

6. MATHEMATICS

PAPER – I

UNIT – I ALGEBRA AND NUMBER THEORY

Group Theory: Groups, Subgroups, Normal Subgroups and Quotient Groups, Homomorphisms and applications, Permutation groups, Conjugacy and Class equation, Simple group, Sylow Theorems.

Ring Theory: Rings, Special Classes of rings, Homomorphisms, Ideals and Quotient rings, Maximal and Prime ideals, Polynomial rings, Principal Ideal Domain, Unique Factorization Domain.

Field : Field of Quotients of an Integral Domain, Polynomials over the rational field, Algebraic Extension of Fields: Irreducible polynomials and Eisenstein Criterion, roots of Polynomial, Splitting field and its degree of extension, Multiple roots, Ruler and Compass Constructions, Symmetric function of roots, Solution of Cubic and Biquadratic Equations.

Number Theory: Integers, g.c.d., Fundamental Theorem of Arithmetic, Euclidean Algorithm, Arithmetical functions (Euler-function, Möbius function-), Dirichlet multiplication, Linear Congruences, Euler-Fermat Theorem, Linear Diophantine Equations, Fermat's Theorem, Fermat Little Theorem, Polynomial Congruence, Lagrange's Theorem, Chinese Remainder Theorem, Wilson's Theorem and Applications.

UNIT –II

ANALYSIS – I

Basic Topology: Finite, Countable and Uncountable sets, Metric Spaces, Topological Spaces, Basis, Closed sets, Open Sets, Limit Points, Properties of Connected Spaces and Compact Spaces, Heine Boril Theorem.

Sequence and Series : Convergent Sequences, Subsequences, Convergence of Monotone Sequences, Cauchy Sequences, Upper and Lower limits of Sequences, Bolzano Weirstrass Theorem, Series of non-negative terms, Convergence tests, Power Series, Cauchy Convergence Criterion, Absolute Convergence, Alternating Series.

Continuity and Differentiability: Properties of Continuous Function, Continuity and Compactness, Continuity and Connectedness, Discontinuity, Monotonic functions, Mean Value Theorem, Taylor Series.

Function of Several Variables: Continuity Differentiability, Extreme Values, Maxima and Minima, Line Integral, Surface Integral, Volume Integral, Applications of Green's Theorem, Stokes Theorem and Gauss Theorem.

UNIT – III COMPLEX ANALYSIS

Analytical Functions: Continuity, Differentiability, Cauchy-Reimann Equations, Analytic Functions, Harmonic Functions.

Bilinear Transformation: Elementary Transformations, Bilinear Transformation, Mapping by Elementary Functions.

Complex Integration: Cauchy- Goursat Theorem, Cauchy Integral Formula, Maximum Modulus Theorem, Liouville's Theorem, Morera's Theorem, Related Problems.

Singularities and Calculus of residues : Series Expansion, Taylor's Series, Laurent's Series, Zeros of Analytic Function, Singularities, Residues, Cauchy's Residue Theorem, Evaluation of Definite Integrals.

UNIT – IV OPERATIONS RESEARCH

Linear Programming: Simplex Method, Computational Procedure, Use of Artificial Variables.

Duality in Linear Programming: General Primal-dual pair, Duality Theorems, Complementary Slackness Theorem, Duality and Simplex Method, Dual Simplex Method.

Games and Strategies : Two-person-Zero Sum Games, Minimax-Maximin Principle, Games with Saddle Points, Mixed Strategies, Graphical Solutions, Dominance Property, Arithmetic Method of $n \times n$ Games, General Solution of $n \times n$ rectangular Games.

Transportation and Assignment: General Transportation Problem, Finding Initial Basic Feasible Solution, Test of Optimality, Transportation Algorithm, Transshipment Problems.

Mathematical Formulation of Assignment Problem, Method of Solution of Assignment Problem, Travelling Salesman Problem.

UNIT – V NUMERICAL ANALYSIS

Root Finding for Non-Linear Equations : Newton's Method, Secant Method, One-point Iteration Method, Multiple Roots, Newton Methods of Non-Linear Systems.

Interpolation Theory : Finite Differences, Newton's Forward and Backward differences, Newton's Divided differences, Lagrange's Interpolation, Errors in data and Forward differences, Hermite Interpolation, Piece-wise linear Interpolation.

Numerical Integration: Newton-cote integration formula, trapezoidal rule, Simpsons' rule, Gaussian quadrature, Asymptotic error formulas and their applications.

Numerical Methods for Ordinary Differential Equations : Euler's Method, Multistep Methods, Midpoint Method, Trapezoidal Method, Single Step Method and Runge-Kutta Method.

82 PAPER-II

UNIT-I ANALYSIS

Riemann stieltjes integral Existence of the integral, Properties of the integral, Fundamental theorem of calculus, change of variables in on integral, Differentiation of integral.

Sequence and series of functions

Uniform convergence of sequence of functions, Cauchy criterion for uniform convergence, weierstrass test for uniform convergence, uniform convergence and continuity, uniform convergence and differentiation, construction of continuous function on the real line which is nowhere differentiable.

Measure Theory Lebesgue outer measure , Properties of outer measure, Measurable sets, Cantor set, Borel set, and sets, Non measurable sets, Measurable functions, Properties of measurable functions.

Lebesgue integration and L_p spaces comparison of Lebesgue and Riemann integral, Lebesgue integral of bounded measurable functions over sets of finite measure, Bounded convergence theorem, Lebesgue integral for nonnegative measurable function. Fatou's Lemma, Monotone convergence theorem, L_p spaces, essential supremum of a function, Minkowski and Holder inequalities, Absolute summable and summable series in a normal linear space completeness in L_p .

UNIT-II FUNCTIONAL ANALYSIS

Normed Linear space Linear spaces, Subspaces, Quotient spaces, properties of norm, Riesz Lemma, Continuity of linear maps, Bounded linear operations, Equivalent norms, Hahn Banach theorem and its consequences.

Banach spaces Uniform boundedness principle, closed graph theorem and its consequences, open mapping theorem and its consequences.

Spaces of Bounded linear functional Duals and transposes, Duals of L_p , $L_p[a,b]$, $C[a,b]$, Weak convergence, weak* convergence, Reflexivity.

Hilbert space Inner product spaces, Orthonormal sets, Gram Schmidt Orthonormalisation, Bessel's Inequality, Riesz Fischer theorem, Projection theorem, Riesz representation theorem.

UNIT-III LINEAR ALGEBRA

Vectorspace, Subspace, Linear Dependence , Independence, Dimension and Basis , Linear Transformation, Range and Kernel, Rank and Nullity, Inverse of Linear Transformation, Linear Map associated with matrix.

Elementary Row Operations, Rank and Nullity of Matrix, Inverse of a Matrix, Determinants and product of Determinants, Eigen values, Eigen vectors, Characteristic roots.

Canonical forms, Triangular form, Nilpotent Transformations, Similarity of Matrices, Quadratic form.

Traces and Transpose, Hermitian, Unitary and Normal Transformation.

UNIT- IV DISCRETE MATHEMATICS

Logic- Fundamentals of logic, Normal forms, Logical Inferences, Methods of proof, Mathematical Induction, Rules of Inferences for quantified propositions.

Lattice and Boolean Algebra – Binary relations, Equivalence relations, poset, Lattice, Hasse Diagram, Algebraic properties of Lattice, Paths and closures, Directed graphs and adjacency matrix, Boolean Algebra, Boolean functions, Minimization of Boolean functions.

Recurrence relation –Generating functions of sequences, Calculating coefficients of generating functions, Recurrence relation, solving recurrence relations by substitution and generating functions. Solution by the method of characteristic roots.

Graph Theory – Trees and their properties, spanning trees, Binary trees, Euler's formula, Euler's circuits, Hamiltonian Graphs

UNIT – V DIFFERENTIAL EQUATIONS

Linear Differential Equations with constant coefficients and variable coefficients, system of Linear Differential Equations. Laplace Transformation : Linearity of the Laplace transformation. Laplace transforms of derivatives and integrals, shifting theorems. Differentiation and integration of transforms. Convolution theorem. Solution of integral equations and systems of differential equations using Laplace Transformation.

Series Solution of differential equations: Power series method, Bessel, Legendre and Hypergeometric equations. Bessel, Legendre functions and their properties . Sturm Liouville problem, Orthogonality of eigen functions. Orthogonality of Bessel functions and Legendre polynomials.

Partial Differential Equations of the 1st order. Lagrange's solution some special types of equations, their solution, Charpit's general method of solution. Partial Differential Equations of second and Higher orders. Classification of linear partial differential equations of second order. Homogeneous and non-homogeneous equations with constant coefficients, Monge's method.

Fourier Series and Fourier Transform, Convergence of Fourier series, Application of Fourier series and Fourier Transforms to Boundary value problems. Solution of Laplace equation, wave equation and heat conduction equations.