்குட்பட்ட மதிப்பு
48.	Free electrons	-	கட்டுறா எலக்ட்ரான்கள்
49.	Horizontal component of the Earth's magnetic field	-	புவி காந்தப்புலத்தின் கிடைத்தளக்கூறு
50.	Hysteresis	-	காந்தத் தயக்கம்
51.	Helical path	-	சுருள்பாதை
52.	superposition principle	-	மேற்பொருந்தல் தத்துவம்
53.	Insulators	-	காப்பான்கள்
54.	Inverter	-	மின் புரட்டி
55.	Inductance	-	மின்தூண்டல் எண்
56.	Inductor	-	மின்தூண்டி
57.	Inclination angle	-	காந்தச் சரிவுக்கோணம்
58.	Intensity of magnetization	-	காந்தமாக்கும் செறிவு
59.	Impedance	-	மின்மறுப்பு
60.	Linear charge density	-	மின்னூட்ட நீள் அடர்த்தி
61.	Lighthing bolt	-	மின்னல் வெட்டு
62.	Laminated Core	-	மென்தகட்டு உள்ளகம்
63.	Lightning conductor	-	மின்னல் கடத்தி
64.	Mutual-induction	-	பரிமாற்று மின்தூண்டல்
65.	Metallic conductor	-	உலோகக் கடத்தி
66.	Moving Coil galvanometer	-	இயங்குசுருள் கால்வானா மீட்டர்
67.	Magnetic meridian	-	காந்த துருவத்தளம்
68.	Magnetic domain	-	காந்தப் பெருங்கூறு, காந்தக் களம்
69.	Magnetic induction	-	காந்தத்தூண்டல்
70.	Magnetising field	-	காந்தமாக்கு புலம்
71.	Magnetic Flux	-	காந்தப்பாயம்
72.	Magnetic susceptibility	-	காந்த ஏற்புத்திறன்
73.	Magnetic permeability	-	காந்த உட்புகுதிறன்
74.	Magnetic flux	-	காந்தப்பாயம்
75.	Magnetic dipole moment	-	காந்த இருமுனை திருப்புத்திறன்
76.	Magnetic declination	-	காந்த ஒதுக்கம்

Glossary.indd 329 14-02-2020 20:15:30

77.	Magnetic dip or inclination	-	காந்தச் சரிவு
78.	Non ohmic conductor	-	ஓம் விதிக்கு உட்படாத கடத்தி
79.	Propagation vector	-	பரவும் வெக்டர்
80.	Phasor	-	கட்ட வெக்டர்
81.	Power factor	-	திறன் காரணி
82.	Potential difference	-	மின்னழுத்த வேறுபாடு
83.	Permittivity	-	விடுதிறன்
84.	Quantization	-	குவாண்டமாக்கல் அல்லது துளிமமாக்கல்
85.	Resonance	-	ஒத்ததிர்வு
86.	Rotor	-	சுழலி
87.	Relative permeability	-	ஒப்புமை உட்புகுதிறன்
89.	Resistors in series	-	மின்தடைகள் தொடரிணைப்பு
90.	Retentivity	-	காந்தப்பற்றுத்திறன்
91.	Surface charge density	-	மின்னூட்டப் பரப்படர்த்தி
92.	Slip rings	-	நழுவு வளையங்கள்
93.	Series and parallel	-	தொடரிணைப்பு, பக்கவினைப்பு
94.	Self-induction	-	தன்மின்தூண்டல்
95.	Successive collisions	-	அடுத்தடுத்த மோதல்கள்
96.	Stator	-	நிலையில்
97.	Superconductors	-	மீக்கடத்திகள்
98.	Semiconductor	-	குறை கடத்தி
99.	Solar spectrum	-	சூரிய நிறமாலை
100.	Shunt resistance	-	இணை மின்தடை
101.	Solenoid	-	வரிச்சுருள்
102.	Temperature coefficient of Resistivity	-	வெப்பநிலை மின்தடை எண்
103.	Toroid	-	வட்ட வரிச்சுருள்
104.	Torsional constant	-	முறுக்குக் கோணம்
105.	Transverse wave	-	குறுக்கலை
106.	Torsion balance	-	முறுக்குத் தராசு
107.	Transformer	-	மின்மாற்றி
108.	Thermistor	-	வெப்பமாறு மின்தடை
109.	Voltage sensitivity	-	மின்னழுத்த உணர்வுநுட்பம்
110.	Volume element	-	பருமக் கூறு
111.	Wave diagram	-	அலை வரைபடம்
112.	Wattful current	-	முழுத்திறன் மின்னோட்டம்
113.	Wattless current	-	சுழித்திறன் மின்னோட்டம்
114.	Winding	-	கம்பிச் சுற்று

Glossary.indd 330 14-02-2020 20:15:30

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

Glossary.indd 331 14-02-2020 20:15:31

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Glossary.indd 332 14-02-2020 20:15:31

ANTI LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

Glossary.indd 333 14-02-2020 20:15:32

ANTI LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Glossary.indd 334 14-02-2020 20:15:32

(NATURAL SINES)

0	0	1	2	3	4	5	6	7	8	9
0	0	0.0017453	0.0034907	0.0052360	0.0069813	0.0087265	0.010472	0.012217	0.013962	0.015707
1	0.017452	0.019197	0.020942	0.022687	0.024432	0.026177	0.027922	0.029666	0.031411	0.033155
2	0.034899	0.036644	0.038388	0.040132	0.041876	0.043619	0.045363	0.047106	0.04885	0.050593
3	0.052336	0.054079	0.055822	0.057564	0.059306	0.061049	0.062791	0.064532	0.066274	0.068015
4	0.069756	0.071497	0.073238	0.074979	0.076719	0.078459	0.080199	0.081939	0.083678	0.085417
5	0.087156	0.088894	0.090633	0.092371	0.094108	0.095846	0.097583	0.09932	0.101056	0.102793
6	0.104528	0.106264	0.107999	0.109734	0.111469	0.113203	0.114937	0.116671	0.118404	0.120137
7	0.121869	0.123601	0.125333	0.127065	0.128796	0.130526	0.132256	0.133986	0.135716	0.137445
8	0.139173	0.140901	0.142629	0.144356	0.146083	0.147809	0.149535	0.151261	0.152986	0.15471
9	0.156434	0.158158	0.159881	0.161604	0.163326	0.165048	0.166769	0.168489	0.170209	0.171929
x	0	1	2	3	4	5	6	7	8	9
10	0.173648	0.175367	0.177085	0.178802	0.180519	0.182236	0.183951	0.185667	0.187381	0.189095
11	0.190809	0.192522	0.194234	0.195946	0.197657	0.199368	0.201078	0.202787	0.204496	0.206204
12	0.207912	0.209619	0.211325	0.21303	0.214735	0.21644	0.218143	0.219846	0.221548	0.22325
13	0.224951	0.226651	0.228351	0.23005	0.231748	0.233445	0.235142	0.236838	0.238533	0.240228
14	0.241922	0.243615	0.245307	0.246999	0.24869	0.25038	0.252069	0.253758	0.255446	0.257133
15	0.258819	0.260505	0.262189	0.263873	0.265556	0.267238	0.26892	0.2706	0.27228	0.273959
16	0.275637	0.277315	0.278991	0.280667	0.282341	0.284015	0.285688	0.287361	0.289032	0.290702
17	0.292372	0.29404	0.295708	0.297375	0.299041	0.300706	0.30237	0.304033	0.305695	0.307357
18	0.309017	0.310676	0.312335	0.313992	0.315649	0.317305	0.318959	0.320613	0.322266	0.323917
19	0.325568	0.327218	0.328867	0.330514	0.332161	0.333807	0.335452	0.337095	0.338738	0.34038
x	0	1	2	3	4	5	6	7	8	9
20	0.34202	0.34366	0.345298	0.346936	0.348572	0.350207	0.351842	0.353475	0.355107	0.356738
21	0.358368	0.359997	0.361625	0.363251	0.364877	0.366501	0.368125	0.369747	0.371368	0.372988
22	0.374607	0.376224	0.377841	0.379456	0.38107	0.382683	0.384295	0.385906	0.387516	0.389124
23	0.390731	0.392337	0.393942	0.395546	0.397148	0.398749	0.400349	0.401948	0.403545	0.405142
24	0.406737	0.40833	0.409923	0.411514	0.413104	0.414693	0.416281	0.417867	0.419452	0.421036
25	0.422618	0.424199	0.425779	0.427358	0.428935	0.430511	0.432086	0.433659	0.435231	0.436802
26	0.438371	0.439939	0.441506	0.443071	0.444635	0.446198	0.447759	0.449319	0.450878	0.452435
27	0.45399	0.455545	0.457098	0.45865	0.4602	0.461749	0.463296	0.464842	0.466387	0.46793
28	0.469472	0.471012	0.472551	0.474088	0.475624	0.477159	0.478692	0.480223	0.481754	0.483282
29	0.48481	0.486335	0.48786	0.489382	0.490904	0.492424	0.493942	0.495459	0.496974	0.498488
x	0	1	2	3	4	5	6	7	8	9
30	0.5	0.501511	0.50302	0.504528	0.506034	0.507538	0.509041	0.510543	0.512043	0.513541
31	0.515038	0.516533	0.518027	0.519519	0.52101	0.522499	0.523986	0.525472	0.526956	0.528438
32	0.529919	0.531399	0.532876	0.534352	0.535827	0.5373	0.538771	0.54024	0.541708	0.543174
33	0.544639	0.546102	0.547563	0.549023	0.550481	0.551937	0.553392	0.554844	0.556296	0.557745
34	0.559193	0.560639	0.562083	0.563526	0.564967	0.566406	0.567844	0.56928	0.570714	0.572146
35	0.573576	0.575005	0.576432	0.577858	0.579281	0.580703	0.582123	0.583541	0.584958	0.586372
36	0.587785	0.589196	0.590606	0.592013	0.593419	0.594823	0.596225	0.597625	0.599024	0.60042
37	0.601815	0.603208	0.604599	0.605988	0.607376	0.608761	0.610145	0.611527	0.612907	0.614285
38	0.615661	0.617036	0.618408	0.619779	0.621148	0.622515	0.62388	0.625243	0.626604	0.627963
39	0.62932	0.630676	0.632029	0.633381	0.634731	0.636078	0.637424	0.638768	0.64011	0.64145
x	0	1	2	3	4	5	6	7	8	9
40	0.642788	0.644124	0.645458	0.64679	0.64812	0.649448	0.650774	0.652098	0.653421	0.654741
41	0.656059	0.657375	0.658689	0.660002	0.661312	0.66262	0.663926	0.66523	0.666532	0.667833
42	0.669131	0.670427	0.671721	0.673013	0.674302	0.67559	0.676876	0.67816	0.679441	0.680721
43	0.681998	0.683274	0.684547	0.685818	0.687088	0.688355	0.68962	0.690882	0.692143	0.693402
44	0.694658	0.695913	0.697165	0.698415	0.699663	0.700909	0.702153	0.703395	0.704634	0.705872
45	0.707107	0.70834	0.709571	0.710799	0.712026	0.71325	0.714473	0.715693	0.716911	0.718126

Glossary.indd 335 14-02-2020 20:15:32

(NATURAL SINES)

46	0.71934	0.720551	0.72176	0.722967	0.724172	0.725374	0.726575	0.727773	0.728969	0.730162
47	0.731354	0.732543	0.73373	0.734915	0.736097	0.737277	0.738455	0.739631	0.740805	0.741976
48	0.743145	0.744312	0.745476	0.746638	0.747798	0.748956	0.750111	0.751264	0.752415	0.753563
49	0.75471	0.755853	0.756995	0.758134	0.759271	0.760406	0.761538	0.762668	0.763796	0.764921
x	0	1	2	3	4	5	6	7	8	9
50	0.766044	0.767165	0.768284	0.7694	0.770513	0.771625	0.772734	0.77384	0.774944	0.776046
51	0.777146	0.778243	0.779338	0.78043	0.78152	0.782608	0.783693	0.784776	0.785857	0.786935
52	0.788011	0.789084	0.790155	0.791224	0.79229	0.793353	0.794415	0.795473	0.79653	0.797584
53	0.798636	0.799685	0.800731	0.801776	0.802817	0.803857	0.804894	0.805928	0.80696	0.80799
54	0.809017	0.810042	0.811064	0.812084	0.813101	0.814116	0.815128	0.816138	0.817145	0.81815
55	0.819152	0.820152	0.821149	0.822144	0.823136	0.824126	0.825113	0.826098	0.827081	0.82806
56	0.829038	0.830012	0.830984	0.831954	0.832921	0.833886	0.834848	0.835807	0.836764	0.837719
57	0.838671	0.83962	0.840567	0.841511	0.842452	0.843391	0.844328	0.845262	0.846193	0.847122
58	0.848048	0.848972	0.849893	0.850811	0.851727	0.85264	0.853551	0.854459	0.855364	0.856267
59	0.857167	0.858065	0.85896	0.859852	0.860742	0.861629	0.862514	0.863396	0.864275	0.865151
x	0	1	2	3	4	5	6	7	8	9
60	0.866025	0.866897	0.867765	0.868632	0.869495	0.870356	0.871214	0.872069	0.872922	0.873772
61	0.87462	0.875465	0.876307	0.877146	0.877983	0.878817	0.879649	0.880477	0.881303	0.882127
62	0.882948	0.883766	0.884581	0.885394	0.886204	0.887011	0.887815	0.888617	0.889416	0.890213
63	0.891007	0.891798	0.892586	0.893371	0.894154	0.894934	0.895712	0.896486	0.897258	0.898028
64	0.898794	0.899558	0.900319	0.901077	0.901833	0.902585	0.903335	0.904083	0.904827	0.905569
65	0.906308	0.907044	0.907777	0.908508	0.909236	0.909961	0.910684	0.911403	0.91212	0.912834
66	0.913545	0.914254	0.91496	0.915663	0.916363	0.91706	0.917755	0.918446	0.919135	0.919821
67	0.920505	0.921185	0.921863	0.922538	0.92321	0.92388	0.924546	0.92521	0.925871	0.926529
68	0.927184	0.927836	0.928486	0.929133	0.929776	0.930418	0.931056	0.931691	0.932324	0.932954
69	0.93358	0.934204	0.934826	0.935444	0.93606	0.936672	0.937282	0.937889	0.938493	0.939094
x	0	1	2	3	4	5	6	7	8	9
70	0.939693	0.940288	0.940881	0.941471	0.942057	0.942641	0.943223	0.943801	0.944376	0.944949
71	0.945519	0.946085	0.946649	0.94721	0.947768	0.948324	0.948876	0.949425	0.949972	0.950516
72	0.951057	0.951594	0.952129	0.952661	0.953191	0.953717	0.95424	0.954761	0.955278	0.955793
73	0.956305	0.956814	0.957319	0.957822	0.958323	0.95882	0.959314	0.959805	0.960294	0.960779
74	0.961262	0.961741	0.962218	0.962692	0.963163	0.96363	0.964095	0.964557	0.965016	0.965473
75	0.965926	0.966376	0.966823	0.967268	0.967709	0.968148	0.968583	0.969016	0.969445	0.969872
76	0.970296	0.970716	0.971134	0.971549	0.971961	0.97237	0.972776	0.973179	0.973579	0.973976
77	0.97437	0.974761	0.975149	0.975535	0.975917	0.976296	0.976672	0.977046	0.977416	0.977783
78	0.978148	0.978509	0.978867	0.979223	0.979575	0.979925	0.980271	0.980615	0.980955	0.981293
79	0.981627	0.981959	0.982287	0.982613	0.982935	0.983255	0.983571	0.983885	0.984196	0.984503
x	0	1	2	3	4	5	6	7	8	9
80	0.984808	0.985109	0.985408	0.985703	0.985996	0.986286	0.986572	0.986856	0.987136	0.987414
81	0.987688	0.98796	0.988228	0.988494	0.988756	0.989016	0.989272	0.989526	0.989776	0.990024
82	0.990268	0.990509	0.990748	0.990983	0.991216	0.991445	0.991671	0.991894	0.992115	0.992332
83	0.992546	0.992757	0.992966	0.993171	0.993373	0.993572	0.993768	0.993961	0.994151	0.994338
84	0.994522	0.994703	0.994881	0.995056	0.995227	0.995396	0.995562	0.995725	0.995884	0.996041
85	0.996195	0.996345	0.996493	0.996637	0.996779	0.996917	0.997053	0.997185	0.997314	0.997441
86	0.997564	0.997684	0.997801	0.997916	0.998027	0.998135	0.99824	0.998342	0.998441	0.998537
87	0.99863	0.998719	0.998806	0.99889	0.998971	0.999048	0.999123	0.999194	0.999263	0.999328
88	0.999391	0.99945	0.999507	0.99956	0.99961	0.999657	0.999701	0.999743	0.999781	0.999816
89	0.999848	0.999877	0.999903	0.999925	0.999945	0.999962	0.999976	0.999986	0.999994	0.999998
90	1	0.999998	0.999994	0.999986	0.999976	0.999962	0.999945	0.999925	0.999903	0.999877

Glossary.indd 336 14-02-2020 20:15:32

(NATURAL COSINES)

x	0	1	2	3	4	5	6	7	8	9
0	1	0.999998	0.999994	0.999986	0.999976	0.999962	0.999945	0.999925	0.999903	0.999877
1	0.999848	0.999816	0.999781	0.999743	0.999701	0.999657	0.99961	0.99956	0.999507	0.99945
2	0.999391	0.999328	0.999263	0.999194	0.999123	0.999048	0.998971	0.99889	0.998806	0.998719
3	0.99863	0.998537	0.998441	0.998342	0.99824	0.998135	0.998027	0.997916	0.997801	0.997684
4	0.997564	0.997441	0.997314	0.997185	0.997053	0.996917	0.996779	0.996637	0.996493	0.996345
5	0.996195	0.996041	0.995884	0.995725	0.995562	0.995396	0.995227	0.995056	0.994881	0.994703
6	0.994522	0.994338	0.994151	0.993961	0.993768	0.993572	0.993373	0.993171	0.992966	0.992757
7	0.992546	0.992332	0.992115	0.991894	0.991671	0.991445	0.991216	0.990983	0.990748	0.990509
8	0.990268	0.990024	0.989776	0.989526	0.989272	0.989016	0.988756	0.988494	0.988228	0.98796
9	0.987688	0.987414	0.987136	0.986856	0.986572	0.986286	0.985996	0.985703	0.985408	0.985109
x	0	1	2	3	4	5	6	7	8	9
10	0.984808	0.984503	0.984196	0.983885	0.983571	0.983255	0.982935	0.982613	0.982287	0.981959
11	0.981627	0.981293	0.980955	0.980615	0.980271	0.979925	0.979575	0.979223	0.978867	0.978509
12	0.978148	0.977783	0.977416	0.977046	0.976672	0.976296	0.975917	0.975535	0.975149	0.974761
13	0.97437	0.973976	0.973579	0.973179	0.972776	0.97237	0.971961	0.971549	0.971134	0.970716
14	0.970296	0.969872	0.969445	0.969016	0.968583	0.968148	0.967709	0.967268	0.966823	0.966376
15	0.965926	0.965473	0.965016	0.964557	0.964095	0.96363	0.963163	0.962692	0.962218	0.961741
16	0.961262	0.960779	0.960294	0.959805	0.959314	0.95882	0.958323	0.957822	0.957319	0.956814
17	0.956305	0.955793	0.955278	0.954761	0.95424	0.953717	0.953191	0.952661	0.952129	0.951594
18	0.951057	0.950516	0.949972	0.949425	0.948876	0.948324	0.947768	0.94721	0.946649	0.946085
19	0.945519	0.944949	0.944376	0.943801	0.943223	0.942641	0.942057	0.941471	0.940881	0.940288
x	0	1	2	3	4	5	6	7	8	9
20	0.939693	0.939094	0.938493	0.937889	0.937282	0.936672	0.93606	0.935444	0.934826	0.934204
21	0.93358	0.932954	0.932324	0.931691	0.931056	0.930418	0.929776	0.929133	0.928486	0.927836
22	0.927184	0.926529	0.925871	0.92521	0.924546	0.92388	0.92321	0.922538	0.921863	0.921185
23	0.920505	0.919821	0.919135	0.918446	0.917755	0.91706	0.916363	0.915663	0.91496	0.914254
24	0.913545	0.912834	0.91212	0.911403	0.910684	0.909961	0.909236	0.908508	0.907777	0.907044
25	0.906308	0.905569	0.904827	0.904083	0.903335	0.902585	0.901833	0.901077	0.900319	0.899558
26	0.898794	0.898028	0.897258	0.896486	0.895712	0.894934	0.894154	0.893371	0.892586	0.891798
27	0.891007	0.890213	0.889416	0.888617	0.887815	0.887011	0.886204	0.885394	0.884581	0.883766
28	0.882948	0.882127	0.881303	0.880477	0.879649	0.878817	0.877983	0.877146	0.876307	0.875465
29	0.87462	0.873772	0.872922	0.872069	0.871214	0.870356	0.869495	0.868632	0.867765	0.866897
x	0	1	2	3	4	5	6	7	8	9
30	0.866025	0.865151	0.864275	0.863396	0.862514	0.861629	0.860742	0.859852	0.85896	0.858065
31	0.857167	0.856267	0.855364	0.854459	0.853551	0.85264	0.851727	0.850811	0.849893	0.848972
32	0.848048	0.847122	0.846193	0.845262	0.844328	0.843391	0.842452	0.841511	0.840567	0.83962
33	0.838671	0.837719	0.836764	0.835807	0.834848	0.833886	0.832921	0.831954	0.830984	0.830012
34	0.829038	0.82806	0.827081	0.826098	0.825113	0.824126	0.823136	0.822144	0.821149	0.820152
35	0.819152	0.81815	0.817145	0.816138	0.815128	0.814116	0.813101	0.812084	0.811064	0.810042
36	0.809017	0.80799	0.80696	0.805928	0.804894	0.803857	0.802817	0.801776	0.800731	0.799685
37	0.798636	0.797584	0.79653	0.795473	0.794415	0.793353	0.79229	0.791224	0.790155	0.789084
38	0.788011	0.786935	0.785857	0.784776	0.783693	0.782608	0.78152	0.78043	0.779338	0.778243
39	0.777146	0.776046	0.774944	0.77384	0.772734	0.771625	0.770513	0.7694	0.768284	0.767165
x	0	1	2	3	4	5	6	7	8	9
40	0.766044	0.764921	0.763796	0.762668	0.761538	0.760406	0.759271	0.758134	0.756995	0.755853
41	0.75471	0.753563	0.752415	0.751264	0.750111	0.748956	0.747798	0.746638	0.745476	0.744312
42	0.743145	0.741976	0.740805	0.739631	0.738455	0.737277	0.736097	0.734915	0.73373	0.732543
43	0.731354	0.730162	0.728969	0.727773	0.726575	0.725374	0.724172	0.722967	0.72176	0.720551
44	0.71934	0.718126	0.716911	0.715693	0.714473	0.71325	0.712026	0.710799	0.709571	0.70834
45	0.707107	0.705872	0.704634	0.703395	0.702153	0.700909	0.699663	0.698415	0.697165	0.695913

Glossary.indd 337 14-02-2020 20:15:32

(NATURAL COSINES)

46	0.694658	0.693402	0.692143	0.690882	0.68962	0.688355	0.687088	0.685818	0.684547	0.683274
47	0.681998	0.680721	0.679441	0.67816	0.676876	0.67559	0.674302	0.673013	0.671721	0.670427
48	0.669131	0.667833	0.666532	0.66523	0.663926	0.66262	0.661312	0.660002	0.658689	0.657375
49	0.656059	0.654741	0.653421	0.652098	0.650774	0.649448	0.64812	0.64679	0.645458	0.644124
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
50	0.642788	0.64145	0.64011	0.638768	0.637424	0.636078	0.634731	0.633381	0.632029	0.630676
51	0.62932	0.627963	0.626604	0.625243	0.62388	0.622515	0.621148	0.619779	0.618408	0.617036
52	0.615661	0.614285	0.612907	0.611527	0.610145	0.608761	0.607376	0.605988	0.604599	0.603208
53	0.601815	0.60042	0.599024	0.597625	0.596225	0.594823	0.593419	0.592013	0.590606	0.589196
54	0.587785	0.586372	0.584958	0.583541	0.582123	0.580703	0.579281	0.577858	0.576432	0.575005
55	0.573576	0.572146	0.570714	0.56928	0.567844	0.566406	0.564967	0.563526	0.562083	0.560639
56	0.559193	0.557745	0.556296	0.554844	0.553392	0.551937	0.550481	0.549023	0.547563	0.546102
57	0.544639	0.543174	0.541708	0.54024	0.538771	0.5373	0.535827	0.534352	0.532876	0.531399
58	0.529919	0.528438	0.526956	0.525472	0.523986	0.522499	0.52101	0.519519	0.518027	0.516533
59	0.515038	0.513541	0.512043	0.510543	0.509041	0.507538	0.506034	0.504528	0.50302	0.501511
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
60	0.5	0.498488	0.496974	0.495459	0.493942	0.492424	0.490904	0.489382	0.48786	0.486335
61	0.48481	0.483282	0.481754	0.480223	0.478692	0.477159	0.475624	0.474088	0.472551	0.471012
62	0.469472	0.46793	0.466387	0.464842	0.463296	0.461749	0.4602	0.45865	0.457098	0.455545
63	0.45399	0.452435	0.450878	0.449319	0.447759	0.446198	0.444635	0.443071	0.441506	0.439939
64	0.438371	0.436802	0.435231	0.433659	0.432086	0.430511	0.428935	0.427358	0.425779	0.424199
65	0.422618	0.421036	0.419452	0.417867	0.416281	0.414693	0.413104	0.411514	0.409923	0.40833
66	0.406737	0.405142	0.403545	0.401948	0.400349	0.398749	0.397148	0.395546	0.393942	0.392337
67	0.390731	0.389124	0.387516	0.385906	0.384295	0.382683	0.38107	0.379456	0.377841	0.376224
68	0.374607	0.372988	0.371368	0.369747	0.368125	0.366501	0.364877	0.363251	0.361625	0.359997
69	0.358368	0.356738	0.355107	0.353475	0.351842	0.350207	0.348572	0.346936	0.345298	0.34366
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
70	0.34202	0.34038	0.338738	0.337095	0.335452	0.333807	0.332161	0.330514	0.328867	0.327218
71	0.325568	0.323917	0.322266	0.320613	0.318959	0.317305	0.315649	0.313992	0.312335	0.310676
72	0.309017	0.307357	0.305695	0.304033	0.30237	0.300706	0.299041	0.297375	0.295708	0.29404
73	0.292372	0.290702	0.289032	0.287361	0.285688	0.284015	0.282341	0.280667	0.278991	0.277315
74	0.275637	0.273959	0.27228	0.2706	0.26892	0.267238	0.265556	0.263873	0.262189	0.260505
75	0.258819	0.257133	0.255446	0.253758	0.252069	0.25038	0.24869	0.246999	0.245307	0.243615
76	0.241922	0.240228	0.238533	0.236838	0.235142	0.233445	0.231748	0.23005	0.228351	0.226651
77	0.224951	0.22325	0.221548	0.219846	0.218143	0.21644	0.214735	0.21303	0.211325	0.209619
78	0.207912	0.206204	0.204496	0.202787	0.201078	0.199368	0.197657	0.195946	0.194234	0.192522
79	0.190809	0.189095	0.187381	0.185667	0.183951	0.182236	0.180519	0.178802	0.177085	0.175367
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
80	0.173648	0.171929	0.170209	0.168489	0.166769	0.165048	0.163326	0.161604	0.159881	0.158158
81	0.156434	0.15471	0.152986	0.151261	0.149535	0.147809	0.146083	0.144356	0.142629	0.140901
82	0.139173	0.137445	0.135716	0.133986	0.132256	0.130526	0.128796	0.127065	0.125333	0.123601
83	0.121869	0.120137	0.118404	0.116671	0.114937	0.113203	0.111469	0.109734	0.107999	0.106264
84	0.104528	0.102793	0.101056	0.09932	0.097583	0.095846	0.094108	0.092371	0.090633	0.088894
85	0.087156	0.085417	0.083678	0.081939	0.080199	0.078459	0.076719	0.074979	0.073238	0.071497
86	0.069756	0.068015	0.066274	0.064532	0.062791	0.061049	0.059306	0.057564	0.055822	0.054079
87	0.052336	0.050593	0.04885	0.047106	0.045363	0.043619	0.041876	0.040132	0.038388	0.036644
88	0.034899	0.033155	0.031411	0.029666	0.027922	0.026177	0.024432	0.022687	0.020942	0.019197
89	0.017452	0.015707	0.013962	0.012217	0.010472	0.0087265	0.0069813	0.0052360	0.0034907	0.0017453
90	0	0.0017453	0.0034907	0.0052360	0.0069813	0.0087265	0.010472	0.012217	0.013962	0.015707

Glossary.indd 338 14-02-2020 20:15:32

(NATURAL TANGENTS)

x	0	1	2	3	4	5	6	7	8	9
0	0	0.0017453	0.0034907	0.0052360	0.0069814	0.0087269	0.010472	0.012218	0.013964	0.015709
1	0.017455	0.019201	0.020947	0.022693	0.024439	0.026186	0.027933	0.029679	0.031426	0.033173
2	0.034921	0.036668	0.038416	0.040164	0.041912	0.043661	0.04541	0.047159	0.048908	0.050658
3	0.052408	0.054158	0.055909	0.05766	0.059411	0.061163	0.062915	0.064667	0.06642	0.068173
4	0.069927	0.071681	0.073435	0.07519	0.076946	0.078702	0.080458	0.082215	0.083972	0.08573
5	0.087489	0.089248	0.091007	0.092767	0.094528	0.096289	0.098051	0.099813	0.101576	0.10334
6	0.105104	0.106869	0.108635	0.110401	0.112168	0.113936	0.115704	0.117473	0.119243	0.121013
7	0.122785	0.124557	0.126329	0.128103	0.129877	0.131652	0.133428	0.135205	0.136983	0.138761
8	0.140541	0.142321	0.144102	0.145884	0.147667	0.149451	0.151236	0.153022	0.154808	0.156596
9	0.158384	0.160174	0.161965	0.163756	0.165549	0.167343	0.169137	0.170933	0.17273	0.174528
x	0	1	2	3	4	5	6	7	8	9
10	0.176327	0.178127	0.179928	0.181731	0.183534	0.185339	0.187145	0.188952	0.19076	0.19257
11	0.19438	0.196192	0.198005	0.19982	0.201635	0.203452	0.205271	0.20709	0.208911	0.210733
12	0.212557	0.214381	0.216208	0.218035	0.219864	0.221695	0.223526	0.22536	0.227194	0.229031
13	0.230868	0.232707	0.234548	0.23639	0.238234	0.240079	0.241925	0.243774	0.245624	0.247475
14	0.249328	0.251183	0.253039	0.254897	0.256756	0.258618	0.26048	0.262345	0.264211	0.266079
15	0.267949	0.269821	0.271694	0.273569	0.275446	0.277325	0.279205	0.281087	0.282971	0.284857
16	0.286745	0.288635	0.290527	0.29242	0.294316	0.296213	0.298113	0.300014	0.301918	0.303823
17	0.305731	0.30764	0.309552	0.311465	0.313381	0.315299	0.317219	0.319141	0.321065	0.322991
18	0.32492	0.32685	0.328783	0.330718	0.332656	0.334595	0.336537	0.338481	0.340428	0.342377
19	0.344328	0.346281	0.348237	0.350195	0.352156	0.354119	0.356084	0.358052	0.360022	0.361995
x	0	1	2	3	4	5	6	7	8	9
20	0.36397	0.365948	0.367928	0.369911	0.371897	0.373885	0.375875	0.377869	0.379864	0.381863
21	0.383864	0.385868	0.387874	0.389884	0.391896	0.39391	0.395928	0.397948	0.399971	0.401997
22	0.404026	0.406058	0.408092	0.41013	0.41217	0.414214	0.41626	0.418309	0.420361	0.422417
23	0.424475	0.426536	0.428601	0.430668	0.432739	0.434812	0.436889	0.438969	0.441053	0.443139
24	0.445229	0.447322	0.449418	0.451517	0.45362	0.455726	0.457836	0.459949	0.462065	0.464185
25	0.466308	0.468434	0.470564	0.472698	0.474835	0.476976	0.47912	0.481267	0.483419	0.485574
26	0.487733	0.489895	0.492061	0.494231	0.496404	0.498582	0.500763	0.502948	0.505136	0.507329
27	0.509525	0.511726	0.51393	0.516138	0.518351	0.520567	0.522787	0.525012	0.52724	0.529473
28	0.531709	0.53395	0.536195	0.538445	0.540698	0.542956	0.545218	0.547484	0.549755	0.55203
29	0.554309	0.556593	0.558881	0.561174	0.563471	0.565773	0.568079	0.57039	0.572705	0.575026
x	0	1	2	3	4	5	6	7	8	9
30	0.57735	0.57968	0.582014	0.584353	0.586697	0.589045	0.591398	0.593757	0.59612	0.598488
31	0.600861	0.603239	0.605622	0.60801	0.610403	0.612801	0.615204	0.617613	0.620026	0.622445
32	0.624869	0.627299	0.629734	0.632174	0.634619	0.63707	0.639527	0.641989	0.644456	0.646929
33	0.649408	0.651892	0.654382	0.656877	0.659379	0.661886	0.664398	0.666917	0.669442	0.671972
34	0.674509	0.677051	0.679599	0.682154	0.684714	0.687281	0.689854	0.692433	0.695018	0.69761
35	0.700208	0.702812	0.705422	0.708039	0.710663	0.713293	0.71593	0.718573	0.721223	0.723879
36	0.726543	0.729213	0.731889	0.734573	0.737264	0.739961	0.742666	0.745377	0.748096	0.750821
37	0.753554	0.756294	0.759041	0.761796	0.764558	0.767327	0.770104	0.772888	0.77568	0.778479
38	0.781286	0.7841	0.786922	0.789752	0.79259	0.795436	0.79829	0.801151	0.804021	0.806898
39	0.809784	0.812678	0.81558	0.818491	0.821409	0.824336	0.827272	0.830216	0.833169	0.83613
x	0	1	2	3	4	5	6	7	8	9
40	0.8391	0.842078	0.845066	0.848062	0.851067	0.854081	0.857104	0.860136	0.863177	0.866227
41	0.869287	0.872356	0.875434	0.878521	0.881619	0.884725	0.887842	0.890967	0.894103	0.897249
42	0.900404	0.903569	0.906745	0.90993	0.913125	0.916331	0.919547	0.922773	0.92601	0.929257
43	0.932515	0.935783	0.939063	0.942352	0.945653	0.948965	0.952287	0.955621	0.958966	0.962322
44	0.965689	0.969067	0.972458	0.975859	0.979272	0.982697	0.986134	0.989582	0.993043	0.996515
45	1	1.0035	1.00701	1.01053	1.01406	1.01761	1.02117	1.02474	1.02832	1.03192

Glossary.indd 339 14-02-2020 20:15:32

(NATURAL TANGENTS)

46	1.03553	1.03915	1.04279	1.04644	1.0501	1.05378	1.05747	1.06117	1.06489	1.06862
47	1.07237	1.07613	1.0799	1.08369	1.08749	1.09131	1.09514	1.09899	1.10285	1.10672
48	1.11061	1.11452	1.11844	1.12238	1.12633	1.13029	1.13428	1.13828	1.14229	1.14632
49	1.15037	1.15443	1.15851	1.16261	1.16672	1.17085	1.175	1.17916	1.18334	1.18754
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
50	1.19175	1.19599	1.20024	1.20451	1.20879	1.2131	1.21742	1.22176	1.22612	1.2305
51	1.2349	1.23931	1.24375	1.2482	1.25268	1.25717	1.26169	1.26622	1.27077	1.27535
52	1.27994	1.28456	1.28919	1.29385	1.29853	1.30323	1.30795	1.31269	1.31745	1.32224
53	1.32704	1.33187	1.33673	1.3416	1.3465	1.35142	1.35637	1.36134	1.36633	1.37134
54	1.37638	1.38145	1.38653	1.39165	1.39679	1.40195	1.40714	1.41235	1.41759	1.42286
55	1.42815	1.43347	1.43881	1.44418	1.44958	1.45501	1.46046	1.46595	1.47146	1.47699
56	1.48256	1.48816	1.49378	1.49944	1.50512	1.51084	1.51658	1.52235	1.52816	1.534
57	1.53986	1.54576	1.5517	1.55766	1.56366	1.56969	1.57575	1.58184	1.58797	1.59414
58	1.60033	1.60657	1.61283	1.61914	1.62548	1.63185	1.63826	1.64471	1.6512	1.65772
59	1.66428	1.67088	1.67752	1.68419	1.69091	1.69766	1.70446	1.71129	1.71817	1.72509
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
60	1.73205	1.73905	1.7461	1.75319	1.76032	1.76749	1.77471	1.78198	1.78929	1.79665
61	1.80405	1.8115	1.81899	1.82654	1.83413	1.84177	1.84946	1.8572	1.86499	1.87283
62	1.88073	1.88867	1.89667	1.90472	1.91282	1.92098	1.9292	1.93746	1.94579	1.95417
63	1.96261	1.97111	1.97966	1.98828	1.99695	2.00569	2.01449	2.02335	2.03227	2.04125
64	2.0503	2.05942	2.0686	2.07785	2.08716	2.09654	2.106	2.11552	2.12511	2.13477
65	2.14451	2.15432	2.1642	2.17416	2.18419	2.1943	2.20449	2.21475	2.2251	2.23553
66	2.24604	2.25663	2.2673	2.27806	2.28891	2.29984	2.31086	2.32197	2.33317	2.34447
67	2.35585	2.36733	2.37891	2.39058	2.40235	2.41421	2.42618	2.43825	2.45043	2.4627
68	2.47509	2.48758	2.50018	2.51289	2.52571	2.53865	2.5517	2.56487	2.57815	2.59156
69	2.60509	2.61874	2.63252	2.64642	2.66046	2.67462	2.68892	2.70335	2.71792	2.73263
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
70	2.74748	2.76247	2.77761	2.79289	2.80833	2.82391	2.83965	2.85555	2.87161	2.88783
71	2.90421	2.92076	2.93748	2.95437	2.97144	2.98868	3.00611	3.02372	3.04152	3.0595
72	3.07768	3.09606	3.11464	3.13341	3.1524	3.17159	3.191	3.21063	3.23048	3.25055
73	3.27085	3.29139	3.31216	3.33317	3.35443	3.37594	3.39771	3.41973	3.44202	3.46458
74	3.48741	3.51053	3.53393	3.55761	3.5816	3.60588	3.63048	3.65538	3.68061	3.70616
75	3.73205	3.75828	3.78485	3.81177	3.83906	3.86671	3.89474	3.92316	3.95196	3.98117
76	4.01078	4.04081	4.07127	4.10216	4.1335	4.1653	4.19756	4.2303	4.26352	4.29724
77	4.33148	4.36623	4.40152	4.43735	4.47374	4.51071	4.54826	4.58641	4.62518	4.66458
78	4.70463	4.74534	4.78673	4.82882	4.87162	4.91516	4.95945	5.00451	5.05037	5.09704
79	5.14455	5.19293	5.24218	5.29235	5.34345	5.39552	5.44857	5.50264	5.55777	5.61397
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
80	5.67128	5.72974	5.78938	5.85024	5.91236	5.97576	6.04051	6.10664	6.17419	6.24321
81	6.31375	6.38587	6.45961	6.53503	6.61219	6.69116	6.77199	6.85475	6.93952	7.02637
82	7.11537	7.20661	7.30018	7.39616	7.49465	7.59575	7.69957	7.80622	7.91582	8.02848
83	8.14435	8.26355	8.38625	8.51259	8.64275	8.77689	8.9152	9.05789	9.20516	9.35724
84	9.51436	9.6768	9.84482	10.0187	10.1988	10.3854	10.5789	10.7797	10.9882	11.2048
85	11.4301	11.6645	11.9087	12.1632	12.4288	12.7062	12.9962	13.2996	13.6174	13.9507
86	14.3007	14.6685	15.0557	15.4638	15.8945	16.3499	16.8319	17.3432	17.8863	18.4645
87	19.0811	19.7403	20.4465	21.2049	22.0217	22.9038	23.8593	24.8978	26.0307	27.2715
88	28.6363	30.1446	31.8205	33.6935	35.8006	38.1885	40.9174	44.0661	47.7395	52.0807
89	57.29	63.6567	71.6151	81.847	95.4895	114.589	143.237	190.984	286.478	572.957
90	∞									

Glossary.indd 340 14-02-2020 20:15:32

Higher Secondary Second Year Physics volume 1

List of Subject Experts in Textbook Writing

Domain Expert & Mentor

Prof. Rita John
Professor and Head
Department of Theoretical physics,
University of madras, Chennai.

Reviewers

Dr. V. Mani
Principal Scientist F, Head (C-MET)
Department of Electronics &
Information Technology, Hyderabad,
Government of India.

Prof. P. Ravindran
Department of Physics,
School of Basic and Applied Sciences,
Central University of Tamil Nadu,
Thiruvarur.

Dr. Rajeev shesha Joshi
Assistant Professor,
School of Physical Sciences,
Central University of Karnataka.

Content Writers

Mr. C. Joseph Prabagar
Assistant Professor,
PG and Research Department of Physics,
Loyola College (Autonomous),
Chennai – 34

Dr. S. S. Naina Mohammed
Assistant Professor,
PG and Research Department of Physics,
Government Arts College
Udumalpet, Tiruppur District.

Dr. P.Balamurugan
Assistant Professor,
PG and Research Department of Physics,
Government Arts College for Men
(Autonomous)
Nandanam, Chennai.

Dr.C. Ramachandra raja
Associate Professor of Physics
Government Arts College,
Kumbakonam.

Tamil Translators

Mr.E.Elangovan
Head Master,
Government Hr Sec School
Ramanayakanpettai,
Vellore District.

Dr.K.Vasudevan
PGT, Government ADW.Hr.Sec.School
Kalangani
Namakkal District.

Mr.V.Balamurugan
PG Assistant in Physics,
Government Hr.Sec.School,
Kadambattur,
Tiruvallur District

Mr. S. Ravishankar
PG Assistant in Physics,
S.R.M. Hr. Sec. School, Ambattur,
Tiruvallur District

Content Readers

Dr.K.Rajarajan
Associate Professor of Physics,
R.V.Government Arts College,
Chengalpattu

Dr.M.Jeyarani Malliga
Associate Professor of Physics,
Bharathi Women's College, Chennai.

Dr.K.Rajakumari
Assistant Professor of Physics,
Queen Mary's College,
Chennai.

Mrs. Bhavani
Assistant Professor of Physics,
Ethiraj College for women, Chennai.

QR CODE Management Team

R. Jaganathan
SGT, PUMS - Ganesapuram,
Polur , Thiruvannamalai.

S. Albert Valavan Babu
B.T, G.H.S, Perumal Kovil,
Paramakudi, Ramanathapuram.

A. Devi Jesintha
B.T, G.H.S, N.M. Kovil, Vellore.

SCERT Subject Coordinators

Mrs. P.Nantha
Senior Lecturer
SCERT, Chennai – 06.

Mrs.D.Shanmuga Sundari
Graduate Assistant
P.U.M.School, Melathulukkankulam
Kariyapatti Union, Virudhunagar
District

Mr.G.Arulraja
Graduate Assistant
MKV Government Boys Hr.Sec.
School
Arani, Thiruvallur District.

Mr. S. Ravishankar
PG Assistant in Physics,
S.R.M. Hr. Sec. School,
Ambattur,
Tiruvallur District

ICT Coordinator

Mr.G.Bergin
PG Assistant (Physics)
GBHSS, Sayalkudi,
Ramanathapuram District.

Typist

Mrs. D. Kavitha
SCERT

Art and Design Team

Layout Design
M. Asker Ali, B. Yogesh, Sridhar Velu

In-House QC
Rajesh Thangappan

Illustration
K. Sasi Kumar
S. Durga Devi

Cover Design
Kathir Arumugam

SCERT Co-ordination
Ramesh Munisamy

This book has been printed on 80 G.S.M.
Elegant Maplitho paper.
Printed by offset at:

PH XII Std Acknowledgement.indd 341 14-02-2020 20:16:48

FREE Ebooks

Download Now

Current Affairs

Explore Now

Free Notes + Mock Tests + Mindmaps + Current Affairs

For TNPSC Group II/IIA, IV & VAO Exams

Web APP

BLOG	FORUM
Your one-stop destination for all exam related information & preparation resources.	Interact with peers & experts, exchange scores & improve your preparation.
Explore Now	Explore Now

www.OliveBoard.in